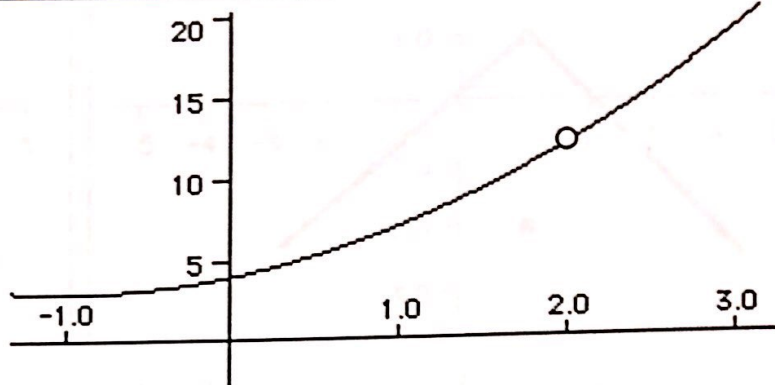


Graphical Approach to Limits - Classwork

Suppose you were to graph

$$f(x) = \frac{x^3 - 8}{x - 2}, \quad x \neq 2$$

For all values of x not equal to 2, you can use standard curve sketching techniques. But the curve is not defined at $x = 2$. There is a hole in the graph. So let's get an idea of the behavior of the curve around $x = 2$.



Set your calculator to 4 decimal accuracy and complete the chart.

x	1.75	1.9	1.99	1.999	2	2.001	2.01	2.1	2.25
$f(x)$									

It should be obvious that as x gets closer and closer to 2, the value of $f(x)$ becomes closer and closer to _____.

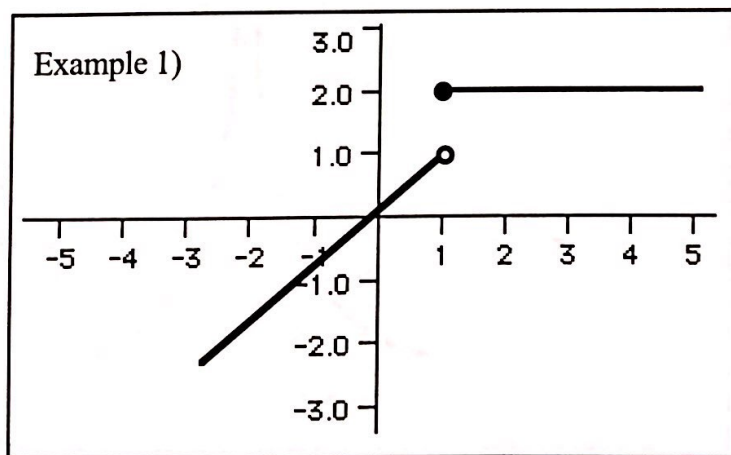
We will say that the **limit** of $f(x)$ as x approaches 2 is 12 and this is written as $\lim_{x \rightarrow 2} f(x) = 12$ or $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2} = 12$.

The informal definition of a limit is "what is happening to y as x gets close to a certain number." In order for a limit to exist, we must be approaching the same y -value as we approach some value c from either the left or the right side. If this does not happen, we say that the limit does not exist (DNE) as we approach c .

If we want the limit of $f(x)$ as we approach some value of c from the left hand side, we will write $\lim_{x \rightarrow c^-} f(x)$.

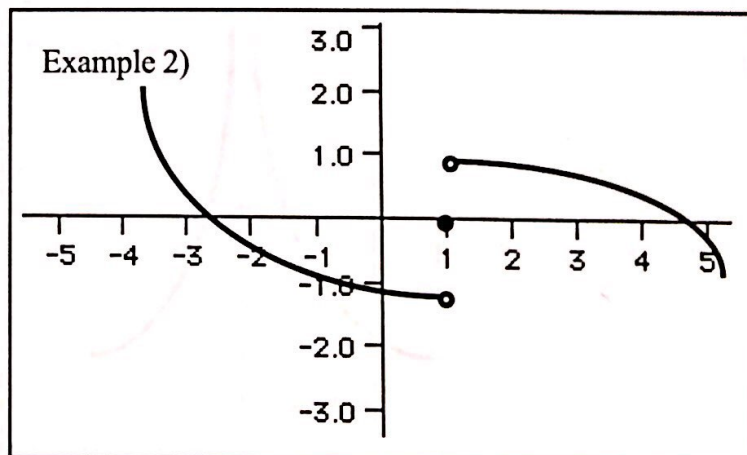
If we want the limit of $f(x)$ as we approach some value of c from the right hand side, we will write $\lim_{x \rightarrow c^+} f(x)$.

