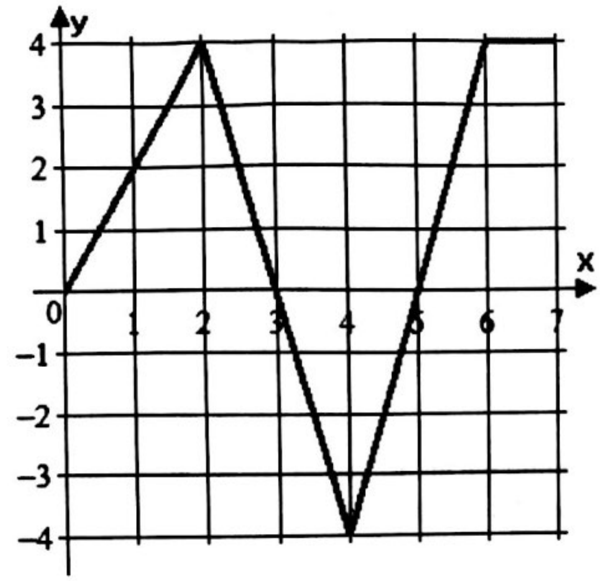
Definite Integrals and the Fundamental Theorem of Calculus

1.The given graph is f(x). Determine the following . . .



a) $\int_0^3 f(x)dx$

b) $\int_3^7 4f(x)dx$

c) $\int_0^7 f(x)dx$

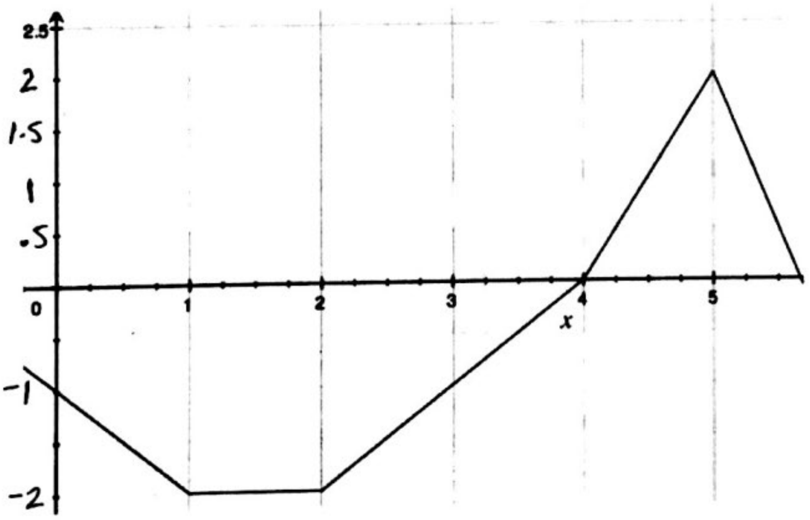
d) $\int_0^7 |f(x)|dx$

e) $\int_5^7 f(x-2)dx$

f) $\int_0^3 f(x)-4dx$

g) $\int_3^0 f(x)dx$

2. Suppose the given graph is f′(x) and f(4) = -3. Find the following . . .



a) f(5)

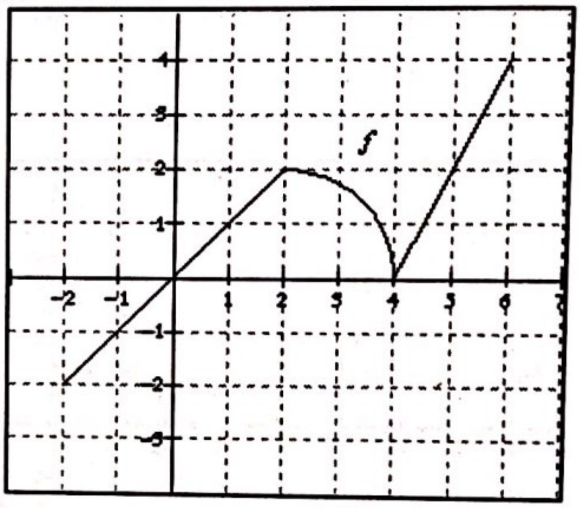
a) f(1)

b) Determine where f(x) has relative extrema on (0,5.75). Justify your answer.

c) Find the absolute minimum and absolute maximum VALUES on [0, 5]. Show work.

Worksheet 3. Graphical Analysis of $F(x)$ Using $F'(x)$

1. Let $F(x) = \int_{2}^{x} f(t)\,dt$. The graph of f on the interval $[-2, 6]$ consists of two line segments and a quarter of a circle, as shown at right.



