

## Geometry <br> Chapter 7

Circles, Arcs, Angles \& Segments


## Glossary for Chapter 7

Arc: An Unbroken part of a circle.
Central Angle: An angle whose vertex is at the center of a circle.
Chord: A line segment whose end points are on the circle.

Circle: Set of all points in a plane at a given distance (radius) from a given point (center).

Circumscribed
Circle: A circle is circumscribed about a polygon if each vertex of the polygon is a point of the circle. That makes each side of the polygon a chord of the circle.

Circumscribed
Polygon: A polygon is circumscribed about a circle if each side of the polygon is tangent to the circle.

Common
Tangents: A line tangent to each of two coplanar circles. If the tangents pass through the segment joining the centers of the circles then they are internal tangents. If they do not pass through the segment joining the centers of the circles then they are external tangents.

Concentric
Circles: Coplanar circles with the same center.
Diameter: The longest chord. A chord that contains the center of the circle.

Inscribed Angle: An angle whose vertex is on the circle and whose sides contain chords of the circle.

Inscribed
Polygon: A polygon is inscribed in a circle if each vertex of the polygon is a point of the circle. That makes each side of the polygon a chord of the circle.

Major Arc: An arc of greater than $180^{\circ}$. You must use three letters to to name a major. Ex. SRT starts at $S$ passes through $R$ and ends at T .

Minor Arc: An arc of less than $180^{\circ}$.
Protractor: A tool used to measure angles.

Secant: A line that contains a chord.

Tangent Line: A line in the same plane of a circle and it contains only one point of the circle.

$\overleftrightarrow{R P}$ is tangent to Circle $H$ at $P$
$\overleftrightarrow{R P}$ is tangent to Circle J at $R$

Tangent Circles: Two coplanar circles that intersect in only one point.


## Different Types of Tangents

Common tangent lines are internal if they intersect the line segment joining the centers and they are external if they do not.


Two internally tangent circles with
a common external tangent



## Central Angle


$\angle 1=$

Angle formed by Two Chords

$\angle 1=$

Angle formed by Two Secants

$\angle 1=$

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|  | Glossary |  |  |  |  |

I hope you realize there is a strong relationship between angles and circles. A circle is the tool used to measure angles. A circle is broken in 360 equal pieces that are called degrees. To measure an angle place the vertex at the center of the circle, the number of degrees on the circle included by the angle is the measure of the angle. Usually the circle is just half of a circle. Half of a circle is known as a semi circle. When the degree marks are placed on the semi circle, the tool is known as a protractor.

$\qquad$ $\mathrm{m} \angle \mathrm{BAD}=$ $\qquad$
$\qquad$
$\mathrm{m} \angle \mathrm{DAC}=$ $\qquad$ $m \angle E A D=$ $\qquad$
$\mathrm{m} \angle E A C=$ $\qquad$
$\mathrm{m} \angle \mathrm{GAC}=$ $\qquad$
$\mathrm{m} \angle \mathrm{GAD}=$ $\qquad$
$\mathrm{m} \angle \mathrm{GAE}=$ $\qquad$

The angles above are known as central angles. Central angles are angles whose vertex is at the center of the circle. The portion of the circle included between the sides is the included arc. We will move the vertex away from the center and discover how the measure of the angle and the measure of the included arc are related.

Note:
A minor arc is less than $180^{\circ}$; a major arc is greater than $180^{\circ}$, you must use three points to name a major arc. When only two points are used to name an arc it must be a minor arc but a minor arc can be named using three points. In the figure below $\overparen{H K}$ is a minor arc but it can also be called $\overparen{H J K}$.


Given: $\odot P$ with $\widetilde{\mathrm{JK}}=50^{\circ}$ \& diameter $\overline{\mathrm{NJ}}$
Find the measure of each arc:

1) $\mathrm{HJ}=$ $\qquad$
2) $\overrightarrow{\mathrm{HK}}=$ $\qquad$
3) $\mathrm{HNK}=$ $\qquad$
4) $\mathrm{HNJ}=$ $\qquad$
5) $\widehat{\mathrm{NHJ}}=$ $\qquad$
6) $\sqrt{\mathrm{NKJ}}=$ $\qquad$
7) $\widehat{\mathrm{JKH}}=$ $\qquad$
8) $\sqrt{N K}=$ $\qquad$
Using the letters in the above diagram, name:
9) 2 equal central angles $\qquad$
10) 2 equal minor arcs $\qquad$
11) Any 2 major arcs $\qquad$

Given: $\odot 0$ with $\widehat{\mathrm{RS}}=70^{\circ} \& \widehat{\mathrm{ST}}=80^{\circ}$

13) $\angle \mathrm{ROS}=$ $\qquad$ 14) $\angle \mathrm{TOS}=$ $\qquad$ 15) $\angle \mathrm{ROT}=$ $\qquad$
Find the measures:

R
16) $\overparen{\mathrm{RT}}=$ $\qquad$
17) $\widehat{\mathrm{RXT}}=$ $\qquad$
18) $\overparen{T X S}=$ $\qquad$

19) Given: $\odot O$ with diameters $\overline{U V} \& \overline{\mathrm{WT}}$; $\widetilde{U T}=$ $\qquad$
20) Given: $\odot \mathrm{L}$ with diameters $\overline{\mathrm{JH}} \& \overline{\mathrm{KI}} ; \widehat{\mathrm{KH}}=$ $\qquad$


B 21) Given: $\odot W$ with diameter $\overline{A B}$; $\widehat{A C}=$

The first move we will make with the vertex is from the center to the circle. This type of angle is known as an inscribed angle. Before we move the vertex, do the problems concerning central angles.

Given: $O A$ with $\widehat{B C}=56^{\circ} \& \angle C A D=78^{\circ} 0$
Find:

$$
\angle \mathrm{CAB}=
$$

$$
\overparen{D C}=
$$

$\qquad$
$\widehat{B E D}=$ $\qquad$ $\widehat{\mathrm{BEC}}=$ $\qquad$


$$
\widetilde{C E D}=
$$

Now let's move the vertex to a point on the circle, an inscribed angle. Try to use what you already know to turn something you don't know into something you do know. Count on what you know not on what you don't know.