In order for a limit to exist at c , $\lim_{x \rightarrow c^-} f(x)$ must equal $\lim_{x \rightarrow c^+} f(x)$ and we say $\lim_{x \rightarrow c} f(x) = L$.



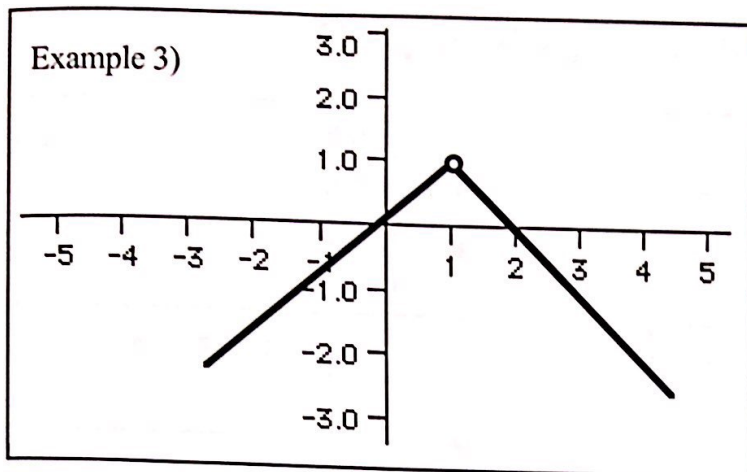
$$\lim_{x \rightarrow 1^-} f(x) = \text{---} \quad \lim_{x \rightarrow 1^+} f(x) = \text{---}$$

$$\lim_{x \rightarrow 1} f(x) = \text{---} \quad f(1) = \text{---}$$



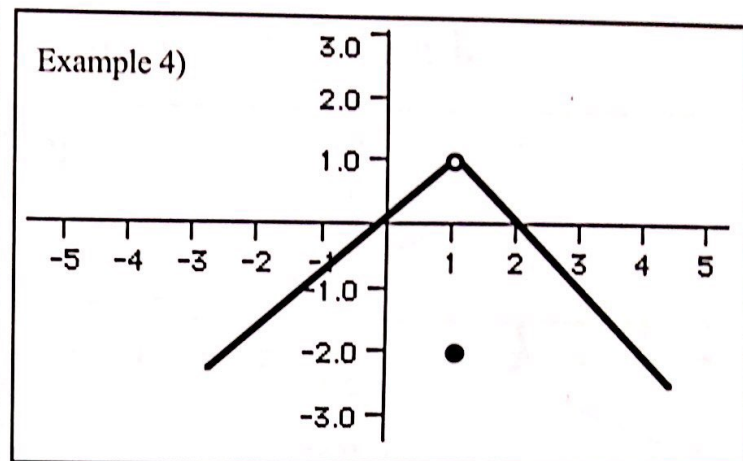
$$\lim_{x \rightarrow 1^-} f(x) = \text{---} \quad \lim_{x \rightarrow 1^+} f(x) = \text{---}$$

$$\lim_{x \rightarrow 1} f(x) = \text{---} \quad f(1) = \text{---}$$



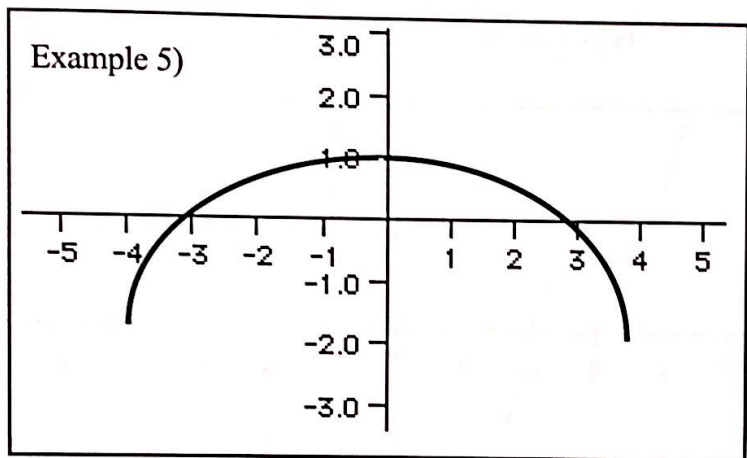
$$\lim_{x \rightarrow 1^-} f(x) = \text{---} \quad \lim_{x \rightarrow 1^+} f(x) = \text{---}$$

