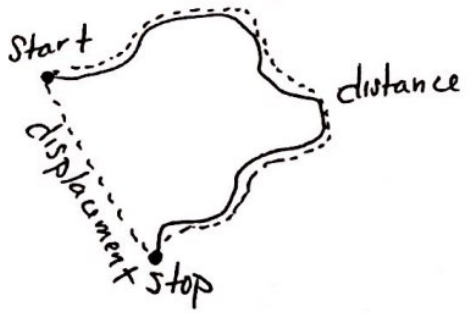
# Distance vs. Displacement

Displacement → how far you are from where you started




Distance ⇒ length of how far actually travelled ⇒ length of path

In AB Calc, moving linearly (back & forth in a line) so displacement is change in position ⇒ $s(b)-s(a)$ on $[a,b]$

How far actually travelled during the time doesn't matter (if go back & forth 50 times but end up where started, displacement is 0)

If want distance though, need to account for any time changing direction & add up separately. One way to do this is to look at speed instead of velocity ⇒ speed $= |v(t)|$

Using calculus, if displacement $= s(b)-s(a)$, then can do

$$\int_a^b v(t)\,dt = \text{displacement}$$

⇒ "signed" area under $v(t)$ graph if given graph.

So displacement can $=0$, be positive (if ending to right of where started) or be negative (if ending left of start).

$$\text{Distance} = \int_a^b |v(t)|\,dt$$

⇒ integral of speed ⇒ $|v(t)|$

Graphically if given $v(t)$, use all areas as positive & add.

Distance is always positive & usually greater than displacement only = displacement if $v(t) > 0$ on $[a,b]$.

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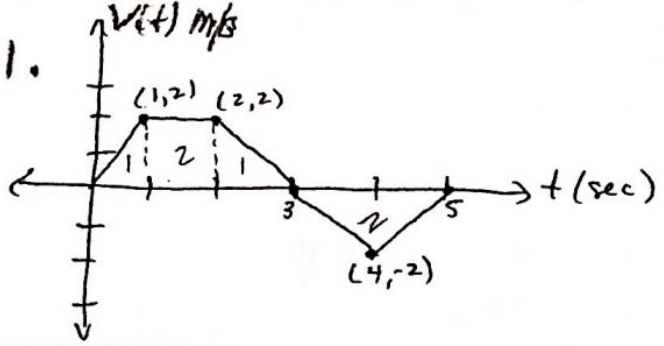
Ex 1.




a) When moving right? left?

b) Displacement $[0,5]$

c) Distance $[0,5]$

d) Find position at $t=5$ if initially at $x=3$.

Solutions

* a) Remember, right $v(t)>0$
left $v(t)<0$

Right: $[0,3)$ b/c $v(t)>0$

Left: $(3,5)$ b/c $v(t)<0$

b) $\int_0^5 v(t)\,dt \Rightarrow$ areas $= 1+2+1-2 = \boxed{2\text{ m}}$

c) $\int_0^5 |v(t)|\,dt = 1+2+1+2 = \boxed{6\text{m}}$

d) Given $s(0)=3$ need $s(5)$ so use FTC

$$\int_0^5 v(t)\,dt = s(5)-s(0)$$

$$2 = s(5)-3$$

$$\boxed{s(5)=5\text{m}}$$

Ex: A body moves with a velocity (m/s) given by $v(t)=\frac{1}{4}t^2-\frac{1}{2}t-2$ for $t\in[0,7]$

a) Find displacement $[0,7]$

* $\int_0^7 \left(\frac{1}{4}t^2-\frac{1}{2}t-2\right)dt = \boxed{7/3\text{ m}}$

b) Find distance travelled $[0,7]$.

* $\int_0^7 \left|\frac{1}{4}t^2-\frac{1}{2}t-2\right|dt \Rightarrow$ Can use calculator! MATH 9 fnInt & type in what integrate

$= \boxed{\frac{47}{3}\text{m}}$

c) Find average velocity on $[0,7]$

* $\frac{1}{7-0}\int_0^7 v(t)\,dt = \boxed{\frac{1}{3}\text{ m/s}}$

d) Find average speed on $[0,7]$

* $\frac{1}{7-0}\int_0^7 |v(t)|\,dt = \boxed{\frac{47}{21}\text{ m/s}}$

Remember avg value $\Rightarrow$

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

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# Distance / Displacement Practice

In Exercises 1–8, the function $v(t)$ is the velocity in m/sec of a particle moving along the $x$-axis. Use analytic methods to do each of the following:

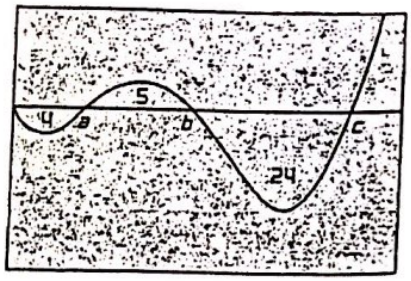
(a) Determine when the particle is moving to the right, to the left, and stopped.