Given: $\odot \mathrm{A}$ with $\widehat{B C}=64^{\circ}$

Find: $\angle C D B=$ $\qquad$

Try your best. If you need a hint, I will draw one line segment that may help. $\rightarrow$


Now use your knowledge about central angles, supplementary angles and triangles to find the answer.

Given: $O A$ with $\widehat{B C}=64^{\circ}$

Find: $\angle C D B=$

## Inscribed Angles

> An inscribed angle is formed by two rays that contain chords with a common end point.
> The vertex of the angle is on the circle.
> An inscribed angle is equal to one half its intercepted arc.
$\angle B A C$ is an inscribed angle. Its intercepted arc is $B C$.
$\angle 1=\frac{1}{2} \widehat{B C}$


Proof:


We will say $\overline{\mathrm{AC}}$ is a diameter, this will help us prove the theorem even when $\overline{\mathrm{AC}}$ is not a $C$ diameter. An auxiliary line from $B$ to the center should lead you to the proof.

What happens if one of the sides of an inscribed angle does not pass through the center?

Answer:
It still equals one half its intercepted arc. See if you can prove this with the help of the following illustrations.
$\angle \mathrm{BAC}$ is an inscribed angle with both sides on the same side of diameter $\overline{A D}$. Prove it is one half of $\widehat{B C}$.

$\angle B A C$ is an inscribed angle with sides on either side of diameter $\overline{A D}$. Prove it is one half of $\widehat{B C}$.

So no matter what an inscribed angle is equal to one half the intercepted arc.
> This type of angle is formed by a tangent and a chord with one end point the point of tangency.
> The vertex of the angle is on the circle, just like an inscribed angle.
> An angle formed by a tangent and a chord is equal to one half its intercepted arc, just like an inscribed angle.


1) $\overparen{B C}=42^{\circ}, \quad \angle 1=$ $\qquad$
2) $\widehat{B C}=63^{\circ}, \angle 1=$ $\qquad$
3) $\angle 1=42^{\circ}, \widehat{B C}=$ $\qquad$
4) $\angle 1=37^{\circ}, \widehat{B C}=$ $\qquad$
5) $\widehat{B C}=\mathrm{p}^{\circ}, \angle 1=$ $\qquad$
6) $\angle 1=h^{\circ}, \overparen{B C}=$ $\qquad$


Problems 1 through 10
7) $\angle 1=(7 x+3)^{\circ}, \overparen{B C}=(15 x+1)^{\circ}, x=$ $\qquad$
8) $\widehat{\mathrm{BAC}}=242^{\circ}, \angle 1=$ $\qquad$
9) $\widehat{\mathrm{BAC}}=\mathrm{s}^{\circ}, \angle 1=$ $\qquad$
10) $\angle 1=\mathrm{p}^{\circ}, \widehat{\mathrm{BAC}}=$ $\qquad$

11) $\angle 1=24^{\circ}, \widehat{B D}=124^{\circ}, \angle 2=$ $\qquad$
12) $\angle \mathrm{BAD}=62^{\circ}, \widehat{\mathrm{BC}}=24^{\circ}, \angle 2=$ $\qquad$

Problems 11 through 14
14) $\angle 1=(x+5)^{\circ}, \angle 2=(3 x-8) \circ, \widehat{B D}=(9 x-16)^{\circ}$
$\qquad$

Given: $\odot \bigcirc$, diameter $\overline{A C}$, tangent $\overleftrightarrow{A D}$ and $\overparen{B C}=42^{\circ}$

Find: All the numbered angles.

$$
\angle 1=
$$

$\angle 2=$ $\qquad$
$\angle 3=$ $\qquad$
$\angle 4=$ $\qquad$
$\qquad$ $\angle 6=$ $\qquad$
$\angle 7=$ $\qquad$
$\angle 8=$ $\qquad$


What will be true of the opposite angles of an inscribed quadrilateral?


Given: $\overline{\mathrm{AB}}$ is a diameter


1) $\angle 2=16^{\circ}, \widehat{B D}=85^{\circ}, \angle 1=$ $\qquad$
2) $\angle \mathrm{BAD}=58^{\circ}, \widehat{\mathrm{BC}}=40^{\circ}, \angle 1=$ $\qquad$
3) $\widehat{B C}: \widehat{B D}=3: 4, \angle 2=15^{\circ}, \angle 1=$ $\qquad$

Given: $\overleftrightarrow{X Y}$ is a tangent
4) $\angle 3=44^{\circ}, \widehat{\mathrm{AB}}=104^{\circ}$

$$
\angle 2=
$$

$\angle \mathrm{A}=$ $\qquad$
$\angle B=$ $\qquad$
$\widehat{B C}=$ $\qquad$

5) $\angle \mathrm{A}=56^{\circ}, \angle \mathrm{B}=62^{\circ}$
$\angle 1=$ $\qquad$
$\angle 2=$ $\qquad$
$\angle 3=$ $\qquad$
$\widehat{A B}=$ $\qquad$

Given: ©O; diameter $\overline{\mathrm{AB}}$; tangent $\overleftrightarrow{\mathrm{ZY}}$

$$
\widehat{B Y}=42^{\circ}, \widehat{B X}=92^{\circ}
$$

Find: All the numbered angles.
$\angle 1=$
$\angle 2=$ $\qquad$
$\angle 3=$ $\qquad$
$\angle 4=$ $\qquad$
$\angle 5=$ $\qquad$
$\qquad$
$\angle 10=$ $\qquad$
$\angle 7=$ $\qquad$
$\angle 8=$ $\qquad$
$\qquad$
Z

$\angle 11=$ $\qquad$


What is true about all the numbered angles in the illustration at the left?

If $A$ is the center of the circle and $\overline{C D}$ is a diameter then


If $C E=5 \mathrm{in}$. and $E D=12 \mathrm{in}$. what is the radius of $\odot A$ ?