$$\lim_{x \rightarrow 1} f(x) = \text{---} \quad f(1) = \text{---}$$



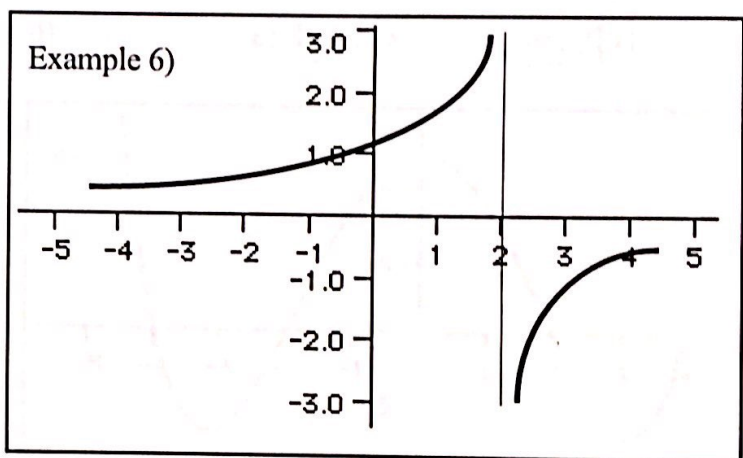
$$\lim_{x \rightarrow 1^-} f(x) = \text{---} \quad \lim_{x \rightarrow 1^+} f(x) = \text{---}$$

$$\lim_{x \rightarrow 1} f(x) = \text{---} \quad f(1) = \text{---}$$



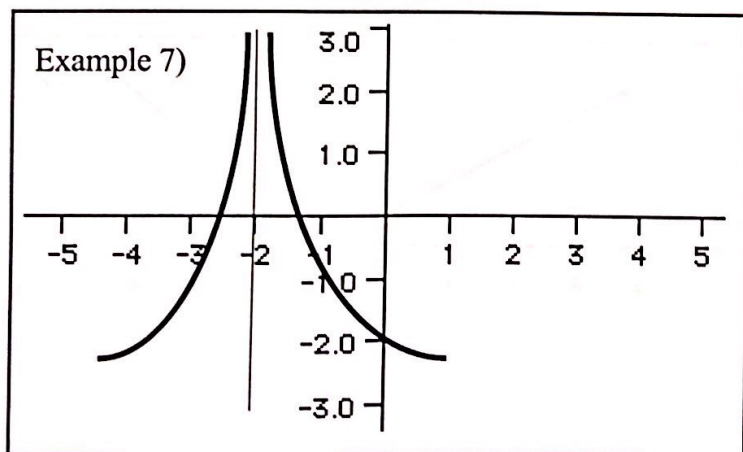
$$\lim_{x \rightarrow 0^-} f(x) = \text{---} \quad \lim_{x \rightarrow 0^+} f(x) = \text{---}$$

$$\lim_{x \rightarrow 0} f(x) = \text{---} \quad f(0) = \text{---}$$



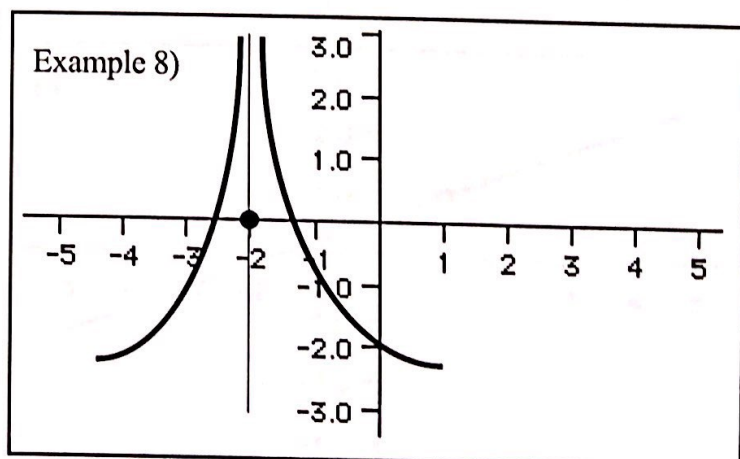
$$\lim_{x \rightarrow 2^-} f(x) = \text{---} \quad \lim_{x \rightarrow 2^+} f(x) = \text{---}$$

$$\lim_{x \rightarrow 2} f(x) = \text{---} \quad f(2) = \text{---}$$



$$\lim_{x \rightarrow -2^-} f(x) = \text{---} \quad \lim_{x \rightarrow -2^+} f(x) = \text{---}$$

