First Derivative Sheet (SWOK) - M3 Answers in Packet

[4] $f'(x) = 3x^2 - 2x - 40 = (3x+10)(x-4) = 0 \Leftrightarrow x = -\frac{10}{3}, 4$.

On $(-\infty, -\frac{10}{3}) \cup (4, \infty)$, $f'(x) > 0$ and f is $\uparrow$ on $(-\infty, -\frac{10}{3}] \cup [4, \infty)$.

On $(-\frac{10}{3}, 4)$, $f'(x) < 0$ and f is $\downarrow$ on $[-\frac{10}{3}, 4]$.

Thus, $f(-\frac{10}{3}) = \frac{2516}{27} \approx 93.2$ is a *LMAX* and $f(4) = -104$ is a *LMIN*.

[5] $f'(x) = 4x^3 - 16x = 4x(x^2-4) = 0 \Leftrightarrow x = -2, 0, 2$.

On $(-\infty, -2) \cup (0, 2)$ $f'(x) < 0$ and f is $\downarrow$ on $(-\infty, -2] \cup [0, 2]$.

On $(-2, 0) \cup (2, \infty)$, $f'(x) > 0$ and f is $\uparrow$ on $[-2, 0] \cup [2, \infty)$.

Thus, $f(0) = 1$ is a *LMAX* and $f(-2) = f(2) = -15$ are *LMIN*.

[7] $f'(x) = 30x^2(x-1)^2 + 2(x-1)(10x^3) = 10x^2(x-1)(5x-3) = 0 \Leftrightarrow x = 0, \frac{3}{5}, 1$.

The sign of f' is determined by the sign of the quadratic $(x-1)(5x-3)$.

f is $\uparrow$ on $(-\infty, \frac{3}{5}] \cup [1, \infty)$ and $\downarrow$ on $[\frac{3}{5}, 1]$.

Thus. $f(\frac{3}{5}) = \frac{216}{625} \approx 0.35$ is a *LMAX* and $f(1) = 0$ is a *LMIN*. See *Figure 7*.

[8] $f'(x) = 4(x^2 - 10x)^3(2x - 10) = 8x^3(x-10)^3(x-5) = 0 \Leftrightarrow x = 0, 5, 10$.

Since each factor is raised to an odd power, f' will change sign at each *CN*.

Hence, $f'(x) < 0$ on $(-\infty, 0) \cup (5, 10)$, and $f'(x) > 0$ on $(0, 5) \cup (10, \infty)$.

f is $\downarrow$ $(-\infty, 0] \cup [5, 10]$ and $\uparrow$ on $[0, 5] \cup [10, \infty)$.

$f(0) = f(10) = 0$ are *LMIN* and $f(5) = 390{,}625$ is a *LMAX*.

[10] $f'(x) = x \cdot \frac{1}{3}(x-5)^{-2/3} + (x-5)^{1/3} = \dfrac{4x - 15}{3(x-5)^{2/3}} = 0 \Leftrightarrow x = \frac{15}{4}$.

f' fails to exist at $x = 5$. On $(-\infty, \frac{15}{4})$, $f'(x) < 0$ and f is $\downarrow$ on $(-\infty, \frac{15}{4}]$.

On $(\frac{15}{4}, 5) \cup (5, \infty)$, $f'(x) > 0$ and f is $\uparrow$ on $[\frac{15}{4}, \infty)$.

Thus, $f(\frac{15}{4}) = -\frac{15}{4}\sqrt[3]{\frac{5}{4}} \approx -4.04$ is a *LMIN*. There is a vertical tangent line at $x = 5$.

[11] $f'(x) = x^{2/3}(2)(x-7) + (x-7)^2(\frac{2}{3})x^{-1/3} = \dfrac{2(x-7)(4x-7)}{3x^{1/3}} = 0 \Leftrightarrow x = 7, \frac{7}{4}$.

f' fails to exist at $x = 0$. f' changes sign at each of its *CN*.

On $(-\infty, 0) \cup (\frac{7}{4}, 7)$, $f'(x) < 0$ and f is $\downarrow$ on $(-\infty, 0] \cup [\frac{7}{4}, 7]$.

On $(0, \frac{7}{4}) \cup (7, \infty)$ $f'(x) > 0$ and f is $\uparrow$ on $[0, \frac{7}{4}] \cup [7, \infty)$. Thus, $f(0) = f(7) = 2$

are *LMIN* and $f(\frac{7}{4}) = \frac{441}{16}\sqrt[3]{\frac{49}{16}} + 2 \approx 42.03$ is a *LMAX*. See *Figure 11*

[14] $f'(x) = -\frac{1}{3}(x^2 - 2x + 1)^{-2/3}(2x - 2) = \dfrac{-2}{3(x-1)^{1/3}} \neq 0$.

f' fails to exist at 1. f is $\uparrow$ on $(-\infty, 1]$ and $\downarrow$ on $[1, \infty)$.

Thus, $f(1) = 8$ is a *LMAX*. There is a cusp at $x = 1$.

[13] $f'(x) = \frac{2}{3}x^3(x^2-4)^{-2/3} + 2x(x^2-4)^{1/3} = \dfrac{8x(x^2-3)}{3(x^2-4)^{2/3}} = 0 \Leftrightarrow x = 0, \pm\sqrt{3}$.

f' fails to exist at $x = \pm 2$. Using a table, we find the following.

