**Notes and practice: Euler’s Method *pronounced (“Oilers”)***

Over the next few days, we will be seeing how to solve certain differential equations (you are given the equation of the derivative and want to find information about the original function). But many differential equations cannot actually be solved.

This is where Euler's Method is used. Euler's Method provides us with an ***approximation*** for the solution of a [differential equation](http://www.mathscoop.com/calculus/differential-equations/). The idea behind Euler's Method is to use the concept of [*local linearity*](http://www.mathscoop.com/calculus/derivatives/local-linearity.php) to join multiple small [line](http://www.mathwarehouse.com/algebra/linear_equation/) segments so that they make up an approximation of the actual curve, as seen below.

|  |  |
| --- | --- |
| **Example:**  The blue line shows the actual graph of a function.   The green curve is an approximation using Euler's method. It is the collection of line segments as a result of Euler's Method. Each time Eulers method is used another point is created and thus another line segment. | Euler Method Example |

**Here, the A0 represents the initial value – notice that it is the actual value since that is where the line is tangent. The A1 is the first approximation – gotten by using the equation of the tangent line from A0. The A2 is the 2nd approximation – you get this by writing an equation of the tangent line assuming that your A1 value is now a value on the curve.**

**Note:** -Generally, the approximation gets less accurate the further you are away from the initial value.   
  
-Better accuracy is achieved when the points in the approximation are closer together (so when the “step size” is as small as possible.

**AP Tip** - Your approximation is going to be higher than the actual value if the function is *concave down (remember, tangent lines would be ABOVE the curve)* and below the actual value if the function is *concave up (again, remember that tangent lines would be BELOW the curve).*

*In a nutshell, you are repeatedly doing tangent line approximations – using your previous answer to get a new one.*

Use your point-slope formula: y – y1=m(x - x1) where x and y are the “new” ones you are finding and x1 and y1 are the previous values you just found. Remember, m = slope, so use your given derivative equation with your previous values each time.

*Example:*

 and (1 ,0) is a point on the solutions curve. Approximate y(3) using 4 steps of equal size.

\*\* To get from x = 1 to x = 3 in 4 steps, you would have steps of length =.

Make a chart to keep track of your approximations!

|  |  |
| --- | --- |
| x | y |
| 1 | 0 |
| 1.5 | 0 |
| 2 | 0.5 |
| 2.5 | 1.5 |
| 3 | 3 |

**Step #1**: Approximate y(1.5) using your tangent line equation at x=1:

Slope at x = 1 . . . use given derivative equation: dy/dx = 2(1-1) = 0

y – 0= 0(1.5 – 1) . . . y(1.5)0

**Step #2**: Approximate y(2) using your tangent line equation at x = 1.5

Slope at x = 1.5 . . . use given derivative equation: dy/dx = 2(1.5-1) = 1

y – 0 = 1(2 – 1.5) . . . y(2) 0.5

**Step #3:** Approximate y(2.5) using your tangent line equation at x = 2

Slope at x=2 . . . use given derivative equation: dy/dx =2(2-1) = 2

y – 0.5 = 2(2.5 – 2) . . . y(2.5)  1.5

**Step #4:** Approximate y(3) using your tangent line equation at x = 2.5

Slope at x=2.5 . . . use given derivative equation: dy/dx = 2(2.5-1) = 3

y – 1.5 = 3(3 – 2.5) . . . y(3) 3

The question was asking for an approximation of y(3) . . . you are done **y(3) 3** !

You do . . .

1. Given  and (1, -1) on the curve, approximate y(1.3) using 3 steps.
2. Given  and (2,0) on the curve, approximate y(1.8) using 2 steps.
3. Given  and (2, 1) on the curve, approximate y(2.6) using 3 steps.

**Answers:**

1. **y(1.3) -0.129 2. y(1.8) -0.19 3. y(2.6) 2.92**