## PROBLEM SET 5.2

In Problems 1–4, show that the indicated function is a solution of the given differential equation; that is, substitute the indicated function for $y$ to see that it produces an equality.

1. $\dfrac{dy}{dx} + \dfrac{x}{y} = 0;\ y = \sqrt{4 - x^2}$

2. $3y^2 \dfrac{dy}{dx} + x = 0;\ y = \left(1 - \dfrac{x^2}{2}\right)^{1/3}$

3. $\dfrac{d^2y}{dx^2} + y = 0;\ y = C_1 \sin x + C_2 \cos x$

4. $\left(\dfrac{dy}{dx}\right)^2 = 1 - y^2;\ y = \sin(x + C)$

ODDS

In Problems 5–14, first find the general solution (involving a constant $C$) for the given differential equation. Then find the particular solution that satisfies the indicated condition. (See Example 2.)

5. $\dfrac{dy}{dx} = 3x^2 + 1;\ y = 4$ at $x = 1$

6. $\dfrac{dy}{dx} = x^{-2} + 2x;\ y = 5$ at $x = 1$

7. $\dfrac{dy}{dx} = \dfrac{x}{2y};\ y = 3$ at $x = 2$

8. $\dfrac{dy}{dx} = \sqrt[3]{\dfrac{x}{y}};\ y = 8$ at $x = 1$

9. $\dfrac{dy}{dt} = t^3y^2;\ y = 1$ at $t = 2$

J. $\dfrac{dy}{dt} = y^3;\ y = 1$ at $t = 1$

11. $\dfrac{ds}{dt} = 3t^2 + 4t - 1;\ s = 5$ at $t = 2$

12. $\dfrac{du}{dt} = u^2(t^2 - 3t);\ u = 4$ at $t = 0$

13. $\dfrac{dy}{dx} = (2x + 1)^4;\ y = 6$ at $x = 0$

14. $\dfrac{dy}{dx} = -y^2x(x^2 + 2)^4;\ y = 1$ at $x = 0$

ODDS

15. Find the $xy$-equation of the curve through $(1, 2)$ whose slope at any point is four times its $x$-coordinate (see Example 1).

16. Find the $xy$-equation of the curve through $(1, 2)$ whose slope at any point is one-half the square of its $y$-coordinate.

ALL

19 $\displaystyle\int \frac{3 \sin x}{1 + 2 \cos x}\, dx$

20 $\displaystyle\int \frac{\sec^2 x}{1 + \tan x}\, dx$

21 $\displaystyle\int \frac{(e^x + 1)^2}{e^x}\, dx$

22 $\displaystyle\int \frac{e^x}{(e^x + 1)^2}\, dx$

2. $\dfrac{-e^{-x}}{+e^{-x}}\, dx$

24 $\displaystyle\int \frac{e^x}{e^x + 1}\, dx$

25 $\displaystyle\int \frac{\cot \sqrt[3]{x}}{\sqrt[3]{x^2}}\, dx$

26 $\displaystyle\int e^x(1 + \tan e^x)\, dx$

27 $\displaystyle\int \frac{1}{\cos 2x}\, dx$

28 $\displaystyle\int (x + \csc 8x)\, dx$

1 The number of bacteria in a culture increases from 5000 to 15,000 in 10 hours. Assuming that the rate of increase is proportional to the number of bacteria present, find a formula for the number of bacteria in the culture at any time $t$. Estimate the number at the end of 20 hours. When will the number be 50,000?

2 The polonium isotope $^{210}Po$ has a half-life of approximately 140 days. If a sample weighs 20 milligrams initially, how much remains after $t$ days? Approximately how much will be left after two weeks?

3 If the temperature is constant, then the rate of change of barometric pressure $p$ with respect to altitude $h$ is proportional to $p$. If $p = 30$ in. at sea level and $p = 29$ in. when $h = 1000$ ft, find the pressure at an altitude of 5000 feet.

4 The population of a city is increasing at the rate of 5% per year. If the present population is 500,000 and the rate of increase is proportional to the number of people, what will the population be in 10 years?

