Practice f' Deriv Test

Exer. 1–16: Find the local extrema of f and the intervals on which f is increasing or is decreasing, and sketch the graph of f.

1 $f(x) = 5 - 7x - 4x^2$

2 $f(x) = 6x^2 - 9x + 5$

3 $f(x) = 2x^3 + x^2 - 20x + 1$

4 $f(x) = x^3 - x^2 - 40x + 8$

5 $f(x) = x^4 - 8x^2 + 1$

6 $f(x) = 4x^3 - 3x^4$

7 $f(x) = 10x^3(x - 1)^2$

8 $f(x) = (x^2 - 10x)^4$

9 $f(x) = x^{4/3} + 4x^{1/3}$

10 $f(x) = x(x - 5)^{1/3}$

11 $f(x) = x^{2/3}(x - 7)^2 + 2$

12 $f(x) = x^{2/3}(8 - x)$

13 $f(x) = x^2\sqrt[3]{x^2 - 4}$

14 $f(x) = 8 - \sqrt[3]{x^2 - 2x + 1}$

15 $f(x) = x\sqrt{x^2 - 9}$

16 $f(x) = x\sqrt{4 - x^2}$

Exer. 17–22: Find the local extrema of f on $[0, 2\pi]$ and the subintervals on which f is increasing or is decreasing. Sketch the graph of f.

17 $f(x) = \cos x + \sin x$

18 $f(x) = \cos x - \sin x$

19 $f(x) = \frac{1}{2}x - \sin x$

20 $f(x) = x + 2\cos x$

21 $f(x) = 2\cos x + \sin 2x$

22 $f(x) = 2\cos x + \cos 2x$

Exer. 23–28: Find the local extrema of f.

23 $f(x) = \sqrt[3]{x^3 - 9x}$

24 $f(x) = \sqrt{x^2 + 4}$

25 $f(x) = (x - 2)^3(x + 1)^4$

26 $f(x) = x^2(x - 5)^4$

27 $f(x) = \dfrac{\sqrt{x - 3}}{x^2}$

28 $f(x) = \dfrac{x^2}{\sqrt{x + 7}}$

Exer. 29–32: Find the local extrema of f on the given interval.

29 $f(x) = \sec \frac{1}{2}x;\quad [-\pi/2, \pi/2]$

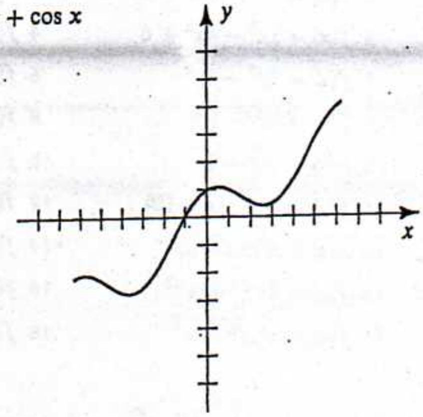
30 $f(x) = \cot^2 x + 2\cot x;\quad [\pi/6, 5\pi/6]$

31 $f(x) = 2\tan x - \tan^2 x;\quad [-\pi/3, \pi/3]$

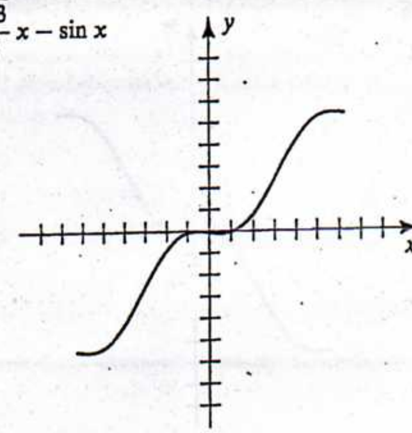
32 $f(x) = \tan x - 2\sec x;\quad [-\pi/4, \pi/4]$

Exer. 33–34: Shown in the figure is a graph of the equation for $-2\pi \le x \le 2\pi$. Determine the x-coordinates of the points that correspond to local extrema.