Interval	$8x$	$(x^2-4)^{2/3}$	(x^2-3)	f'	f
$(-\infty, -2)$	−	+	+	−	$\downarrow$
$(-2, -\sqrt{3})$	−	+	+	−	$\downarrow$
$(-\sqrt{3}, 0)$	−	+	−	+	$\uparrow$
$(0, \sqrt{3})$	+	+	−	−	$\downarrow$
$(\sqrt{3}, 2)$	+	+	+	+	$\uparrow$
$(2, \infty)$	+	+	+	+	$\uparrow$

Thus, f is $\uparrow$ on $[-\sqrt{3}, 0] \cup [\sqrt{3}, \infty)$ and f is $\downarrow$ on $(-\infty, -\sqrt{3}] \cup [0, \sqrt{3}]$.

$f(0) = 0$ is a *LMAX* and $f(\pm\sqrt{3}) = -3$ are *LMIN*.

There are no extrema at $x = \pm 2$, but there are vertical tangent lines at these values.

[16] $f'(x) = -x^2(4-x^2)^{-1/2} + (4-x^2)^{1/2} = \dfrac{2(2-x^2)}{(4-x^2)^{1/2}} = 0$ at $x = \pm\sqrt{2}$.

f' fails to exist at $x = \pm 2$. Since the denominator is positive on $(-2, 2)$, $f'(x) < 0$ on $(-2, -\sqrt{2}) \cup (\sqrt{2}, 2)$ and $f'(x) > 0$ on $(-\sqrt{2}, \sqrt{2})$. Thus, f is $\downarrow$ on $[-2, -\sqrt{2}] \cup [-\sqrt{2}, 2]$ and f is $\uparrow$ on $[-\sqrt{2}, \sqrt{2}]$. $f(-\sqrt{2}) = -2$ is a *LMIN* and $f(\sqrt{2}) = 2$ is

a *LMAX*. *Note*: $x = \pm 2$ are endpoints of the domain of f.

[17] $f'(x) = \cos x - \sin x = 0$ if $x = \frac{\pi}{4}, \frac{5\pi}{4}$. Since $\cos x > \sin x$ on $[0, \frac{\pi}{4}) \cup (\frac{5\pi}{4}, 2\pi]$,

it follows that $f'(x) > 0$ on these intervals. Hence, f is $\uparrow$ on $[0, \frac{\pi}{4}] \cup [\frac{5\pi}{4}, 2\pi]$.

Similarly, $\cos x < \sin x$ on $(\frac{\pi}{4}, \frac{5\pi}{4})$ and f is $\downarrow$ on $[\frac{\pi}{4}, \frac{5\pi}{4}]$.

Thus, $f(\frac{\pi}{4}) = \sqrt{2}$ is a *LMAX* and $f(\frac{5\pi}{4}) = -\sqrt{2}$ is a *LMIN*.

[19] $f'(x) = \frac{1}{2} - \cos x = 0$ if $x = \frac{\pi}{3}, \frac{5\pi}{3}$. Since $\cos x > \frac{1}{2}$ on $[0, \frac{\pi}{3}) \cup (\frac{5\pi}{3}, 2\pi]$,

$f'(x) < 0$ on these intervals and f is $\downarrow$ on $[0, \frac{\pi}{3}] \cup [\frac{5\pi}{3}, 2\pi]$.

Similarly, $\cos x < \frac{1}{2}$ on $(\frac{\pi}{3}, \frac{5\pi}{3})$ and f is $\uparrow$ on $[\frac{\pi}{3}, \frac{5\pi}{3}]$. Thus,

$f(\frac{\pi}{3}) = \frac{\pi}{6} - \frac{1}{2}\sqrt{3} \approx -0.34$ is a *LMIN* and $f(\frac{5\pi}{3}) = \frac{5\pi}{6} + \frac{1}{2}\sqrt{3} \approx 3.48$ is a *LMAX*.

[20] $f'(x) = 1 - 2\sin x = 0$ if $x = \frac{\pi}{6}, \frac{5\pi}{6}$. Since $\sin x < \frac{1}{2}$ on $[0, \frac{\pi}{6}) \cup (\frac{5\pi}{6}, 2\pi]$,

$f'(x) > 0$ on these intervals and f is $\uparrow$ on $[0, \frac{\pi}{6}] \cup [\frac{5\pi}{6}, 2\pi]$. Similarly,

$\sin x > \frac{1}{2}$ on $(\frac{\pi}{6}, \frac{5\pi}{6})$ and f is $\downarrow$ on $[\frac{\pi}{6}, \frac{5\pi}{6}]$. Thus, $f(\frac{\pi}{6}) = \frac{\pi}{6} + \sqrt{3} \approx 2.26$ is a *LMAX*

and $f(\frac{5\pi}{6}) = \frac{5\pi}{6} - \sqrt{3} \approx 0.886$ is a *LMIN*.

[22] $f'(x) = -2\sin x - 2\sin 2x = -2\sin x - 4\sin x\cos x = -2\sin x(1 + 2\cos x) = 0 \Leftrightarrow$

$\sin x = 0$ or $\cos x = -\frac{1}{2}$ if $x = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, 2\pi$. Since f' changes sign at each *CN*,

it follows that $f'(x) < 0$ on $(0, \frac{2\pi}{3}) \cup (\pi, \frac{4\pi}{3})$ and $f'(x) > 0$ on $(\frac{2\pi}{3}, \pi) \cup (\frac{4\pi}{3}, 2\pi)$.

Hence f is $\downarrow$ on $[0, \frac{2\pi}{3}] \cup [\pi, \frac{4\pi}{3}]$ and $\uparrow$ on $[\frac{2\pi}{3}, \pi] \cup [\frac{4\pi}{3}, 2\pi]$.

$f(0) = f(2\pi) = 3$ are endpoint maxima. *Note*: These are not local extrema since

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there is no *open* interval in $[0, 2\pi]$ that contains 0 or 2π.

$f(\pi) = -1$ is a *LMAX*. $f(\frac{2\pi}{3}) = f(\frac{4\pi}{3}) = -\frac{3}{2}$ are *LMIN*.