DeMana 3.6

Find the derivative:

1. $y = \sin(3x+1)$ 3. $y = \cos(\sqrt{3}\,x)$ 5. $y = 5\cot\left(\frac{2}{x}\right)$

7. $y = \cos(\sin x)$ 9. $y = (x+\sqrt{x})^{-2}$ 11. $y = \sin^{-5}x - \cos^3 x$

13. $y = \sin^3 x \tan 4x$ 15. $y = \frac{3}{\sqrt{2x+1}}$ 17. $y = \sin^2(3x-2)$ 19. $y = (1+\cos^2 7x)^3$

21. $S = \cos\left(\frac{\pi}{2} - 3t\right)$ 23. $S = \frac{4}{3\pi}\sin 3t + \frac{4}{5\pi}\cos 5t$ 25. $r = \tan(2-\theta)$

27. $r = \sqrt{\theta \sin\theta}$ 29. Find y'': $y = \tan x$ 31. Find y'': $y = \cot(3x-1)$

56. Suppose f and g and their derivatives have the following values at $x=2$ and $x=3$

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	8	2	$1/3$	-3
3	3	-4	2π	5

*Evaluate the derivatives with respect to x of the following

a) $2f(x)$ at $x=2$ b) $f(x)+g(x)$ at $x=3$ c) $f(x)g(x)$ at $x=3$

d) $\frac{f(x)}{g(x)}$ at $x=2$ e) $f(g(x))$ at $x=2$ f) $\sqrt{f(x)}$ at $x=2$

g) $\frac{1}{g^2(x)}$ at $x=3$ h) $\sqrt{f^2(x)+g^2(x)}$ at $x=2$

57.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	1	1	5	$1/3$
1	3	-4	$-1/3$	$-8/3$

a) $5f(x) - g(x)$, $x=1$ b) $f(x)g^3(x)$, $x=0$ c) $\frac{f(x)}{g(x)+1}$, $x=1$

d) $f(g(x))$, $x=0$ e) $g(f(x))$, $x=0$ f) $(g(x)+f(x))^{-2}$, $x=1$

g) $f(x+g(x))$, $x=0$

3.6 Chain Rule (pp. 141–149)

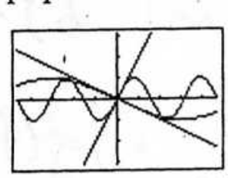
#1-31 odd, 56, 57, 62

Quick Review 3.6

1. $\sin(x^2+1)$
2. $\sin(49x^2+1)$
3. $49x^2+1$
4. $7x^2+7$
5. $\sin\dfrac{x^2+1}{7x}$
6. $g(f(x))$
7. $g(h(f(x)))$
8. $h(g(f(x)))$
9. $f(h(h(x)))$
10. $f(g(h(x)))$