33 $y = \frac{1}{2}x + \cos x$



34 $y = \dfrac{\sqrt{3}}{2}x - \sin x$



Exer. 35–40: Sketch the graph of a differentiable function f that satisfies the given conditions.

35 $f(0) = 3$; $f(-2) = f(2) = -4$; $f'(0)$ is undefined; $f'(-2) = f'(2) = 0$; $f'(x) > 0$ if $-2 < x < 0$ or $x > 2$; $f'(x) < 0$ if $x < -2$ or $0 < x < 2$

36 $f(3) = 5$; $f(5) = 0$; $f'(5)$ is undefined; $f'(3) = 0$; $f'(x) > 0$ if $x < 3$ or $x > 5$; $f'(x) < 0$ if $3 < x < 5$

37 $f(3) = 5$; $f(5) = 0$; $f'(3) = f'(5) = 0$; $f'(x) > 0$ if $x < 3$ or $x > 5$; $f'(x) < 0$ if $3 < x < 5$

38 $f(0) = 3$; $f(-2) = f(2) = -4$; $f'(-2) = f'(0) = f'(2) = 0$; $f'(x) > 0$ if $-2 < x < 0$ or $x > 2$; $f'(x) < 0$ if $x < -2$ or $0 < x < 2$

39 $f(-5) = 4$; $f(0) = 0$; $f(5) = -4$; $f'(-5) = f'(0) = f'(5) = 0$; $f'(x) > 0$ if $|x| > 5$; $f'(x) < 0$ if $0 < |x| < 5$

40 $f(a) = a$ and $f'(a) = 0$ for $a = 0, \pm1, \pm2, \pm3$; $f'(x) > 0$ for all other values of x

$\boxed{c}$ **Exer. 41–42: Graph f on $[-2, 2]$. (a) Use the graph to estimate the local extrema of f. (b) Estimate where f is increasing or is decreasing.**

41 $f(x) = \dfrac{x^2 - 1.5x + 2.1}{0.3x^4 + 2.3x + 2.7}$

42 $f(x) = \dfrac{10\cos 2x}{x^2 + 4}$

$\boxed{c}$ **Exer. 43–44: Graph f' on $[-1, 1]$. Estimate the x-coordinates of the local extrema of f and classify each local extremum.**

43 $f'(x) = x - \cos \pi x - \sin x$

44 $f'(x) = 6x^3 - 3x^2 - 1.3x + 0.5$

 Practice 2nd Deriv Test

Exer. 1–18: Find the local extrema of f, using the second derivative test whenever applicable. Find the intervals on which the graph of f is concave upward or is concave downward, and find the x-coordinates of the points of inflection. Sketch the graph of f.

1 $f(x) = x^3 - 2x^2 + x + 1$

2 $f(x) = x^3 + 10x^2 + 25x - 50$

3 $f(x) = 3x^4 - 4x^3 + 6$

4 $f(x) = 8x^2 - 2x^4$

5 $f(x) = 2x^6 - 6x^4$

6 $f(x) = 3x^5 - 5x^3$

7 $f(x) = (x^2 - 1)^2$

8 $f(x) = x^4 - 4x^3 + 10$

9 $f(x) = \sqrt[3]{x} - 1$

10 $f(x) = 2 - \sqrt[3]{x^2}$

11 $f(x) = \sqrt[3]{x^2}(3x + 10)$

12 $f(x) = x^{2/3}(1 - x)$

13 $f(x) = x^2(3x - 5)^{1/3}$

14 $f(x) = x\sqrt[3]{3x + 2}$

15 $f(x) = 8x^{1/3} + x^{4/3}$

16 $f(x) = 6x^{1/2} + x^{3/2}$

17 $f(x) = x^2\sqrt{9 - x^2}$

18 $f(x) = x\sqrt{4 - x^2}$

Exer. 25–28: Use the second derivative test to find the local extrema of f on the given interval. (See Exercises 29–32 of Section 4.3.)