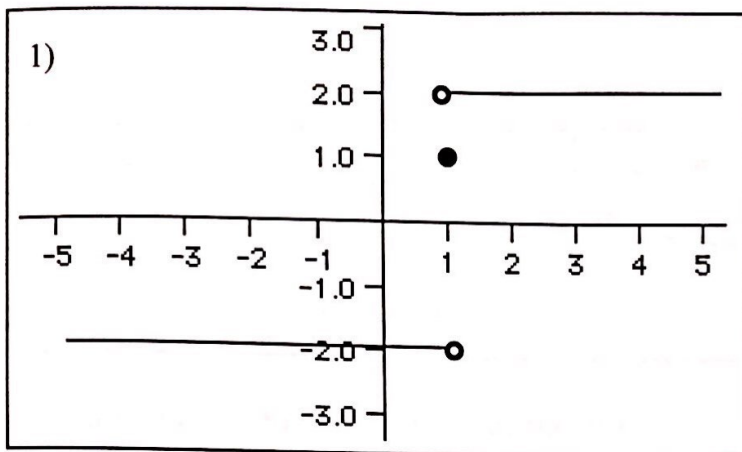
$$\lim_{x \rightarrow -2} f(x) = \text{---} \quad f(-2) = \text{---}$$



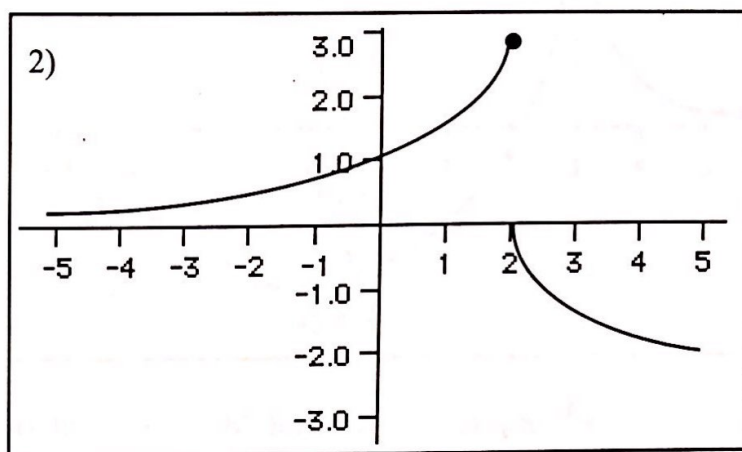
$$\lim_{x \rightarrow -2^-} f(x) = \text{---} \quad \lim_{x \rightarrow -2^+} f(x) = \text{---}$$

