3.5 – Solving Polynomials: Division Techniques (All Zeros)

– **real zero** → any type of zero that does NOT contain "i"

- **rational zero** – zeros that are whole #'s, terminating decimals, or repeating decimals which can be written as a fraction (I call these "pretty" zeros)
- **irrational zero** – zeros that are non-terminate decimals, non-repeat decimals, or contain squareroots

– **imaginary zero** → zeros that contain the imaginary unit of "i" where these zeros do NOT touch the x-axis

(aka – Rational)

Common Fractional Zeros

$0.5 \rightarrow \frac{1}{2}$	$1.5 \rightarrow \frac{3}{2}$
$0.\bar{3} \rightarrow \frac{1}{3}$	$0.\bar{6} \rightarrow \frac{2}{3}$
$0.25 \rightarrow \frac{1}{4}$	$0.75 \rightarrow \frac{3}{4}$
$1.\bar{3} \rightarrow \frac{4}{3}$	$1.\bar{6} \rightarrow \frac{5}{3}$
$2.5 \rightarrow \frac{5}{2}$	$3.5 \rightarrow \frac{7}{2}$

Steps to Find All Zeros of a Polynomial

1.) Put polynomial P(x) in Y1 = and have Y2 = 0, find ANY or ALL rational zero(s) (using intersection).

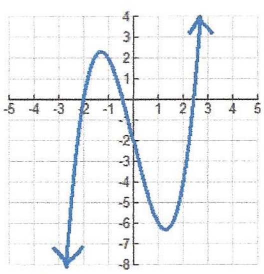
2.) Use synthetic division to divide any rational zero into the polynomial P(x) (remainder should = 0). (zero found in Step # 1 will = c … so don't change the sign in the half box when doing synthetic ÷)

3.) Repeat Step # 2 (using continuous synthetic division) until have used all rational zeros for P (x). Goal – To get quotient down to either a LINEAR EQUATION or QUADRATIC EQUATION. * For quadratic equations – you will have to use the Quadratic Formula *

4.) When listing all zeros – remember… degree of polynomial P (x) = # of zeros for polynomial P (x).

Example 1: Using the given polynomial P (x) and its graph, find all the zeros for P (x).

a.) $P(x) = x^3 - 5x - 2$ → all zeros of P (x) = $\boxed{-2,\ 1 \pm \sqrt{2}}$



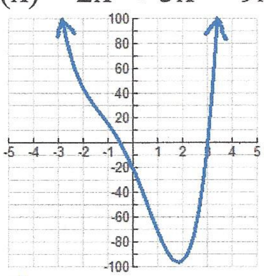
Rational zero = −2

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -5 & -2 \\ & \downarrow & -2 & 4 & 2 \\ \hline & 1 & -2 & -1 & 0 \end{array}$$

$x^2 - 2x - 1 = 0$

$$\frac{2 \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)} = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

b.) $P(x) = 2x^4 + 3x^3 - 9x^2 - 47x - 21$ → all zeros of P (x) = $\boxed{-\frac{1}{2},\ 3,\ -2 \pm i\sqrt{3}}$



Rational zeros = $-\frac{1}{2}$, 3

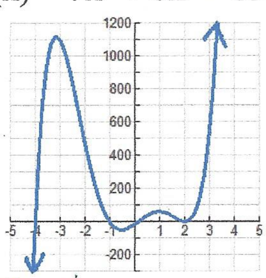
$$\begin{array}{r|rrrrr} -\frac{1}{2} & 2 & 3 & -9 & -47 & -21 \\ & \downarrow & -1 & -1 & 5 & 21 \\ \hline 3 & 2 & 2 & -10 & -42 & 0 \\ & \downarrow & 6 & 24 & 42 & \\ \hline & 2 & 8 & 14 & 0 & \end{array}$$

$2x^2 + 8x + 14 = 0$

$x^2 + 4x + 7 = 0$

$$\frac{-4 \pm \sqrt{(4)^2 - 4(1)(7)}}{2(1)} = \frac{-4 \pm \sqrt{-12}}{2} = \frac{-4 \pm 2i\sqrt{3}}{2} = -2 \pm i\sqrt{3}$$

c.) $P(x) = 7x^5 + 6x^4 - 85x^3 + 40x^2 + 108x - 16$ → all zeros of P (x) = $\boxed{-4,\ -1,\ 2\text{(m=2)},\ \frac{1}{7}}$