(b) Find the particle's displacement for the given time interval.

(c) Find the total distance traveled by the particle.

1. $v(t) = 5\cos t, \quad 0 \le t \le 2\pi$
2. $v(t) = 6\sin 3t, \quad 0 \le t \le \pi/2$
3. $v(t) = 49 - 9.8t, \quad 0 \le t \le 10$
4. $v(t) = 6t^2 - 18t + 12, \quad 0 \le t \le 2$
5. $v(t) = 5\sin^2 t \cos t, \quad 0 \le t \le 2\pi$
6. $v(t) = \sqrt{4 - t}, \quad 0 \le t \le 4$
7. $v(t) = e^{\sin t}\cos t, \quad 0 \le t \le 2\pi$

In Exercises 12–16, a particle moves along the $x$-axis (units in cm). Its initial position at $t = 0$ sec is $x(0) = 15$. The figure shows the graph of the particle's velocity $v(t)$. The numbers are the *areas* of the enclosed regions.



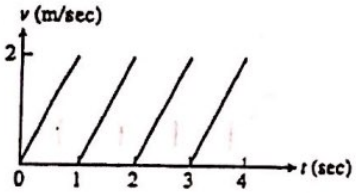

12. What is the particle's displacement between $t = 0$ and $t = c$?
13. What is the total distance traveled by the particle in the same time period?
14. Give the positions of the particle at times $a$, $b$, and $c$.
15. Approximately where does the particle achieve its greatest positive acceleration on the interval $[0, b]$?
16. Approximately where does the particle achieve its greatest positive acceleration on the interval $[0, c]$?

In Exercises 17–20, the graph of the velocity of a particle moving on the $x$-axis is given. The particle starts at $x = 2$ when $t = 0$.

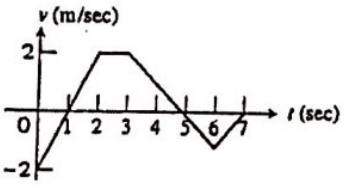
(a) Find where the particle is at the end of the trip.

(b) Find the total distance traveled by the particle.

17.




19.




1. The velocity $v$ of an object travelling on a straight line is given by $v = 2t - t^2$ m/sec, $0 \le t \le 4$. Find in the indicated time interval:

   **a.** The distance travelled by the object. **b.** The displacement of the object.

2. An object travelling on the $x$-axis has at time $t$ a velocity of $v(t) = \cos[(\pi/3)t]$ m/sec.
   a. What is its direction and speed at time $t = 0$ sec?
   b. What is its direction and speed at time $t = 3$ sec?
   c. Find the distance it has travelled from time $t = 0$ sec to time $t = 3$ sec.
   d. Where is it at time $t = 3$ sec relative to where it was at time $t = 0$ sec? (Hint : Find displacement.)

3. The velocity $v$ of an object travelling on a straight line is given by $v = |t - 5|$ m/sec, $0 \le t \le 8$. Find in the indicated time interval:

   **a.** The distance travelled by the object. **b.** The displacement of the object.

4. A body moves on the $x$-axis with acceleration $a(t) = d^2x/dt^2 = 6t$ m/sec$^2$. It starts at time $t = 0$ with initial velocity $v_0 = -3$ m/sec.
   a. Find the velocity $v(t)$ as a function of $t$.
   b. Find the total distance $s$ travelled by the body from time $t = 0$ sec to time $t = 4$ sec.
   c. Where is its position at time $t = 4$ sec relative to its position at time $t = 0$ sec?

5. An object moves on a straight line with velocity $v(t) = 2e^{-t}$ km/h for $t \ge 0$.
   a. Find the distance $s(t)$ the object has moved as a function of time $t$.
   b. How far does the object move throughout eternity?

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