25 $f(x) = \sec \frac{1}{2}x$; $[-\pi/2, \pi/2]$

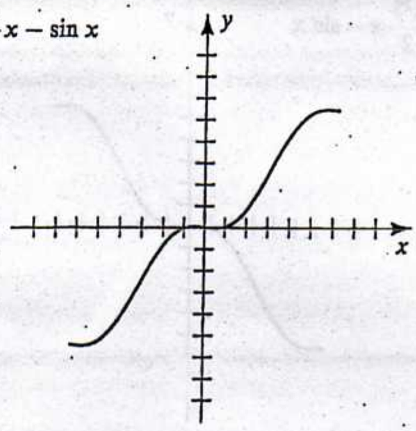
26 $f(x) = \cot^2 x + 2 \cot x$; $[\pi/6, 5\pi/6]$

27 $f(x) = 2 \tan x - \tan^2 x$; $[-\pi/3, \pi/3]$

28 $f(x) = \tan x - 2 \sec x$; $[-\pi/4, \pi/4]$

Use 2nd deriv test to find extrema:

30 $y = \frac{\sqrt{3}}{2}x - \sin x$



Exer. 31–38: Sketch the graph of a continuous function f that satisfies all of the stated conditions.

31 $f(0) = 1$; $f(2) = 3$; $f'(0) = f'(2) = 0$;
$f'(x) < 0$ if $|x - 1| > 1$; $f'(x) > 0$ if $|x - 1| < 1$;
$f''(x) > 0$ if $x < 1$; $f''(x) < 0$ if $x > 1$

32 $f(0) = 4$; $f(2) = 2$; $f(5) = 6$; $f'(0) = f'(2) = 0$;
$f'(x) > 0$ if $|x - 1| > 1$; $f'(x) < 0$ if $|x - 1| < 1$;
$f''(x) < 0$ if $x < 1$ or if $|x - 4| < 1$;
$f''(x) > 0$ if $|x - 2| < 1$ or if $x > 5$

33 $f(0) = 2$; $f(2) = f(-2) = 1$; $f'(0) = 0$;
$f'(x) > 0$ if $x < 0$; $f'(x) < 0$ if $x > 0$;
$f''(x) < 0$ if $|x| < 2$; $f''(x) > 0$ if $|x| > 2$

34 $f(1) = 4$; $f'(x) > 0$ if $x < 1$; $f'(x) < 0$ if $x > 1$;
$f''(x) > 0$ for every $x \neq 1$

35 $f(-2) = f(6) = -2$; $f(0) = f(4) = 0$; $f(2) = f(8) = 3$;
f' is undefined at 2 and 6; $f'(0) = 1$;
$f'(x) > 0$ throughout $(-\infty, 2)$ and $(6, \infty)$;
$f'(x) < 0$ if $|x - 4| < 2$;
$f''(x) < 0$ throughout $(-\infty, 0)$, $(4, 6)$, and $(6, \infty)$;
$f''(x) > 0$ throughout $(0, 2)$ and $(2, 4)$

36 $f(0) = 2$; $f(2) = 1$; $f(4) = f(10) = 0$; $f(6) = -4$;
$f'(2) = f'(6) = 0$;
$f'(x) < 0$ throughout $(-\infty, 2)$, $(2, 4)$, $(4, 6)$, and $(10, \infty)$;
$f'(x) > 0$ throughout $(6, 10)$;
$f'(4)$ and $f'(10)$ do not exist;
$f''(x) > 0$ throughout $(-\infty, 2)$, $(4, 10)$, and $(10, \infty)$;
$f''(x) < 0$ throughout $(2, 4)$

37 If n is an odd integer, then $f(n) = 1$ and $f'(n) = 0$; if n is an even integer, then $f(n) = 0$ and $f'(n)$ does not exist; if n is any integer, then

(a) $f'(x) > 0$ whenever $2n < x < 2n + 1$

(b) $f'(x) < 0$ whenever $2n - 1 < x < 2n$

(c) $f''(x) < 0$ whenever $2n < x < 2n + 2$

38 $f(x) = x$ if $x = -1, 2, 4$, or 8;
$f'(x) = 0$ if $x = -1, 4, 6$, or 8;
$f'(x) < 0$ throughout $(-\infty, -1)$, $(4, 6)$, and $(8, \infty)$;
$f'(x) > 0$ throughout $(-1, 4)$ and $(6, 8)$;
$f''(x) > 0$ throughout $(-\infty, 0)$, $(2, 3)$, and $(5, 7)$;
$f''(x) < 0$ throughout $(0, 2)$, $(3, 5)$, and $(7, \infty)$