$$\lim_{x \rightarrow -2} f(x) = \text{---} \quad f(-2) = \text{---}$$

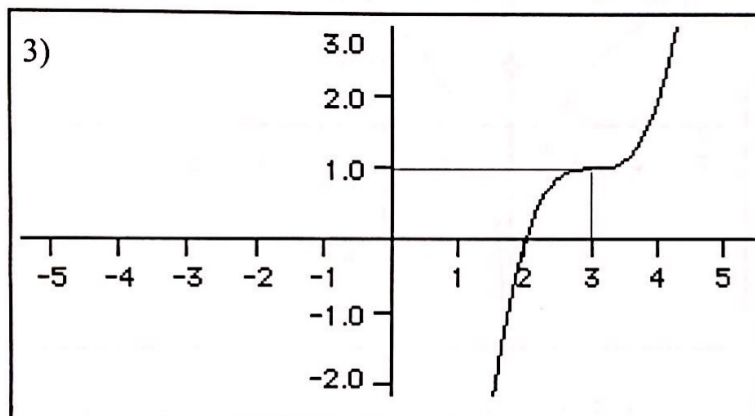
Graphical Approach to Limits - Homework



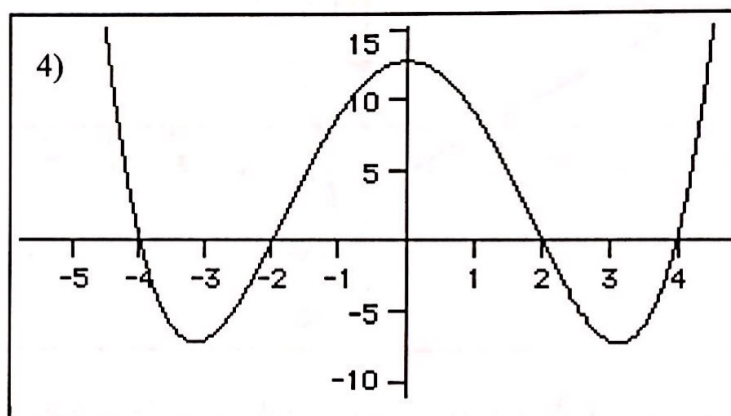
- a) $\lim_{x \rightarrow 1^-} f(x)$ b) $\lim_{x \rightarrow 1^+} f(x)$ c) $\lim_{x \rightarrow 1} f(x)$
 d) $f(1)$ e) $\lim_{x \rightarrow -\infty} f(x)$ f) $\lim_{x \rightarrow \infty} f(x)$



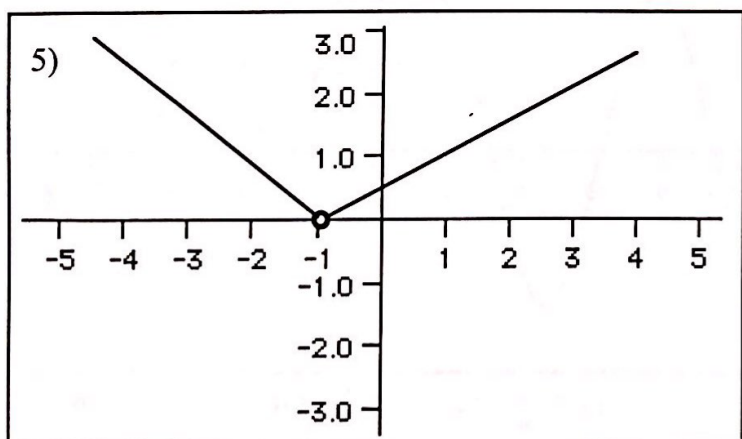
- a) $\lim_{x \rightarrow 2^-} f(x)$ b) $\lim_{x \rightarrow 2^+} f(x)$ c) $\lim_{x \rightarrow 2} f(x)$
 d) $f(2)$ e) $\lim_{x \rightarrow -\infty} f(x)$ f) $\lim_{x \rightarrow \infty} f(x)$



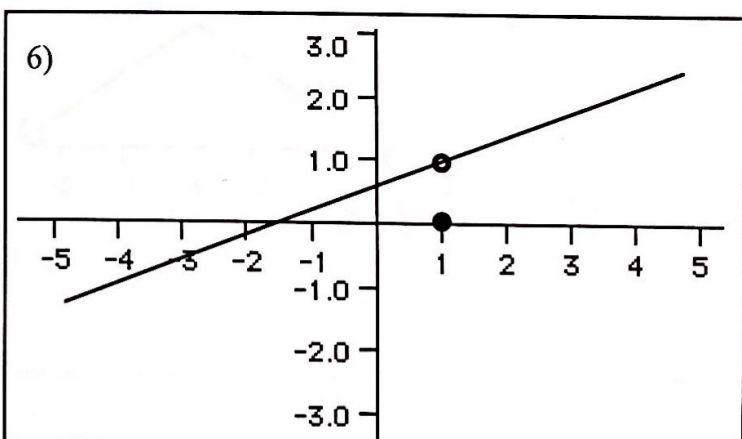
- a) $\lim_{x \rightarrow 3^-} f(x)$ b) $\lim_{x \rightarrow 3^+} f(x)$ c) $\lim_{x \rightarrow 3} f(x)$
 d) $f(3)$ e) $\lim_{x \rightarrow -\infty} f(x)$ f) $\lim_{x \rightarrow \infty} f(x)$