Section 3.6 Exercises

1. $3\cos(3x+1)$
2. $-5\cos(7-5x)$
3. $-\sqrt{3}\sin(\sqrt{3}x)$
4. $(2-3x^2)\sec^2(2x-x^3)$
5. $\dfrac{10}{x^2}\csc^2\left(\dfrac{2}{x}\right)$
6. $\dfrac{2\sin x}{(1+\cos x)^2}$
7. $-\sin(\sin x)\cos x$
8. $\sec(\tan x)\tan(\tan x)\sec^2 x$
9. $-2(x+\sqrt{x})^{-3}\left(1+\dfrac{1}{2\sqrt{x}}\right)$
10. $\dfrac{\csc x}{\csc x+\cot x}$
11. $-5\sin^{-6}x\cos x+3\cos^2 x\sin x$
12. $8x^3(2x-5)^3+3x^2(2x-5)^4$ $=x^2(2x-5)^3(14x-15)$
13. $4\sin^3 x\sec^2 4x+3\sin^2 x\cos x\tan 4x$
14. $2\sec x\sqrt{\sec x+\tan x}$
15. $-3(2x+1)^{-3/2}$
16. $(1+x^2)^{-3/2}$
17. $6\sin(3x-2)\cos(3x-2)=3\sin(6x-4)$
18. $-4(1+\cos 2x)\sin 2x$
19. $-42(1+\cos^2 7x)^2\cos 7x\sin 7x$
20. $\dfrac{5}{2}(\tan 5x)^{-1/2}\sec^2 5x$
21. $3\sin\left(\dfrac{\pi}{2}-3t\right)$
22. $4t\sin(\pi-4t)+\cos(\pi-4t)$
23. $\dfrac{4}{\pi}\cos 3t-\dfrac{4}{\pi}\sin 5t$
24. $\dfrac{3\pi}{2}\cos\dfrac{3\pi t}{2}-\dfrac{7\pi}{4}\sin\dfrac{7\pi t}{4}$
25. $-\sec^2(2-\theta)$
26. $2\sec^3 2\theta+2\sec 2\theta\tan^2 2\theta$
27. $\dfrac{\theta\cos\theta+\sin\theta}{2\sqrt{\theta\sin\theta}}$
28. $\sqrt{\sec\theta}\,(\theta\tan\theta+2)$
29. $2\sec^2 x\tan x$
30. $2\csc^2 x\cot x$
31. $18\csc^2(3x-1)\cot(3x-1)$
32. $2\sec^2\dfrac{x}{3}\tan\dfrac{x}{3}$
33. $\dfrac{5}{2}$
34. 1
35. $-\dfrac{\pi}{4}$
36. 5π
37. 0
38. -8
39. (a) $-6\sin(6x+2)$ (b) $-6\sin(6x+2)$
40. (a) $2x\cos(x^2+1)$ (b) $2x\cos(x^2+1)$
41. $y=-x+2\sqrt{2}$
42. $y=\sqrt{3}x+2$
43. $y=-\dfrac{1}{2}x-\dfrac{1}{2}$
44. $y=2x-\sqrt{3}$
45. $y=x+\dfrac{1}{4}$
46. $y=x-4$
47. $y=\sqrt{3}x+2-\dfrac{\pi}{\sqrt{3}}$
48. $y=2$
49. (a) $\dfrac{\cos t}{2t+1}$
    (b) $\dfrac{d}{dt}\left(\dfrac{dy}{dx}\right)=-\dfrac{(2t+1)(\sin t)+2\cos t}{(2t+1)^2}$
    (c) $\dfrac{d}{dx}\left(\dfrac{dy}{dx}\right)=-\dfrac{(2t+1)(\sin t)+2\cos t}{(2t+1)^3}$
    (d) part (c)
50. Since the radius goes through $(0, 0)$ and $(2\cos t, 2\sin t)$, it has slope given by $\tan t$. But $\dfrac{dy}{dx}=\dfrac{dy/dt}{dx/dt}=-\dfrac{\cos t}{\sin t}=-\cot t$, which is the negative reciprocal of $\tan t$. This means that the radius and the tangent are perpendicular.
51. 5
52. 3
53. $\dfrac{1}{2}$
54. $y=mx$
55. Tangent: $y=\pi x-\pi+2$;
    Normal: $y=-\dfrac{1}{\pi}x+\dfrac{1}{\pi}+2$
56. (a) $\dfrac{2}{3}$ (b) $2\pi+5$
    (c) $15-8\pi$ (d) $\dfrac{37}{6}$
    (e) -1 (f) $\dfrac{1}{12\sqrt{2}}$
    (g) $\dfrac{5}{32}$ (h) $-\dfrac{5}{3\sqrt{17}}$
57. (a) 1 (b) 6
    (c) 1 (d) $-\dfrac{1}{9}$
    (e) $-\dfrac{40}{3}$ (f) -6
    (g) $-\dfrac{4}{9}$
58. The slope of $y=\sin(2x)$ at the origin is 2. The slope of $y=-\sin\dfrac{x}{2}$ at the origin is $-\dfrac{1}{2}$. So the lines tangent to the two curves at the origin are perpendicular.



$[-4.7, 4.7]$ by $[-3.1, 3.1]$