39 Prove that the graph of a quadratic function has no point of inflection. State conditions for which the graph is always

(a) concave upward (b) concave downward

40 Prove that the graph of a polynomial function of degree 3 has exactly one point of inflection.

c **Exer. 41–42: Graph f on $[-1, 1]$. (a) Estimate where the graph of f is concave upward or is concave downward. (b) Estimate the x-coordinate of each point of inflection.**

41 $f(x) = 4x^5 + x^4 + 3x^2 - 2x + 1$

42 $f(x) = (x - 0.1)^2\sqrt{1.08 - 0.9x^2}$

c **Exer. 43–44: Graph f'' on $[0, 3]$. (a) Estimate where the graph of f is concave upward or is concave downward. (b) Estimate the x-coordinate of each point of inflection.**

43 $f''(x) = x^4 - 5x^3 + 7.57x^2 - 3.3x + 0.4356$

44 $f''(x) = 2.1 \sin \pi x + 1.4 \cos x - 0.6$

Ans. - 1st Deriv. Test

[3] $f'(x) = 6x^2 + 2x - 20 = 2(3x-5)(x+2) = 0 \Leftrightarrow x = \frac{5}{3}, -2$. On $(-\infty, -2) \cup (\frac{5}{3}, \infty)$, $f'(x) > 0$ and f is $\uparrow$ on $(-\infty, -2] \cup [\frac{5}{3}, \infty)$. On $(-2, \frac{5}{3})$, $f'(x) < 0$ and f is $\downarrow$ on $[-2, \frac{5}{3}]$. Thus, $f(-2) = 29$ is a *LMAX* and $f(\frac{5}{3}) = -\frac{548}{27}$ is a *LMIN*.

[6] $f'(x) = 12x^2 - 12x^3 = 12x^2(1-x) = 0 \Leftrightarrow x = 0, 1$.
On $(-\infty, 0) \cup (0, 1)$, $f'(x) > 0$ and f is $\uparrow$ on $(-\infty, 1]$.
On $(1, \infty)$, $f'(x) < 0$ and f is $\downarrow$ on $[1, \infty)$. Thus, $f(1) = 1$ is a *LMAX*.

[9] $f'(x) = \frac{4}{3}(x^{1/3} + x^{-2/3}) = \dfrac{4(x+1)}{3x^{2/3}} = 0 \Leftrightarrow x = -1$. f' fails to exist at $x = 0$.
The sign of f' is determined by the factor $(x+1)$. On $(-\infty, -1)$, $f'(x) < 0$ and f is $\downarrow$ on $(-\infty, -1]$. On $(-1, 0) \cup (0, \infty)$, $f'(x) > 0$ and f is $\uparrow$ on $[-1, \infty)$.
Thus, $f(-1) = -3$ is a *LMIN*. There is a vertical tangent line at $x = 0$.

[12] $f'(x) = -x^{2/3} + \frac{2}{3}x^{-1/3}(8-x) = \dfrac{16-5x}{3x^{1/3}} = 0 \Leftrightarrow x = \frac{16}{5}$. f' fails to exist at $x = 0$. The sign of f' changes at each critical number. On $(-\infty, 0) \cup (\frac{16}{5}, \infty)$, $f'(x) < 0$ and f is $\downarrow$ on $(-\infty, 0] \cup [\frac{16}{5}, \infty)$. On $(0, \frac{16}{5})$, $f'(x) > 0$ and f is $\uparrow$ on $[0, \frac{16}{5}]$. Thus, $f(0) = 0$ is a *LMIN* and $f(\frac{16}{5}) = \frac{24}{5}(\frac{16}{5})^{2/3} \approx 10.42$ is a *LMAX*.