Given: $\odot 0 ; \overparen{\mathrm{ST}}=120^{\circ} ; \widehat{\mathrm{RS}}=80^{\circ}$
Find:

1) $\angle \mathrm{ROS}=$ $\qquad$ 2) $\angle T O S=$ $\qquad$
2) $\angle \mathrm{ROT}=$ $\qquad$
3) $\widehat{\mathrm{RST}}=$ $\qquad$
4) $\overparen{\mathrm{RT}}=$ $\qquad$ 6) $\widehat{\mathrm{RSX}}=$ $\qquad$

5) $\widehat{\mathrm{BC}}=70^{\circ}, \angle 1=$ $\qquad$
6) $\widehat{B C}=d^{\circ}, \angle 1=$ $\qquad$

C 9) $\angle 1=39^{\circ}, \widehat{\mathrm{BC}}=$ $\qquad$
10) $\angle 1=r^{\circ}, \widehat{B C}=$ $\qquad$
11) $\angle 1=(2 \mathrm{x}-4)^{\circ}, \widehat{\mathrm{BC}}=(3 \mathrm{x}+11)^{\circ}, \mathrm{x}=$ $\qquad$
12) $\angle 1=(3 \mathrm{x}+20)^{\circ}, \widehat{\mathrm{BC}}=(11 \mathrm{x}-15)^{\circ}, \angle 1=$ $\qquad$
13) Given: ๑O, diameter $\overline{\mathrm{AC}}$, tangent $\overleftrightarrow{A D}$ and $\overleftrightarrow{\mathrm{BC}}=74^{\circ}$

Find: All the numbered angles.
$\angle 1=$ $\qquad$

$$
\angle 2=
$$

$\angle 3=$ $\qquad$ $\angle 4=$ $\qquad$
$\angle 5=$ $\qquad$

$$
\angle 6=
$$

$\angle 7=$ $\qquad$
$\angle 8=$ $\qquad$

14) Given: $\odot 0$, tangent $\overleftrightarrow{X Y}, \angle 1=40^{\circ}$ and $\overparen{B C}=130^{\circ}$

Find:

$$
\begin{array}{ll}
\overparen{\mathrm{AC}}= & \angle 2= \\
\angle 3= \\
\angle 5= & \angle 4= \\
& \widehat{\mathrm{ABC}}=
\end{array}
$$


15) Point $A$ lies on $\odot O$. How many chords contain point $A$ ? $\qquad$

How many diameters contain point $A$ ? $\qquad$

Given: $\odot$; Given: $\odot$; $\widehat{T S}=70^{\circ} ; \widehat{T U}=115^{\circ}$

Find the measures:

1) $\angle \mathrm{TRU}=$ $\qquad$ 2) $\angle \mathrm{SRU}=$

2) $\widehat{\mathrm{GFE}}=250^{\circ}, \angle 1=$ $\qquad$
3) Given: $\odot P$, diameter $\overline{W V}$, tangent $\overleftrightarrow{T W}$ and $\overparen{W E}=140^{\circ}$

Find: All the numbered angles.

$$
\begin{aligned}
& \angle 1=\square \\
& \angle 3= \\
& \angle 5= \\
& \angle 4=
\end{aligned}
$$


8) Given: $\odot$ P, tangent $\overleftrightarrow{T W}, \angle 2=70^{\circ}$ and $\overparen{\mathrm{SK}}=80^{\circ}$

Find:
$\widehat{\mathrm{SW}}=\square$
$\angle 3=$
$\angle 5=$
$\angle 1=$ $\qquad$
$\angle 5=$

$$
\widehat{W S K}=
$$

$\qquad$


So far, we have examined angles whose vertex is at the center of a circle and on the circle. We will now examine angles whose vertex is inside the circle.
> The angle is formed by intersecting chords. (That's how it got its name.)
> The vertex will be a point in the interior of the circle.
> The angle will equal one half the sum of the intercepted arcs.


Proof:
An auxiliary line from C to D should lead you to the proof.


Note: This rule still works if the vertex is at the center and it's very easy to prove it does.

In the illustration E is now the center of the circle.

## Intersecting Chords Worksheet

1) $\widehat{\mathrm{BC}}=24^{\circ}, \widehat{\mathrm{AD}}=52^{\circ}, \angle 1=$
2) $\widehat{\mathrm{BC}}=37^{\circ}, \widehat{\mathrm{AD}}=64^{\circ}, \angle 1=$ $\qquad$
3) $\angle 1=42^{\circ}, \widehat{A D}=67^{\circ}, \widehat{B C}=$ $\qquad$

4) $\angle 1=37^{\circ}, \widehat{B C}=16^{\circ}, \widehat{A D}=$ $\qquad$
5) $\widehat{B C}=\mathrm{p}^{\circ}, \widehat{\mathrm{AD}}=\mathrm{g}^{\circ}, \angle 1=$ $\qquad$

6) $\angle 1=s^{\circ}, \widehat{A D}=r^{\circ}, \widehat{B C}=$ $\qquad$

B



7) $\angle 1=a^{\circ}, \widehat{B C}=b^{\circ}, \widehat{A D}=$
8) $\overparen{A B}=106^{\circ}, \overparen{C D}=120^{\circ}, \angle 1=$ $\qquad$

9) $\angle 1=43^{\circ}, \widehat{A B}=124^{\circ}, \widehat{C D}=$ $\qquad$
10) $\angle 1=x^{\circ}, \widehat{C D}=y^{\circ}, \widehat{A B}=$ $\qquad$

11) $\angle 1=(4 x+3)^{\circ}, \widehat{A D}=(5 x+4)^{\circ}, \widehat{B C}=(4 x-6)^{\circ}, x=$ $\qquad$
12) $\angle 1=(4 x-8)^{\circ}, \widehat{A B}=(10 x-10)^{\circ}, \overparen{C D}=(12 x+26)^{\circ}, x=$ $\qquad$


13) $\widehat{\mathrm{BC}}=35.5^{\circ}, \widehat{\mathrm{AD}}=65.5^{\circ}, \angle 1=$ $\qquad$
14) $\widehat{B C}=41 \frac{1}{2}^{\circ}, \widehat{A D}=62 \frac{3}{4}^{\circ}, \angle 1=$ $\qquad$

15) $\angle 1=48^{\circ}, \widehat{A D}=72^{\circ}, \widehat{B C}=$ $\qquad$
16) $\angle 1=36.7^{\circ}, \widehat{B C}=19.3^{\circ}, \widehat{A D}=$ $\qquad$

17) $\angle 1=6 y^{\circ}, \widehat{A D}=3 y^{\circ}, \widehat{B C}=$

18) $\angle 1=\frac{5 x}{2}, \overparen{B C}=3 x^{\circ}, \overparen{A D}=$
19) $\widehat{\mathrm{AB}}=110^{\circ}, \mathrm{CD}=132^{\circ}, \angle 1=$ $\qquad$