Rational zeros = −4, −1, 2 (m=2)

$$\begin{array}{r|rrrrrr} -4 & 7 & 6 & -85 & 40 & 108 & -16 \\ & \downarrow & -28 & 88 & -12 & -112 & 16 \\ \hline -1 & 7 & -22 & 3 & 28 & -4 & 0 \\ & \downarrow & -7 & 29 & -32 & 4 & \\ \hline 2 & 7 & -29 & 32 & -4 & 0 & \\ & \downarrow & 14 & -30 & 4 & & \\ \hline 2 & 7 & -15 & 2 & 0 & & \\ & \downarrow & 14 & -2 & & & \\ \hline & 7 & -1 & 0 & & & \end{array}$$

$7x - 1 = 0$

$7x = 1$

$x = \frac{1}{7}$

Example 2: Find all the zeros of each polynomial P (x).

a.) $P(x) = x^3 + 2x^2 - 2x - 1$ $RZ = 1$

$$\begin{array}{r|rrrr} 1 & 1 & 2 & -2 & -1 \\ & \downarrow & 1 & 3 & 1 \\ \hline & 1 & 3 & 1 & 0 \end{array}$$

$x^2 + 3x + 1 = 0$

$$\frac{-3 \pm \sqrt{(3)^2 - 4(1)(1)}}{2(1)} = \frac{-3 \pm \sqrt{5}}{2}$$

All Zeros of P (x) = $\boxed{1, \frac{-3 \pm \sqrt{5}}{2}}$

b.) $P(x) = 3x^3 + 14x^2 + 23x + 10$ $RZ = -\frac{2}{3}$

$$\begin{array}{r|rrrr} -\frac{2}{3} & 3 & 14 & 23 & 10 \\ & \downarrow & -2 & -8 & -10 \\ \hline & 3 & 12 & 15 & 0 \end{array}$$

$3x^2 + 12x + 15 = 0$

$x^2 + 4x + 5 = 0$

$$\frac{-4 \pm \sqrt{(4)^2 - 4(1)(5)}}{2(1)} = \frac{-4 \pm \sqrt{-4}}{2}$$

$$= \frac{-4 \pm 2i}{2} = -2 \pm i$$

All Zeros of P (x) = $\boxed{-\frac{2}{3}, -2 \pm i}$

c.) $P(x) = 2x^4 - 10x^3 + 3x^2 + 36x - 27$ $RZ = 3 \text{ (mult)}$

$$\begin{array}{r|rrrrr} 3 & 2 & -10 & 3 & 36 & -27 \\ & \downarrow & 6 & -12 & -27 & 27 \\ \hline 3 & 2 & -4 & -9 & 9 & 0 \\ & \downarrow & 6 & 6 & -9 & \\ \hline & 2 & 2 & -3 & 0 & \end{array}$$

$2x^2 + 2x - 3 = 0$

$$\frac{-2 \pm \sqrt{(2)^2 - 4(2)(-3)}}{2(2)} = \frac{-2 \pm \sqrt{28}}{4}$$

$$\frac{-2 \pm 2\sqrt{7}}{4} = \frac{-1 \pm \sqrt{7}}{2}$$

All Zeros of P (x) = $\boxed{3 \text{ (mult)}, \frac{-1 \pm \sqrt{7}}{2}}$

d.) $P(x) = 3x^5 + 7x^4 + x^3 - 17x^2 - 30x$ $RZ = -2, 0, \frac{5}{3}$

$$\begin{array}{r|rrrrrr} -2 & 3 & 7 & 1 & -17 & -30 & 0 \\ & \downarrow & -6 & -2 & 2 & 30 & 0 \\ \hline 0 & 3 & 1 & -1 & -15 & 0 & 0 \\ & \downarrow & 0 & 0 & 0 & 0 & \\ \hline \frac{5}{3} & 3 & 1 & -1 & -15 & 0 & \\ & \downarrow & 5 & 10 & 15 & & \\ \hline & 3 & 6 & 9 & 0 & & \end{array}$$

$3x^2 + 6x + 9 = 0$

$x^2 + 2x + 3 = 0$

$$\frac{-2 \pm \sqrt{(2)^2 - 4(1)(3)}}{2(1)} = \frac{-2 \pm \sqrt{-8}}{2}$$

$$= \frac{-2 \pm 2i\sqrt{2}}{2} = -1 \pm i\sqrt{2}$$

All Zeros of P (x) = $\boxed{-2, 0, \frac{5}{3}, -1 \pm i\sqrt{2}}$