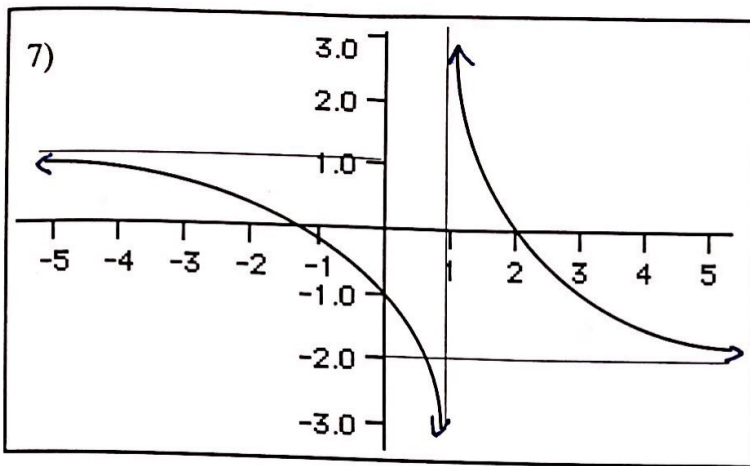
- a) $\lim_{x \rightarrow 0^-} f(x)$ b) $\lim_{x \rightarrow 0^+} f(x)$ c) $\lim_{x \rightarrow 0} f(x)$
 d) $f(0)$ e) $\lim_{x \rightarrow -\infty} f(x)$ f) $\lim_{x \rightarrow \infty} f(x)$



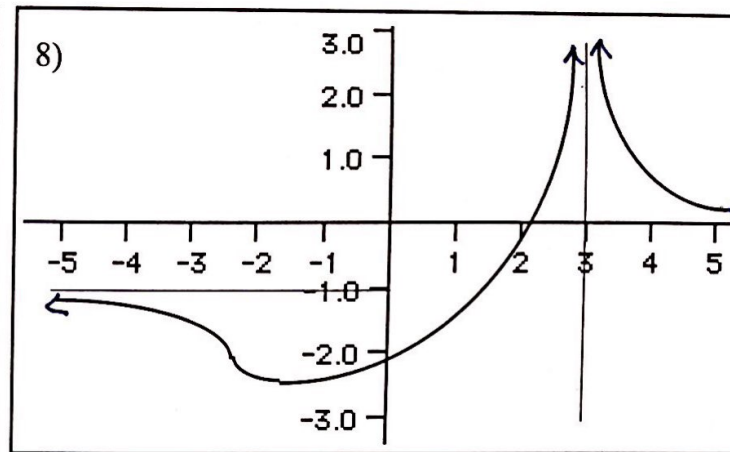
- a) $\lim_{x \rightarrow -1^-} f(x)$ b) $\lim_{x \rightarrow -1^+} f(x)$ c) $\lim_{x \rightarrow -1} f(x)$
 d) $f(-1)$ e) $\lim_{x \rightarrow -\infty} f(x)$ f) $\lim_{x \rightarrow \infty} f(x)$



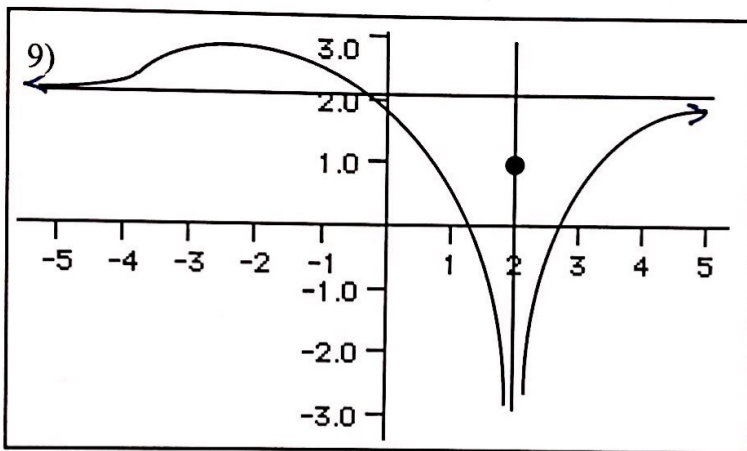
- a) $\lim_{x \rightarrow 1^-} f(x)$ b) $\lim_{x \rightarrow 1^+} f(x)$ c) $\lim_{x \rightarrow 1} f(x)$
 d) $f(1)$ e) $\lim_{x \rightarrow -\infty} f(x)$ f) $\lim_{x \rightarrow \infty} f(x)$