20) $\angle 1=48^{\circ}, \widehat{\mathrm{AB}}=136^{\circ}, \widehat{\mathrm{CD}}=$

21) $\angle 1=6 x^{\circ}, \overparen{C D}=2 x^{\circ}, \overparen{A B}=$ $\qquad$

22) $\angle 1=(5 x-8)^{\circ}, \widehat{A D}=(5 x+2)^{\circ}, \widehat{B C}=(4 x-4)^{\circ}, x=$ $\qquad$
23) $\angle 1=(3 x-2)^{\circ}, \widehat{A B}=(16 x+2)^{\circ}, \overparen{C D}=(5 x+11)^{\circ}, x=$ $\qquad$


It is time to move the vertex again. We will now examine angles whose vertex is outside the circle.
$>$ The angle is formed by intersecting secants. (I think you may have guessed that.)
$>$ The vertex of the angle will be a point of the exterior of the circle. (outside the circle)
$>$ The angle will equal one half the difference of the intercepted arcs.
$\angle B E C$ is an angle formed by intersecting secants, its intercepted arcs are $\overparen{B C} \& \overparen{A D}$.
$\angle 1=\frac{1}{2}(\overparen{B C}-\overparen{A D})$


Proof:
An auxiliary line from B to D should lead you to the proof.


## Vertex Outside the Circle

Angle formed by a Secant \& Tangent
$>$ The angle is formed by a secant and a tangent.
> The vertex of the angle will be a point of the exterior of the circle. (outside the circle)
> The angle will equal one half the difference of the intercepted arcs.


## Angle formed by Two Tangents

> The angle is formed by two intersecting tangents.
> The vertex of the angle will be a point of the exterior of the circle. (outside the circle)
> The angle will equal one half the difference of the intercepted arcs.


## Vertex of Angle Outside the Circle Worksheet

1) $\widehat{B C}=150^{\circ}, \widehat{E D}=52^{\circ} ; \angle 1=$
2) $\overparen{A B}=85^{\circ}$; $\widehat{A E}=32^{\circ}$,

$$
\angle 2=
$$

$\qquad$
3) $\angle 1=42^{\circ}, \widehat{E D}=67^{\circ} ; \widehat{B C}=$
4) $\angle 2=15^{\circ}, \widehat{\mathrm{AB}}=58^{\circ}$; $\widehat{\mathrm{AE}}=$ $\qquad$
5) $\widehat{B C}=\mathrm{p}^{\circ}, \overrightarrow{E D}=\mathrm{g}^{\circ} ; \angle 1=$ $\qquad$ D
A
)
$\qquad$
$\qquad$
$\qquad$
6) $\angle 1=s^{\circ}, \overparen{E D}=h^{\circ}$; $\widehat{B C}=$ $\qquad$
7) $\angle 2=13^{\circ}, \widehat{A E}=28^{\circ}, \widehat{A C}=150^{\circ}, \widehat{E D}=32^{\circ} ; \angle 1=$ $\qquad$
8) $\angle 1=(2 x-1)^{\circ}, \overparen{E D}=(x+12)^{\circ}, \widehat{B C}=(3 x+46)^{\circ}$; $x$ $\qquad$ $\& \angle 1=$ $\qquad$
9) $\angle 2=15^{\circ}, \widehat{\mathrm{AB}}: \widehat{\mathrm{AE}}=5: 2$; $\widehat{\mathrm{AE}}=$ $\qquad$ -
$\overrightarrow{D C} \& \overrightarrow{D A}$ are tangents
10) $\overparen{\mathrm{AC}}=105^{\circ} ; \angle 1=$ $\qquad$ D $<1$
11) $\widehat{\mathrm{ABC}}=238^{\circ} ; \angle 1=$ $\qquad$
12) $\angle 1=42^{\circ}$; $\overparen{A C}=$ $\qquad$
13) $\angle 1=62^{\circ}$; $\widehat{\mathrm{ABC}}=$ $\qquad$
14) $\angle 1=x^{\circ}$; $\widehat{A C}=$ $\qquad$
15) $\widehat{\mathrm{ABC}}: \widehat{\mathrm{AC}}=3: 2 ; \angle 1=$

1) $\angle 1=22^{\circ}, \widehat{A D}=57^{\circ}$,

$$
\angle 2=
$$

$\qquad$
2) $\angle 2=63^{\circ}, \widehat{B C}=93^{\circ}$,

$$
\angle 1=
$$

$\qquad$

3) $\angle 1=30^{\circ}, \angle 2=80^{\circ}, \widehat{\mathrm{AD}}=$ $\qquad$ \& $\overparen{B C}=$
4) $\angle 1=a^{\circ}, \angle 2=b^{\circ}, \widehat{A D}=$ $\qquad$ $\& \overrightarrow{B C}=$ $\qquad$


Wait till you see this! $\boldsymbol{\rightarrow}$




Given: $\odot 0$, diameter $\overline{\mathrm{BE}}$, tangents $\overrightarrow{\mathrm{TS}} \& \overleftrightarrow{\mathrm{DS}}$,

$$
\widehat{B C}=52^{\circ}, \overparen{D E}=48^{\circ}, \overparen{\mathrm{AB}}=28^{\circ}, \widehat{\mathrm{AF}}=134^{\circ}
$$

Find all numbered angles. (The whole nine yards.)