[15] $f'(x) = x \cdot \frac{1}{2}(x^2-9)^{-1/2}(2x) + (x^2-9)^{1/2} = \dfrac{2x^2-9}{\sqrt{x^2-9}} = 0 \Leftrightarrow x = \pm\sqrt{\frac{9}{2}}$,
which are not in the domain of f. f' fails to exist at ± 3.
f' is positive throughout its domain, and hence f is $\uparrow$ on $(-\infty, -3] \cup [3, \infty)$.
There are no extrema. There are vertical tangent lines at $x = \pm 3$.

[18] $f'(x) = -\sin x - \cos x = 0 \Leftrightarrow \sin x = -\cos x$ if $x = \frac{3\pi}{4}, \frac{7\pi}{4}$.
Since $-\sin x < \cos x$ on $[0, \frac{3\pi}{4}) \cup (\frac{7\pi}{4}, 2\pi]$, $f'(x) < 0$ on these intervals.
(To see this, consider the graphs of $y = -\sin x$ and $y = \cos x$.) Hence, f is $\downarrow$ on $[0, \frac{3\pi}{4}] \cup [\frac{7\pi}{4}, 2\pi]$. Similarly, $-\sin x > \cos x$ on $(\frac{3\pi}{4}, \frac{7\pi}{4})$ and f is $\uparrow$ on $[\frac{3\pi}{4}, \frac{7\pi}{4}]$.
Thus, $f(\frac{3\pi}{4}) = -\sqrt{2}$ is a *LMIN* and $f(\frac{7\pi}{4}) = \sqrt{2}$ is a *LMAX*.

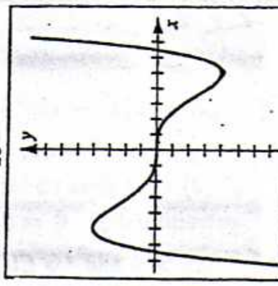
[21] $f'(x) = -2\sin x + 2\cos 2x = -2\sin x + 2(1 - 2\sin^2 x) =$
$(2 - 4\sin x)(\sin x + 1) = 0 \Leftrightarrow \sin x = \frac{1}{2}$ or $\sin x = -1$ if $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$.
Since $(\sin x + 1) \geq 0$ for all x, the sign of $f'(x)$ is determined by $2(1 - 2\sin x)$.
Following the solution to Exercise 20, f is $\uparrow$ on $[0, \frac{\pi}{6}] \cup [\frac{5\pi}{6}, 2\pi]$ and $\downarrow$ on $[\frac{\pi}{6}, \frac{5\pi}{6}]$.
Thus, $f(\frac{\pi}{6}) = \frac{3}{2}\sqrt{3}$ is a *LMAX* and $f(\frac{5\pi}{6}) = -\frac{3}{2}\sqrt{3}$ is a *LMIN*.

[24] $f'(x) = \dfrac{x}{\sqrt{x^2+4}} = 0 \Leftrightarrow x = 0$. $f'(x) < 0$ for $x < 0$ and $f'(x) > 0$ for $x > 0$.
Thus, $f(0) = 2$ is a *LMIN*.

[27] $f'(x) = \dfrac{-3x+12}{2x^3\sqrt{x-3}} = 0 \Leftrightarrow x = 4$.
f' fails to exist at $x = 0$ and 3, but 0 is not in the domain of f.
If $3 < x < 4$, $f'(x) > 0$ and if $x > 4$, $f'(x) < 0$. Thus, $f(4) = \frac{1}{16}$ is a *LMAX*.

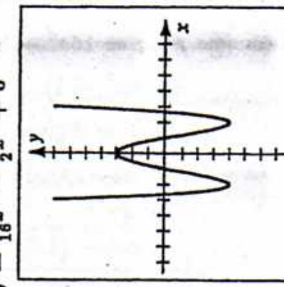
[30] $f'(x) = -2\cot x \csc^2 x - 2\csc^2 x = -2\csc^2 x(\cot x + 1) = 0$ on $[\frac{\pi}{6}, \frac{5\pi}{6}] \Rightarrow x = \frac{3\pi}{4}$
since $\csc^2 x \neq 0$. f' fails to exist at $x = \frac{\pi}{2}$ but $\frac{\pi}{2}$ is not in the domain of f. Since the sign of f' changes from negative to positive at $x = \frac{3\pi}{4}$, $f(\frac{3\pi}{4}) = -1$ is a *LMIN*.