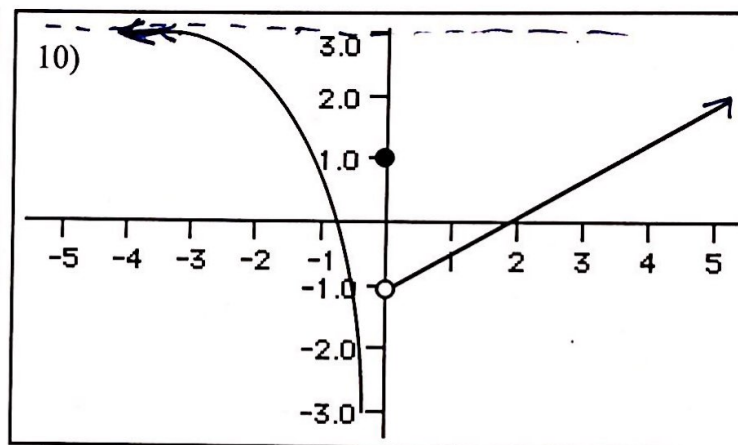
- a) $\lim_{x \rightarrow 1^-} f(x)$ b) $\lim_{x \rightarrow 1^+} f(x)$ c) $\lim_{x \rightarrow 1} f(x)$
 d) $f(1)$ e) $\lim_{x \rightarrow -\infty} f(x)$ f) $\lim_{x \rightarrow \infty} f(x)$



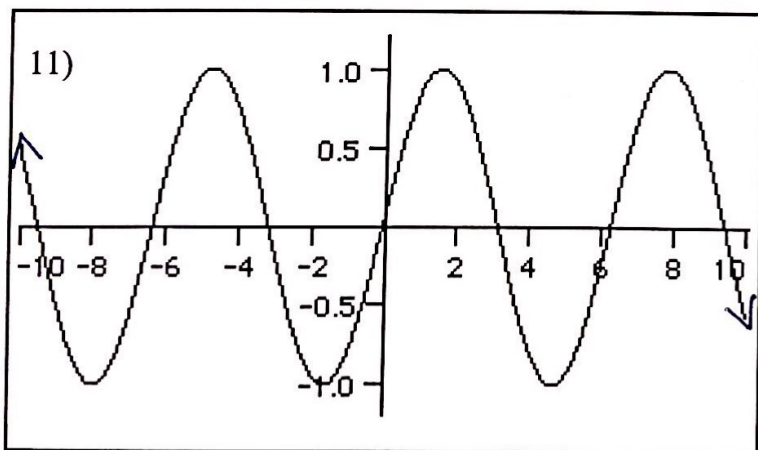
- a) $\lim_{x \rightarrow 3^-} f(x)$ b) $\lim_{x \rightarrow 3^+} f(x)$ c) $\lim_{x \rightarrow 3} f(x)$
 d) $f(3)$ e) $\lim_{x \rightarrow -\infty} f(x)$ f) $\lim_{x \rightarrow \infty} f(x)$