## A Little More Practice

Given: $\odot \bigcirc$ with diameter $\overline{\mathrm{BE}} ; \overrightarrow{\mathrm{TC}} \& \overleftrightarrow{\mathrm{DS}}$ are tangents;
$\widehat{E F}=10^{\circ} ; \widehat{B A}=30^{\circ}$;
$\widehat{B C}=50^{\circ} ; \overparen{D E}=60^{\circ}$;

$T$
8 B
$\qquad$ $\angle 2=$ $\qquad$ $\angle 3=$ $\qquad$
$\angle 4=$ $\qquad$
$\qquad$ $\angle 6=$ $\qquad$
$\qquad$ $\angle 8=$ $\qquad$ $\angle 9=$ $\qquad$
$\angle 10=$ $\qquad$ $\angle 11=$ $\qquad$
12) $\widehat{\mathrm{AD}}=46^{\circ}$; $\widehat{\mathrm{BC}}=22^{\circ} ; \angle 1=$ $\qquad$

13) $\widehat{B C}=14^{\circ} ; \angle 1=28^{\circ} ; \widehat{A D}=$ $\qquad$

14) $\overline{\mathrm{AD}}$ is a diameter; $\angle 1=10^{\circ} ; \mathrm{DC}=100^{\circ}$;

15) $\angle 1=(3 x-1)^{\circ}$, $\widehat{R S}=(5 x+11)^{\circ}$
$\angle 1=$ $\qquad$
$\widehat{B C}=$
$\angle 2=$ $\qquad$
16) $\widehat{T U}=105^{\circ} ; \sqrt{W W}=57^{\circ}$

$$
\angle 1=
$$

$\qquad$

17) $\angle 1=31^{\circ} ; \widehat{T U}=106^{\circ}$

$$
\widetilde{W W}=
$$

$\qquad$
18) $\angle 1=x^{\circ} ; \sqrt{W W}=y^{\circ}$
$T \mathbb{T}=$ $\qquad$
$\overrightarrow{B C} \& \overrightarrow{B A}$ are tangents
19) $\widehat{\mathrm{ADC}}=275^{\circ}$

$$
\angle 1=
$$

$\qquad$


## Practice for Test

Given: $\odot$ O; $\overline{\mathrm{EB}}$ is a diameter; $\widehat{\mathrm{AB}}=48^{\circ} ; \widehat{\mathrm{AF}}=106^{\circ} ; \overparen{\mathrm{DC}}=50^{\circ} ; \widehat{\mathrm{BC}}=60^{\circ}$


Find all numbered angles.
$\qquad$ $\angle 2=$ $\qquad$
$\angle 3=$ $\qquad$
$\angle 4=$ $\qquad$ $\angle 5=$ $\qquad$ $\angle 6=$ $\qquad$

Given: tangent $\overleftrightarrow{\mathrm{PJ}}, \angle 4=52^{\circ}$

$$
\text { and } \overparen{\mathrm{HK}}=112^{\circ}
$$

Find:

1) $\widehat{\mathrm{KJ}}=$ $\qquad$
2) $\angle 2=$ $\qquad$
3) $\angle 3=$ $\qquad$ 4) $\angle 1=$
4) $\angle 5=$ $\qquad$
5) $\widehat{\mathrm{JHK}}=$ $\qquad$


6) $\angle 1=32 ; ~ \widehat{K N}=13 ; ~ \widehat{M L}=$
7) $\angle 1=2 \mathrm{x}+8 ; \overparen{\mathrm{LM}}=10 \mathrm{x}-12 ; \overparen{\mathrm{KN}}=4 \mathrm{x}-6$;

$$
\mathrm{x}=\square \quad \angle 1=
$$

12) $\angle 1=42^{\circ} ; \widehat{\mathrm{KL}}=140$; $\widehat{\mathrm{NM}}=104$;
$\widehat{K N}=$ $\qquad$

Note:
A minor arc is less than $180^{\circ}$; a major arc is greater than $180^{\circ}$, you must use three points to name a major arc. When only two points are used to name an arc it must be a minor arc but a minor arc can be named using three points. In the figure below $\overparen{H K}$ is a minor arc but it can also be called $\overparen{H J K}$.

3) $\mathrm{HK}=140^{\circ}$
4) $\widehat{\mathrm{HNK}}=$
5) $\mathrm{HNJ}=270^{\circ}$
6) $\widehat{\mathrm{NHJ}}=$
7) $\widehat{N K J}=180^{\circ}$
8) $\widehat{\mathrm{JKH}}=$
9) $\overparen{N K}=130^{\circ}$

Using the letters in the above diagram, name:
10) 2 equal central angles
11) 2 equal minor arcs $\widehat{N H} \& \hat{H J}$
12) Any 2 major arcs

Given: $\odot 0$ with $\widehat{\mathrm{RS}}=70^{\circ} \& \widehat{\mathrm{ST}}=80^{\circ}$

13) $\angle \mathrm{ROS}=70^{\circ}$
14) $\angle \mathrm{TOS}=$
15) $\angle \mathrm{ROT}=150^{\circ}$
16) $\widehat{\mathrm{RT}}=$
17) $\widehat{\mathrm{RXT}}=210^{\circ}$
18) $\widehat{T X S}=$

19) Given: $\odot O$ with diameters $\overline{U V} \& \overline{W T}$; $\overparen{U T}=60^{\circ}$
20) Given: $\odot \mathrm{L}$ with diameters $\overline{\mathrm{JH}} \& \overline{\mathrm{KI}} ; \widehat{\mathrm{KH}}=$

$B$ 21) Given: $\odot W$ with diameter $\overline{A B} ; \overparen{A C}=133^{\circ}$

The first move we will make with the vertex is from the center to the circle. This type of angle is known as an inscribed angle. Before we move the vertex, do the problems concerning central angles.

Given: $\bigcirc$ A with $\widehat{B C}=56^{\circ} \& \angle C A D=78^{\circ}$
Find:

$$
\begin{array}{ll}
\angle \mathrm{CAB}=56^{\circ} & \overparen{\mathrm{DC}}= \\
\overparen{\mathrm{DB}}=134^{\circ} & \angle \mathrm{DAB}= \\
\widehat{\mathrm{BED}}=226^{\circ} & \overparen{\mathrm{BEC}}=
\end{array}
$$



$$
\overparen{C E D}=282^{\circ}
$$

Now let's move the vertex to a point on the circle, an inscribed angle. Try to use what you already know to turn something you don't know into something you do know. Count on what you know not on what you don't know.


Given: $\odot \mathrm{A}$ with $\widehat{B C}=64^{\circ}$

Find: $\angle C D B=$

Try your best. If you need a hint, I will draw one line segment that may help. $\rightarrow$


Now use your knowledge about central angles, supplementary angles and triangles to find the answer.