5 Agronomists use the assumption that a quarter acre of land is required to provide food for one person and estimate that there are 10 billion acres of tillable land in the world. Hence a maximum population of 40 billion people can be sustained if no other food source is available. The world population at the beginning of 1980 was approximately 4.5 billion. Assuming that the population increases at a rate of 2% per year and the rate of increase is proportional to the number of people, when will the maximum population be reached?

6 A metal plate that has been heated cools from 180 °F to 150 °F in 20 minutes when surrounded by air at a temperature of 60 °F. Use Newton's law of cooling (see Example 3) to approximate its temperature at the end of one hour of cooling. When will the temperature be 100 °F?

7 An outdoor thermometer registers a temperature of 40 °F. Five minutes after it is brought into a room where the temperature is 70 °F, the thermometer registers 60 °F. When will it register 65 °F?

8 The rate at which salt dissolves in water is directly proportional to the amount that remains undissolved. If 10 pounds of salt are placed in a container of water and 4 pounds dissolve in 20 minutes, how long will it take for two more pounds to dissolve?

9 According to Kirchhoff's first law for electrical circuits, $V = RI + L(dI/dt)$, where the constants $V$, $R$, and $L$ denote the electromotive force, the resistance, and the inductance, respectively, and $I$ denotes the current at time $t$. If the electromotive force is terminated at time $t = 0$ and if the current is $I_0$ at the instant of removal, prove that $I = I_0e^{-Rt/L}$.

29 $\displaystyle\int \frac{\tan e^{-3x}}{e^{3x}}\, dx$

30 $\displaystyle\int e^{\cos x} \sin x\, dx$

31 $\displaystyle\int \frac{\cos^2 x}{\sin x}\, dx$

32 $\displaystyle\int \frac{\tan^2 2x}{\sec 2x}\, dx$

33 $\displaystyle\int \frac{\cos x \sin x}{\cos^2 x - 1}\, dx$

34 $\displaystyle\int (\tan 3x + \sec 3x)\, dx$

35 $\displaystyle\int (1 + \sec x)^2\, dx$

36 $\displaystyle\int \csc x\,(1 - \csc x)\, dx$

**ANS Practice Sheet 5.2 (Diff Eq)**

5. $y = x^3 + x + 2$  7. $V = \sqrt{\frac{1}{2}x^2 + 7}$  9. $y = \frac{-4}{t^4 - 20}$  11. $s = t^3 + 2t^2 - t - 9$

13. $y = 1/10((2x+1)^5+59)$  15. $y = 2x^2$

1. $y = 5000e^{(1/10)(\ln 3)t}$; $y(20) = 45000$; $t = 20.959$ hr  2. 18.661 mg  3. 25.322 in

4. $P(t) = 500000e^{.05t}$; 824,361 people  5. $P(t) = 4.5e^{0.02t}$; $t = 109.240$ yrs

6. $T = 120e^{(\frac{1}{20}\ln\frac{3}{4})t} + 60$; $110.625^0$F; 76.377 min  7. $T = -30e^{1/5\ln(1/3)t} + 70$; 8.155 min

8. 35.875 min total or 15.875 additional minutes

19. $\frac{-3}{2}\ln|1+2\cos x| + c$  20. $\ln|1+\tan x| + c$  21. $e^x + 2x - e^{-x} + c$  22. $-\frac{1}{e^x+1} + c$

23. $\ln|e^x + e^{-x}| + c$  24. $\ln|e^x + 1| + c$  25. $3\ln\left|\sin x^{\frac{1}{3}}\right| + c$  26. $e^x + \ln|\sec(e^x)| + c$

27. $\frac{1}{2}\ln|\sec 2x + \tan 2x| + c$  28. $\frac{1}{2}x^2 + \frac{1}{8}\ln|\csc 8x - \cot 8x| + c$  29. $\frac{-1}{3}\ln|\sec e^{-3x}| + c$

30. $-e^{\cos x} + c$  31. $\ln|\csc x - \cot x| + \cos x + c$  32. $\frac{1}{2}\ln|\sec 2x + \tan 2x| - \frac{1}{2}\sin 2x + c$

33. $-\ln|\sin x| + c$  34. $\frac{1}{3}\ln|\sec 3x| + \frac{1}{3}\ln|\sec 3x + \tan 3x| + c$  35. $x + 2\ln|\sec x + \tan x| + \tan x + c$

36. $\ln|\csc x - \cot x| + \cot x + c$