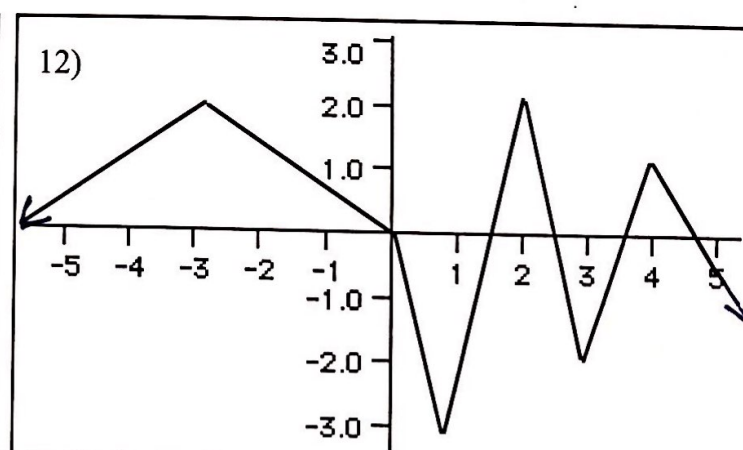
- a) $\lim_{x \rightarrow 2^-} f(x)$ b) $\lim_{x \rightarrow 2^+} f(x)$ c) $\lim_{x \rightarrow 2} f(x)$
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- a) $\lim_{x \rightarrow 0^-} f(x)$ b) $\lim_{x \rightarrow 0^+} f(x)$ c) $\lim_{x \rightarrow 0} f(x)$
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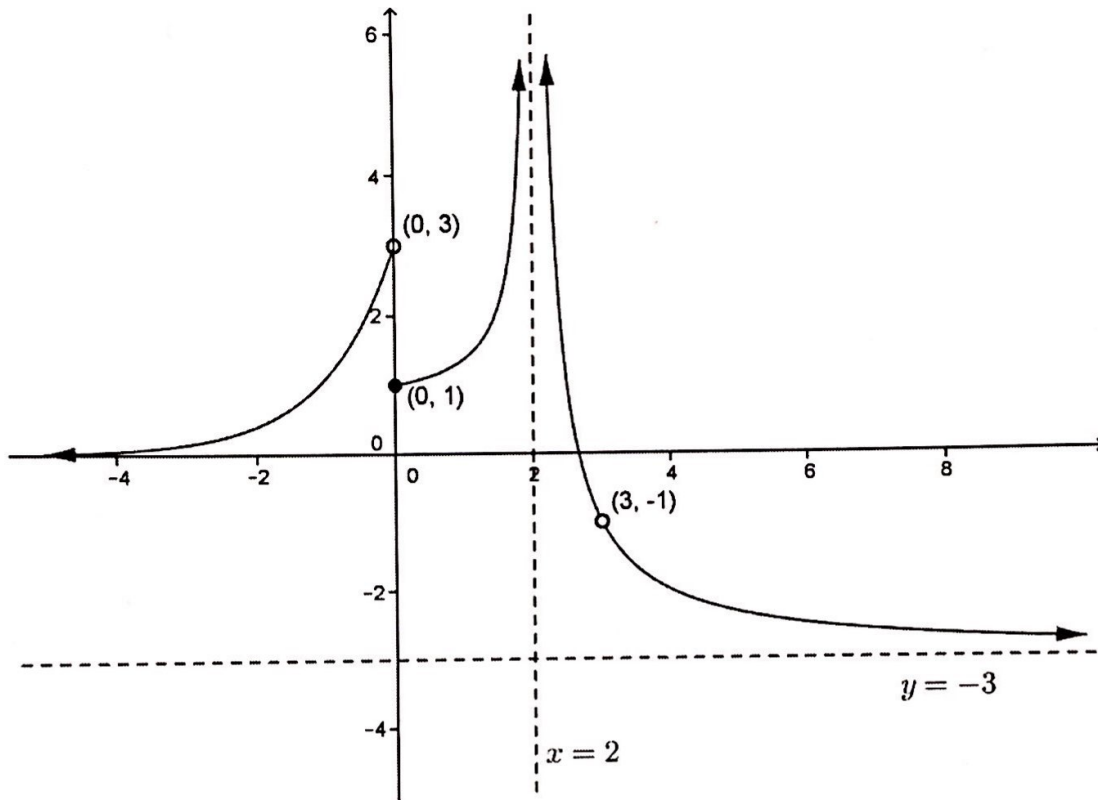


- a) $\lim_{x \rightarrow 0^-} f(x)$ b) $\lim_{x \rightarrow 0^+} f(x)$ c) $\lim_{x \rightarrow 0} f(x)$
 d) $f(0)$ e) $\lim_{x \rightarrow -\infty} f(x)$ f) $\lim_{x \rightarrow \infty} f(x)$



- a) $\lim_{x \rightarrow 0^-} f(x)$ b) $\lim_{x \rightarrow 0^+} f(x)$ c) $\lim_{x \rightarrow 0} f(x)$
 d) $f(0)$ e) $\lim_{x \rightarrow -\infty} f(x)$ f) $\lim_{x \rightarrow \infty} f(x)$

Part II: This part of the activity concentrates on writing correct notation for limit statements as well as making the connection between limits and graphical behavior. Shown below is the graph of a function $f(x)$. There are eleven limit statements, including one-sided and two-sided limits, based on the labeled points and lines on this graph. Write at least ten of these limit statements.



1. _____

6. _____

2. _____

7. _____

3. _____

8. _____

4. _____

9. _____

5. _____

10. _____

11. _____