Given: $\odot A$ with $\widehat{B C}=64^{\circ}$

Find: $\angle C D B=$

1) $\widehat{\mathrm{BC}}=42^{\circ}, \angle 1=21^{\circ}$
2) $\widehat{\mathrm{BC}}=63^{\circ}, \angle 1=$
3) $\angle 1=42^{\circ}, \widehat{B C}=84^{\circ}$
4) $\angle 1=37^{\circ}, \widehat{B C}=$
5) $\overparen{\mathrm{BC}}=\mathrm{p}^{0}, \angle 1=\frac{\mathrm{p}^{0}}{2}$
6) $\angle 1=h^{\circ}, \widehat{B C}=$


Problems 1 through 10
7) $\angle 1=(7 x+3)^{\circ}, \widehat{B C}=(15 x+1)^{\circ}, x=5$
8) $\widehat{\mathrm{BAC}}=242^{\circ}, \angle 1=$
9) $\widehat{\mathrm{BAC}}=\mathrm{s}^{\circ}, \angle 1=\frac{(360-\mathrm{s})^{\circ}}{2}$
10) $\angle 1=\mathrm{p}^{\circ}, \widehat{\mathrm{BAC}}=$

11) $\angle 1=24^{\circ}, \widehat{B D}=124^{\circ}, \angle 2=38^{\circ}$
12) $\angle \mathrm{BAD}=62^{\circ}, \widehat{\mathrm{BC}}=24^{\circ}, \angle 2=$
D 13) $\widehat{\mathrm{BC}}: \widehat{\mathrm{BD}}=2: 5, \angle 1=14^{\circ}$,

$$
\angle 2=21^{\circ}
$$

Problems 11 through 14
14) $\angle 1=(x+5)^{\circ}, \angle 2=(3 x-8)^{\circ}, \widehat{B D}=(9 x-16)^{\circ}$

$$
x=
$$

Given: $\odot \bigcirc$, diameter $\overline{\mathrm{AC}}$, tangent $\overleftrightarrow{\mathrm{AD}}$ and $\widehat{B C}=42^{\circ}$

Find: All the numbered angles.
$\angle 1=90^{\circ}$
$\angle 2=$
$\angle 3=69^{\circ}$
$\angle 4=$
$\angle 5=42^{\circ}$
$\angle 6=$
$\angle 7=69^{\circ}$
$\angle 8=$

What will be true of the opposite angles of an inscribed quadrilateral?


What will be true of the arcs between parallel chords?

Given: $\overline{A B} \| \overline{C D}$

What is true about $\widehat{A D} \& \widehat{B C}$ and why?

Given: $\overline{\mathrm{AB}}$ is a diameter


1) $\angle 2=16^{\circ}, \widehat{B D}=85^{\circ}, \angle 1=26.5^{\circ}$
2) $\angle \mathrm{BAD}=58^{\circ}, \widehat{\mathrm{BC}}=40^{\circ}, \angle 1=$
3) $\widehat{B C}: \overparen{B D}=3: 4, \angle 2=15^{\circ}, \angle 1=5^{\circ}$

Given: $\overleftrightarrow{X Y}$ is a tangent
4) $\angle 3=44^{\circ}, \widehat{A B}=104^{\circ}$
$\angle 2=$
$\angle \mathrm{A}=44^{\circ}$
$\angle B=$
$\widehat{B C}=88^{\circ}$

5) $\angle \mathrm{A}=56^{\circ}, \angle \mathrm{B}=62^{\circ}$

$$
\angle 1=62^{\circ}
$$

$$
\angle 2=
$$

$\angle 3=56^{\circ}$
$\widehat{A B}=$


Given: ©O; diameter $\overline{\mathrm{AB}}$; tangent $\overleftrightarrow{\mathrm{ZY}}$

$$
\widehat{B Y}=42^{\circ}, \widehat{B X}=92^{\circ}
$$

Find: All the numbered angles.


What is true about all the numbered angles in the illustration at the left?

If $A$ is the center of the circle and $\overline{C D}$ is a diameter then
$\angle 1$ must be equal to
Any angle inscribed in a semi circle must
be a


If $C E=5 \mathrm{in}$. and $E D=12 \mathrm{in}$. what is the radius of $\odot A$ ? 6.5 in .

## Central \& Inscribed Angles Practice for Quiz

Given: $\odot \mathrm{O} ; \widehat{\mathrm{ST}}=120^{\circ} ; \widehat{\mathrm{RS}}=80^{\circ}$
Find:

1) $\angle \mathrm{ROS}=80^{\circ}$
2) $\angle T O S=$
3) $\angle \mathrm{ROT}=160^{\circ}$
4) $\overparen{\mathrm{RST}}=$
5) $\widehat{\mathrm{RT}}=160^{\circ}$
6) $\widehat{\mathrm{RSX}}=$

7) $\angle 1=(2 \mathrm{x}-4)^{\circ}, \widehat{\mathrm{BC}}=(3 \mathrm{x}+11)^{\circ}, \mathrm{x}=19$

$$
4 x-8=3 x+11 ; x-8=11
$$

12) $\angle 1=(3 \mathrm{x}+20)^{\circ}, \widehat{\mathrm{BC}}=(11 \mathrm{x}-15)^{\circ}, ~ \angle 1=$
13) Given: $\odot 0$, diameter $\overline{A C}$, tangent $\overleftrightarrow{A D}$ and $\overparen{B C}=74^{\circ}$

Find: All the numbered angles.
$\angle 1=90^{\circ}$
$\angle 2=$
$\angle 3=53^{\circ}$
$\angle 4=$
$\angle 5=74^{\circ}$
$\angle 6=$
$\angle 7=53^{\circ}$
$\angle 8=$


Given: $\odot$ R; $\widehat{T S}=70^{\circ} ; \widehat{T U}=115^{\circ}$
Find the measures:

1) $\angle \mathrm{TRU}=115^{\circ}$
2) $\angle \mathrm{SRU}=$
3) $\widehat{\mathrm{SUT}}=290^{\circ}$
4) $\widehat{\mathrm{STU}}=$

5) $\widehat{\mathrm{GFE}}=250^{\circ}, \angle 1=55^{\circ}$
6) $\angle 1=(3 x+21)^{\circ}, \widehat{E G}=(10 x-10)^{\circ}, \angle 1=$
7) Given: $\odot P$, diameter $\overline{W V}$, tangent $\overleftrightarrow{T W}$ and $\widetilde{W E}=140^{\circ}$