[33] $y' = \frac{1}{2} - \sin x = 0$ on $[-2\pi, 2\pi] \Rightarrow x = -\frac{11\pi}{6}, -\frac{7\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$.
$\sin x > \frac{1}{2}$ on $(\frac{\pi}{6}, \frac{5\pi}{6}) \cup (-\frac{11\pi}{6}, -\frac{7\pi}{6})$ and $f'(x) < 0$ on these intervals.
$\sin x < \frac{1}{2}$ on $[-2\pi, -\frac{11\pi}{6}) \cup (-\frac{7\pi}{6}, \frac{\pi}{6}) \cup (\frac{5\pi}{6}, 2\pi]$ and $f'(x) > 0$ on these intervals.
Thus, there are *LMIN* at $x = -\frac{7\pi}{6}, \frac{5\pi}{6}$ and *LMAX* at $x = -\frac{11\pi}{6}, \frac{\pi}{6}$.



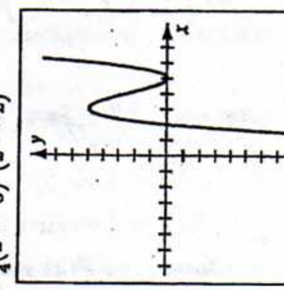
[39] $f(x) = \frac{6}{3125}x^5 - \frac{2}{25}x^3$

Figure 39



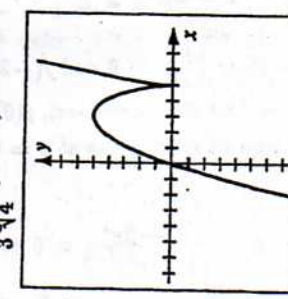
[38] $f(x) = \frac{7}{16}x^4 - \frac{7}{2}x^2 + 3$

Figure 38



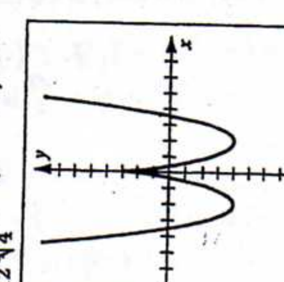
[37] $f(x) = \frac{5}{4}(x-5)^2(x-2)$

Figure 37



[36] $f(x) = \frac{5}{3\sqrt[3]{4}}x(x-5)^{2/3}$

Figure 36



[35] $f(x) = \frac{7}{12\sqrt[3]{4}}x^{2/3}(x^2-16) + 3$

Figure 35

Ans. 4.4

[3] $f'(x) = 12x^3 - 12x^2 = 12x^2(x-1) = 0 \Leftrightarrow x = 0, 1.$

$f''(x) = 36x^2 - 24x = 12x(3x-2)$. $f''(1) = 12 > 0 \Rightarrow f(1) = 5$ is a *LMIN*.
$f''(0) = 0$ gives no information. By the first derivative test, $x = 0$ is not an extremum. $f''(x) = 0$ at $x = 0, \frac{2}{3}$. f'' changes sign at each of these points.
$f''(x) > 0$ and f is *CU* on $(-\infty, 0) \cup (\frac{2}{3}, \infty)$. $f''(x) < 0$ and f is *CD* on $(0, \frac{2}{3})$.
Thus, there are *PI* at $x = 0, \frac{2}{3}$.

[6] $f'(x) = 15x^4 - 15x^2 = 15x^2(x^2-1) = 0 \Leftrightarrow x = 0, \pm 1$. $f''(x) = 60x^3 - 30x = 30x(2x^2-1)$. $f''(\pm 1) = \pm 30 \Rightarrow f(1) = -2$ is a *LMIN* and $f(-1) = 2$ is a *LMAX*.
$f''(0) = 0$ gives no information. By the first derivative test,
$f(0)$ not an extremum. The table indicates *PI* at $x = 0, \pm\sqrt{1/2}$.