(a) Find $F(0)$ and $F(4)$.

(b) Determine the interval where $F(x)$ is increasing. Justify your answer.

(c) Find the critical numbers of $F(x)$ and determine if each corresponds to a relative minimum value, a relative maximum value, or neither. Justify your answers.

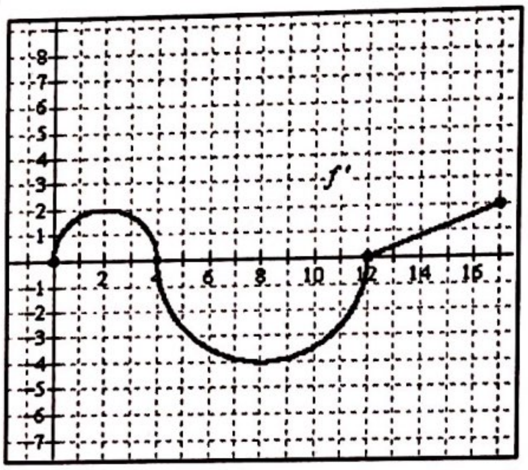
(d) Find the absolute extreme values of $F(x)$ and the x-values at which they occur. Justify your answers.

(e) Find the x-coordinates of the inflection points of $F(x)$. Justify your answer.

(f) Determine the intervals where the graph of $F(x)$ is concave down. Justify your answer.

Let $H(x)$ $\int^{x+2}$ ...

3. The graph below is of the function $f'(x)$, the derivative of the function $f(x)$, on the interval $0 \le x \le 17$. The graph consists of two semicircles and one line segment. Horizontal tangents are located at $x = 2$ and $x = 8$, and a vertical tangent is located at $x = 4$.



(a) On what intervals is $f(x)$ increasing? Justify your answer.

(b) For what values of x does $f(x)$ have a relative minimum value? Justify.

(c) On what intervals, for $0 < x < 17$, is the graph of $f(x)$ concave up? Justify.

(d) For what values of x, for $0 < x < 17$, is $f''(x)$ undefined?

(e) Identify the x-coordinates of all points of inflection of $f(x)$. Justify.

(f) For what value of x does $f(x)$ reach its absolute maximum value? Justify.

(g) If $f(4) = 3$, find $f(12)$.

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Using Fundamental Theorem of Calculus

1.

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \le t \le 12$. The radius of the balloon is 30 feet when $t = 5$. (Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)

(a) Estimate the radius of the balloon when $t = 5.4$ using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.

(b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.

(c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t)\,dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t)\,dt$ in terms of the radius of the balloon.

(d) Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t)\,dt$? Give a reason for your answer.

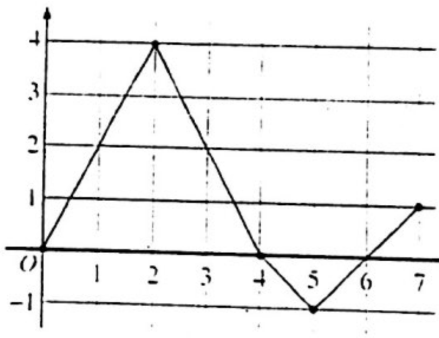
2. Let f be a function defined on the closed interval $[0,7]$. The graph of f, consisting of four line segments, is shown above. Let g be the function given by $g(x) = \int_2^x f(t)\,dt$.

(a) Find $g(3)$, $g'(3)$, and $g''(3)$.

(b) Find the average rate of change of g on the interval $0 \le x \le 3$.

(c) For how many values c, where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part (b)? Explain your reasoning.

(d) Find the x-coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.



Graph of f

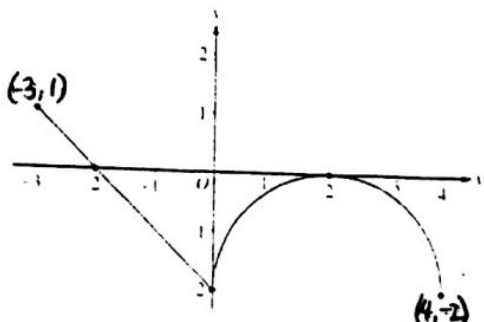
3. Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.

(a) On what intervals, if any, is f increasing? Justify your answer.

(b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

(c) Find an equation for the line tangent to the graph of f at the point $(0,3)$.

(d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.



Graph of f'