Find: All the numbered angles.

$$
\begin{array}{ll}
\angle 1=20^{\circ} & \angle 2= \\
\angle 3=70^{\circ} & \angle 4= \\
\angle 5=90^{\circ} &
\end{array}
$$


8) Given: $\odot P$, tangent $\overleftrightarrow{T W}, \angle 2=70^{\circ}$ and $\overparen{S K}=80^{\circ}$

Find:

$$
\begin{array}{ll}
\widehat{S W}=160^{\circ} & \angle 1= \\
\angle 3=70^{\circ} & \angle 4= \\
\angle 5=70^{\circ} & \overparen{W S K}=
\end{array}
$$



1) $\widehat{\mathrm{BC}}=24^{\circ}, \widehat{\mathrm{AD}}=52^{\circ}, \angle 1=38^{\circ}$
2) $\angle 1=42^{\circ}, \widehat{A D}=67^{\circ}, \widehat{B C}=17^{\circ}$
3) $\angle 1=37^{\circ}, \widehat{B C}=16^{\circ}, \widehat{A D}=$
4) $\overparen{B C}=\mathrm{p}^{\circ}, \overparen{\mathrm{AD}}=\mathrm{g}^{\circ}, \angle 1=\frac{\mathrm{p}^{\circ}+\mathrm{g}^{\circ}}{2}$

5) $\angle 1=s^{\circ}, \widehat{A D}=r^{\circ}, \widehat{B C}=$
6) $\angle 1=a^{\circ}, \widehat{B C}=b^{\circ}, \widehat{A D}=(2 a-b)^{\circ}$
7) $\overparen{\mathrm{AB}}=106^{\circ}, \widehat{\mathrm{CD}}=120^{\circ}, \angle 1=$
8) $\angle 1=43^{\circ}, \widehat{A B}=124^{\circ}, \widehat{C D}=150^{\circ}$

9) $\angle 1=(4 x+3)^{\circ}, \widehat{\mathrm{AD}}=(5 \mathrm{x}+4)^{\circ}, \widehat{\mathrm{BC}}=(4 \mathrm{x}-6)^{\circ}, \mathrm{x}=8$ $8 x+6=9 x-2 ; 6=x-2$
10) $\angle 1=(4 \mathrm{x}-8)^{\circ}, \widehat{\mathrm{AB}}=(10 \mathrm{x}-10)^{\circ}, \widehat{C D}=(12 \mathrm{x}+26)^{\circ}, \mathrm{x}=$

11) $\widehat{\mathrm{BC}}=35.5^{\circ}, \widehat{\mathrm{AD}}=65.5^{\circ}, \angle 1=50.5^{\circ}$
12) $\widehat{\mathrm{BC}}=41 \frac{1}{2}^{\circ}, \widehat{\mathrm{AD}}=62 \frac{3}{4}^{\circ}, \angle 1=$
13) $\angle 1=48^{\circ}, \widehat{\mathrm{AD}}=72^{\circ}, \widehat{\mathrm{BC}}=24^{\circ}$
14) $\angle 1=36.7^{\circ}, \widehat{B C}=19.3^{\circ}, \widehat{A D}=$
15) $\widehat{\mathrm{BC}}=6 \mathrm{x}^{\circ}, \widehat{\mathrm{AD}}=10 \mathrm{x}^{\circ}, \angle 1=8 \mathrm{x}^{\circ}$

16) $\angle 1=6 y^{\circ}, \widehat{A D}=3 y^{\circ}, \widehat{B C}=$
17) $\angle 1=\frac{5 x}{2}, \overparen{B C}=3 x^{\circ}, \overparen{A D}=2 x^{\circ}$
18) $\widehat{\mathrm{AB}}=110^{\circ}, \widehat{\mathrm{CD}}=132^{\circ}, \angle 1=$
19) $\angle 1=48^{\circ}, \widehat{A B}=136^{\circ}, \widehat{C D}=128^{\circ}$
20) $\angle 1=6 x^{\circ}, \overparen{C D}=2 x^{\circ}, \overparen{A B}=$

21) $\angle 1=(5 x-8)^{\circ}, \widehat{A D}=(5 x+2)^{\circ}, \widehat{B C}=(4 x-4)^{\circ}, x=14$ $10 x-16=9 x-2 ; x-16=-2$
22) $\angle 1=(3 x-2)^{\circ}, \widehat{A B}=(16 x+2)^{\circ}, \widehat{C D}=(5 x+11)^{\circ}, x=$

## Vertex of Angle Outside the Circle Worksheet

1) $\widehat{B C}=150^{\circ}, E D=52^{\circ} ; \angle 1=49^{\circ}$
2) $\widehat{A B}=85^{\circ} ; \widehat{A E}=32^{\circ}$,

$$
\angle 2=
$$


3) $\angle 1=42^{\circ}, \widehat{E D}=67^{\circ}$; $\widehat{B C}=151^{\circ}$
4) $\angle 2=15^{\circ}, \widehat{\mathrm{AB}}=58^{\circ}$; $\widehat{\mathrm{AE}}=$

5) $\overparen{B C}=\mathrm{p}^{\circ}, \overparen{E D}=\mathrm{g}^{\circ} ; \angle 1=\frac{\mathrm{p}^{\circ}-\mathrm{g}^{\circ}}{2}$
6) $\angle 1=s^{\circ}, \overparen{E D}=h^{\circ}$; $\overparen{B C}=$
7) $\angle 2=13^{\circ}, \widehat{\mathrm{AE}}=28^{\circ}, \widehat{\mathrm{AC}}=150^{\circ}, \widehat{\mathrm{ED}}=32^{\circ} ; \angle 1=32^{\circ}$
8) $\angle 1=(2 x-1)^{\circ}, \widehat{E D}=(x+12)^{\circ}, \widehat{B C}=(3 x+46)^{\circ} ; x=\& \angle 1=$
9) $\angle 2=15^{\circ}, \widehat{A B}: \widehat{\mathrm{AE}}=5: 2 ; \widehat{\mathrm{AE}}=20^{\circ}$ $5 x-2 x=30 ; 3 x=30 ; x=10^{2 x}$
$\overrightarrow{D C} \& \overrightarrow{D A}$ are tangents
10) $\overparen{\mathrm{AC}}=105^{\circ} ; \angle 1=$
11) $\widehat{\mathrm{ABC}}=238^{\circ} ; \angle 1=58^{\circ}$
12) $\angle 1=42^{\circ}$; $\overparen{A C}=$