Interval	$30x$	$2x^2-1$	f''	Concavity
$(-\infty, -\sqrt{1/2})$	−	+	−	*CD*
$(-\sqrt{1/2}, 0)$	−	−	+	*CU*
$(0, \sqrt{1/2})$	+	−	−	*CD*
$(\sqrt{1/2}, \infty)$	+	+	+	*CU*

[9] $f'(x) = \frac{1}{5}x^{-4/5}$ is undefined when $x = 0$, otherwise $f'(x) > 0$. No local extrema.
$f''(x) = -\frac{4}{25}x^{-9/5}$. $f''(x) > 0$ and f is *CU* on $(-\infty, 0)$.
$f''(x) < 0$ and f is *CD* on $(0, \infty)$. *PI* at $x = 0$.

[12] $f'(x) = \dfrac{2-5x}{3x^{1/3}} = 0 \Leftrightarrow x = \frac{2}{5}$. f' fails to exist at $x = 0$. $f''(x) = \dfrac{-2(5x+1)}{9x^{4/3}}$.
$f''(\frac{2}{5}) < 0 \Rightarrow f(\frac{2}{5}) = (0.4)^{2/3}(0.6) \approx 0.33$ is a *LMAX*. Since $f''(0)$ is undefined, use the first derivative test to show that $f(0) = 0$ is a *LMIN*. Since $9x^{4/3} > 0$ for $x \neq 0$, there is no *PI* at $x = 0$. $f''(x) > 0$ and f is *CU* on $(-\infty, -\frac{1}{5})$.
$f''(x) < 0$ and f is *CD* on $(-\frac{1}{5}, 0) \cup (0, \infty)$. *PI* at $x = -\frac{1}{5}$.
Note: f is not *CD* at $x = 0$ since f' does not exist at $x = 0$. Also, the *PI* is not noticeable in the sketch of the graph since the concavity change is slight.

[15] $f'(x) = \dfrac{8+4x}{3x^{2/3}} = 0 \Leftrightarrow x = -2$. f' fails to exist at $x = 0$. $f''(x) = \dfrac{4(x-4)}{9x^{5/3}} \Rightarrow$
$f''(-2) = \frac{2}{3}\sqrt[3]{2} > 0$ and $f(-2) = -6\sqrt[3]{2} \approx -7.55$ is a *LMIN*. $f''(0)$ is undefined.
By the first derivative test, $f(0) = 0$ is not a local extremum.
The sign of f'' changes at $x = 0, 4$. $f''(x) > 0$ and f is *CU* on $(-\infty, 0) \cup (4, \infty)$.
$f''(x) < 0$ and f is *CD* on $(0, 4)$. *PI* at $x = 0, 4$.

[18] $f'(x) = \dfrac{4-2x^2}{(4-x^2)^{1/2}} = 0 \Leftrightarrow x = \pm\sqrt{2}$. $f''(x) = \dfrac{2x(x^2-6)}{(4-x^2)^{3/2}}$.
$f''(-\sqrt{2}) = 4 > 0 \Rightarrow f(-\sqrt{2}) = -2$ is a *LMIN*. $f''(\sqrt{2}) = -4 < 0 \Rightarrow$
$f(\sqrt{2}) = 2$ is a *LMAX*. $x = \pm 2$ are endpoints and cannot be local extrema.
The only value where f'' changes sign *in the domain* of f is $x = 0$.
$f''(x) > 0$ and f is *CU* on $(-2, 0)$. $f''(x) < 0$ and f is *CD* on $(0, 2)$. *PI* at $x = 0$.

[21] The *CN* are $x = \frac{\pi}{3}, \frac{5\pi}{3}$. $f''(x) = \sin x$. $f''(\frac{5\pi}{3}) = -\frac{\sqrt{3}}{2} < 0 \Rightarrow$
$f(\frac{5\pi}{3}) = \frac{5\pi}{6} + \frac{\sqrt{3}}{2}$ is a *LMAX*. $f''(\frac{\pi}{3}) = \frac{\sqrt{3}}{2} > 0 \Rightarrow f(\frac{\pi}{3}) = \frac{\pi}{6} - \frac{\sqrt{3}}{2}$ is a *LMIN*.

