AFM : Exponential/Logarithmic Modeling Notes Name Key

I. Formulas for compounding interest. *(Refer to 6.2 page 389 – 393 for help)*

a) For "n" compoundings per year:	**Compounding Continuously**
$A = P(1+\frac{r}{n})^{nt}$	$A = Pe^{rt}$ or $N(t) = N_0 e^{kt}$

*Example:

1. A sum of $10,000 is invested at an annual rate of 8%. Find the balance in the account after 5 years if:
   a) Compounded quarterly. $10000(1+\frac{.08}{4})^{(4)(5)} = \$14859.47$

   b) Compounded continuously. $10000e^{.08(5)} = \$14918.25$

   c) Which is the better investment? How much MORE do you make with that investment?
   Continuously → $58.78

   d) How long will it take your money to double with these options?
   quarterly: $20000 = 10000(1+\frac{.08}{4})^{4t}$ $t = 8.75$ yrs
   Cont: $20000 = 10000e^{.08t}$ $t = 8.66$ yrs

II. Exponential Growth/Decay Models *(Refer to 6.6 page 421 – 425 for help)*

a) Growth	**b) Decay**
$A = Pe^{rt}$ $r =$ pos	$A = Pe^{rt}$ $r =$ neg.

*Example:

2. In 1990, the population of Africa was 643 million and by 2006 it had grown to 906 million.
   a) Find the exponential growth function that models the data for t years after 1990. (Hint: find the relative growth rate first)
   $906 = 643e^{r(16)}$
   $\frac{906}{643} = e^{16r}$
   $\ln\left(\frac{906}{643}\right) = 16r$
   $r = \frac{\ln\left(\frac{906}{643}\right)}{16} = .02143$
   $A = 643e^{.02143t}$

   b) By which year will Africa's population reach 2000 million, or two billion?
   $2000 = 643e^{.02143t}$
   $\ln\left(\frac{2000}{643}\right) = .02143t$
   $t = 52.95$ yrs
   2042

III. Half-life *(Refer to 6.6 page 424 for help)*

Similar to: exp decay $\frac{1}{2}P = Pe^{rt}$ $\frac{1}{2} = e^{rt}$ $\ln\frac{1}{2} = rt$ $r = \frac{\ln 1/2}{t}$

*Example:

3. Strontium-90 is a waste product from nuclear reactors. As a consequence of fallout from atmospheric nuclear tests, we all have a measurable amount of strontium-90 in our bones.
   a) The half-life of strontium-90 is 28 years, meaning that after 28 years a given amount of substance will have decayed to half the original amount. Find the exponential decay model for strontium 90. (Hint: find decay rate first)
   $r = \frac{\ln 1/2}{28} \Rightarrow r = -.024755$
   $A = Pe^{-.024755t}$

   b) Suppose that a nuclear accident occurs and releases 60 grams of strontium-90 into the atmosphere. How long will it take for strontium-90 to decay to a level of 10 grams?
   $10 = 60e^{-.024755t}$ $\frac{\ln\left(\frac{1}{6}\right)}{-.024755} = t = \boxed{72.38 \text{ yrs}}$

4. Polonium-210 has a half-life of 140 days. Suppose a sample of this substance has a starting mass of 300 mg.

a) Find the function that models the amount of the sample remaining at any time t days.

$r = \frac{\ln 1/2}{140}$

$A = 300e^{-.00495t}$

b) Find the mass remaining after one year.

$300e^{-.00495(365)} = \boxed{49.237mg}$

c) How long will it take for the sample to decay to 200mg?

$200 = 300e^{-.00495t}$ $\quad \ln(2/3) = -.00495t$

$\boxed{t = 81.89 \text{ days}}$

5. The half-life of a radioactive substance is 153 days. How many days will it take for 70% of the substance to decay?

$.7 = e^{-.00453t}$ $\quad r = \frac{\ln 1/2}{153} = -.00453$

$\ln .7 / -.00453 = t = \boxed{78.73 \text{ days}}$

IV. Logistic Growth Model

$y = \frac{m}{1+ae^{rt}}$
Limiting size (carrying capacity) = m

*Example:

6. In a learning theory project, psychologists discovered that $f(t) = \frac{0.8}{1+e^{-0.2t}}$ is a model for describing the proportion of correct responses, *f(t)*, after *t* learning trials.

a) Find the proportion of correct responses prior to learning trials taking place. $\rightarrow t = 0$

$\frac{.8}{1+e^0} = \frac{.8}{2} = \boxed{.4}$

b) Find the proportion of correct responses after 10 learning trials.

$\frac{.8}{1+e^{-0.2(10)}} = \boxed{.7}$

c) What is the limiting size (also called the "carrying capacity") of the proportion of correct responses, as continued learning trials take place? $\boxed{.8}$