13) $\angle 1=62^{\circ}$; $\widehat{\mathrm{ABC}}=242^{\circ}$
14) $\angle 1=x^{\circ}$; $\widehat{A C}=$
15) $\widehat{\mathrm{ABC}}: \widehat{\mathrm{AC}}=3: 2 ; \angle 1=108^{\circ}$
$3 x+2 x=180 ; 5 x=180 ; x=36 ; \overparen{A C}=72^{\circ}$

1) $\angle 1=22^{\circ}, \widehat{A D}=57^{\circ}$,

$$
\angle 2=79^{\circ}
$$

2) $\angle 2=63^{\circ}, \widehat{B C}=93^{\circ}$,
$\angle 1=$

3) $\angle 1=30^{\circ}, \angle 2=80^{\circ}, \widehat{\mathrm{AD}}=50^{\circ} \& \widehat{\mathrm{BC}}=110^{\circ}$
4) $\angle 1=a^{\circ}, \angle 2=b^{\circ}, \widehat{A D}=\& \widehat{B C}=$


Wait till you see this! $\rightarrow$


Given: $\odot$, diameter $\overline{\mathrm{BE}}$, tangents $\overrightarrow{\mathrm{TS}} \& \overleftrightarrow{\mathrm{DS}}$,

$$
\widehat{B C}=52^{\circ}, \widehat{\mathrm{DE}}=48^{\circ}, \widehat{\mathrm{AB}}=28^{\circ}, \widehat{\mathrm{AF}}=134^{\circ}
$$

Find all numbered angles. (The whole nine yards.)

$\angle 1=24^{\circ}$
$\angle 2=$
$\angle 3=14^{\circ}$
$\angle 4=$
$\angle 5=18^{\circ}$
$\angle 6=$
$\angle 7=19^{\circ}$
$\angle 8=$
$\angle 9=66^{\circ}$
$\angle 10=$
$\angle 11=100^{\circ}$

## A Little More Practice

Given: $\odot \bigcirc$ with diameter $\overline{\mathrm{BE}} ; \overrightarrow{\mathrm{TC}} \& \overleftrightarrow{\mathrm{DS}}$ are tangents;

$$
\widehat{E F}=10^{\circ} ; \widehat{B A}=30^{\circ} ;
$$

$$
\widehat{B C}=50^{\circ} ; \widehat{D E}=60^{\circ} ;
$$


13) $\widehat{\mathrm{BC}}=14^{\circ} ; \angle 1=28^{\circ} ; \widehat{\mathrm{AD}}=42^{\circ}$

14) $\overline{\mathrm{AD}}$ is a diameter; $\angle 1=10^{\circ} ; \overparen{\mathrm{DC}}=100^{\circ}$;

15) $\angle 1=(3 x-1)$; $\widehat{R S}=(5 x+11)^{\circ}$ $6 x-2=5 x+11 ; x-2=11 ; x=13$
$\angle 1=38^{\circ}$

$$
\begin{aligned}
& \overparen{B C}= \\
& \angle 2= \\
& \widehat{A D C}=
\end{aligned}
$$

16) $\overparen{T U}=105^{\circ} ; \sqrt{W W}=57^{\circ}$

$$
\angle 1=
$$


17) $\angle 1=31^{\circ} ; \overparen{T U}=106^{\circ}$

$$
\widehat{V W}=44^{\circ}
$$


18) $\angle 1=x^{\circ}$; $\widehat{V W}=y^{\circ}$

$$
\overparen{T U}=
$$

$\overrightarrow{B C} \& \overrightarrow{B A}$ are tangents
19) $\widehat{\mathrm{ADC}}=275^{\circ}$

$$
\angle 1=95^{\circ}
$$



Given: $\odot$ O; $\overline{\mathrm{EB}}$ is a diameter; $\widehat{\mathrm{AB}}=48^{\circ} ; \widehat{\mathrm{AF}}=106^{\circ} ; \overparen{\mathrm{DC}}=50^{\circ} ; \widehat{\mathrm{BC}}=60^{\circ}$


Find all numbered angles.
$\angle 1=$ $\qquad$ $\angle 2=$
$\angle 3=$ $\qquad$
$\angle 4=$
$\angle 5=$ $\qquad$


Given: tangent $\overleftrightarrow{\mathrm{PJ}}, \angle 4=52^{\circ}$

$$
\text { and } \overparen{\mathrm{HK}}=112^{\circ}
$$

Find:

1) $\widehat{\mathrm{KJ}}=$ $\qquad$
2) $\angle 2=$
3) $\angle 3=$ $\qquad$ 4) $\angle 1=$
4) $\angle 5=72^{\circ}$
5) $\widehat{\mathrm{HK}}=$


6) $\angle 1=23^{\circ} ; \overparen{\mathrm{RS}}=13^{\circ}$; $\overparen{\mathrm{QT}}=$ $\qquad$
7) $\angle 1=44 ; \overparen{\mathrm{RQ}}=103$; $\overparen{\mathrm{ST}}=$
8) $\angle 1=6 \mathrm{x}-1 ; \overparen{\mathrm{RS}}=5 \mathrm{x}-2 ; \overparen{\mathrm{QT}}=5 \mathrm{x}+12$;
$12 x-2=10 x+10 ; 2 x-2=10$

$$
x=6
$$

$$
\overparen{\mathrm{RS}}=28^{\circ}
$$

10) $\angle 1=32^{\circ} ; \widehat{\mathrm{KN}}=13^{\circ} ; \widehat{\mathrm{ML}}=$
11) $\angle 1=2 x+8 ; \overparen{L M}=10 x-12 ; \overparen{K N}=4 x-6$;
$4 x+16=10 x-12-(4 x-6) ; 4 x+16=6 x-6 ; 16=2 x-6$

$$
x=\frac{11}{} \quad \angle 1=30^{\circ}
$$

12) $\angle 1=42^{\circ} ; \widehat{\mathrm{KL}}=140^{\circ}$; $\widehat{\mathrm{NM}}=104^{\circ}$;

K
 1 N
$\widehat{K N}=$