[24] The *CN* in $(0, 2\pi)$ are $x = \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$. $f''(x) = -2\cos x - 4\cos 2x$.
$f''(\pi) = -2 < 0 \Rightarrow f(\pi) = -1$ is a *LMAX*.
$f''(\frac{2\pi}{3}) = f''(\frac{4\pi}{3}) = 3 > 0 \Rightarrow f(\frac{2\pi}{3}) = -\frac{3}{2}$ and $f(\frac{4\pi}{3}) = -\frac{3}{2}$ are *LMIN*.

[27] The only *CN* in $(-\frac{\pi}{3}, \frac{\pi}{3})$ is $x = \frac{\pi}{4}$. $f''(x) = 4\sec^2 x \tan x(1 - \tan x) - 2\sec^4 x \Rightarrow$
$f''(\frac{\pi}{4}) = -8 < 0 \Rightarrow f(\frac{\pi}{4}) = 1$ is a *LMAX*.

[30] The *CN* on $(-2\pi, 2\pi)$ are $x = -\frac{11\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{11\pi}{6}$. $f''(x) = \sin x$.
Since $f''(-\frac{11\pi}{6}) = f''(\frac{\pi}{6}) = \frac{1}{2} > 0$, $f(-\frac{11\pi}{6}) = -\frac{11\sqrt{3}\pi}{12} - \frac{1}{2} \approx -5.49$ and
$f(\frac{\pi}{6}) = \frac{\sqrt{3}\pi}{12} - \frac{1}{2} \approx -0.05$ are *LMIN*. Since $f''(-\frac{\pi}{6}) = f''(\frac{11\pi}{6}) = -\frac{1}{2} < 0$,
$f(-\frac{\pi}{6}) = \frac{1}{2} - \frac{\sqrt{3}\pi}{12} \approx 0.05$ and $f(\frac{11\pi}{6}) = \frac{11\sqrt{3}\pi}{12} + \frac{1}{2} \approx 5.49$ are *LMAX*.

[31]

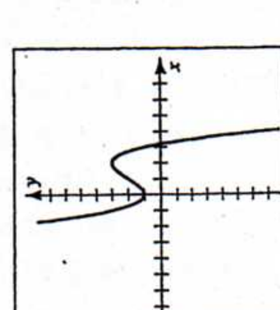


Figure 31

[32]

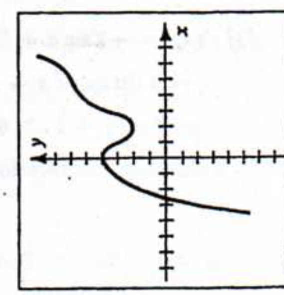
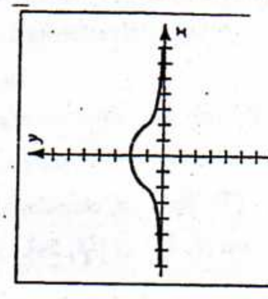


Figure 32


33

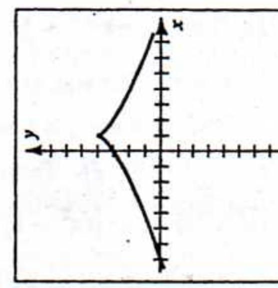


Figure 34

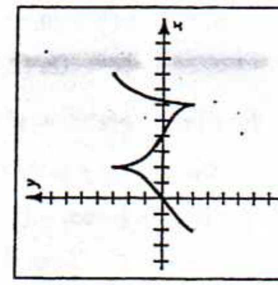


Figure 35

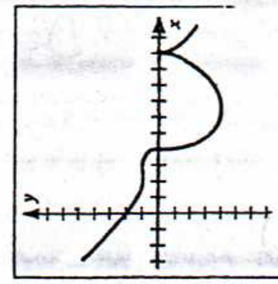


Figure 36