

Period Functions

The graph of a **periodic function** shows a repeating pattern. The distance from 1 point on the graph to the point where the pattern begins repeating is called the **period**.

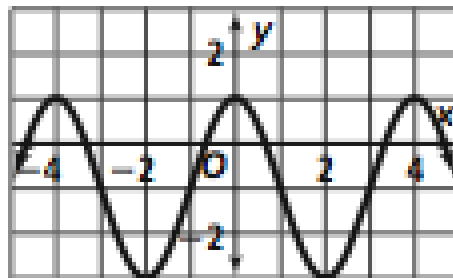
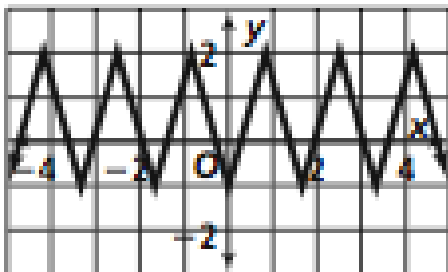
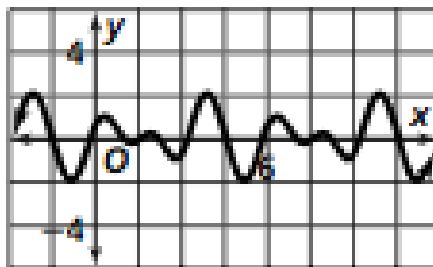
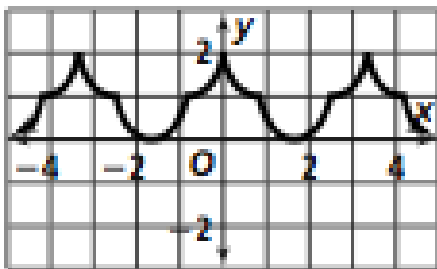
Periodic Function: repeats a pattern of y-values at regular intervals

Cycle: one complete pattern, a cycle may begin at any point on the graph

Period: the horizontal (x) length of one cycle

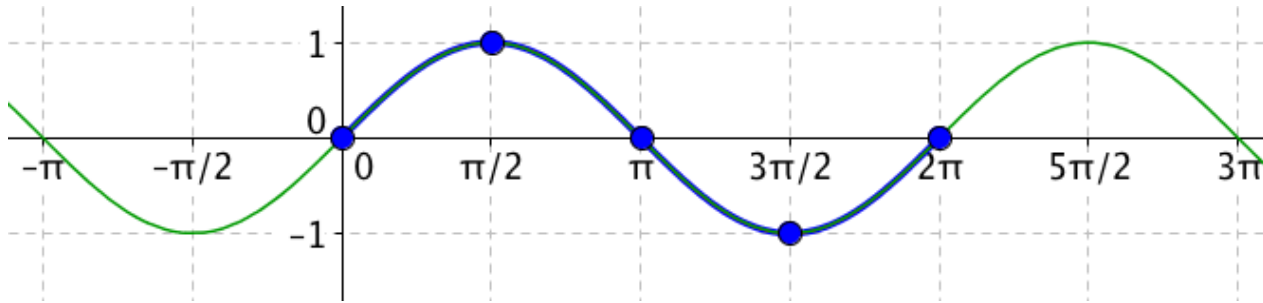
Amplitude: half the distance between the maximum and minimum y-values of a function

Examples:



Graphing Sine

Sine Function: $y = \sin x$ (amplitude = 1, period = 2π)



We will graph the angle measure (the x value) in radians.

To graph by hand we will find 5 key points. These points are the maximum, the minimum, and the x-intercepts. We will usually graph only 1 cycle.

The graph of a sine function is called a **sine curve**.

For $y = a \sin bx$ with $a \neq 0$, $b > 0$ and x in radians:

- $|a|$ is the amplitude of the function
- if a is negative the graph flips over the x-axis
- b is the number of cycles in the interval 0 to 2π
- $\frac{2\pi}{b}$ is the period of the function

Example: Sketch one cycle of $y = \frac{1}{2} \sin 2x$

$|a| = \frac{1}{2}$, so the amplitude is $\frac{1}{2}$

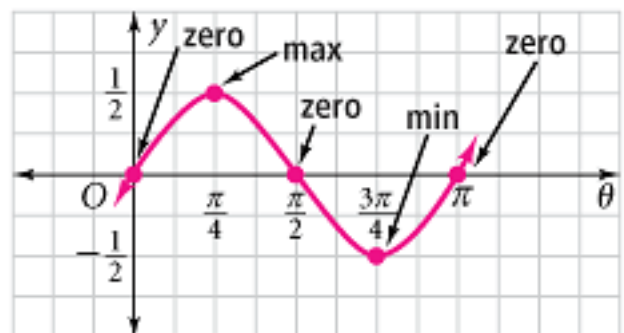
$b = 2$ so there are 2 cycles from 0 to 2π

$\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$ so the period is π

Divide the period into fourths.

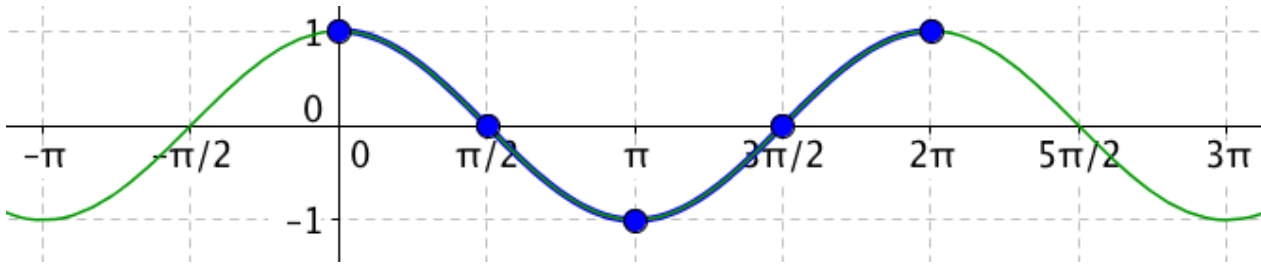
Using the values of the amplitude and period plot the pattern

zero-max-zero-min-zero.



Graphing Cosine

Cosine Function: $y = \cos x$ (amplitude = 1, period = 2π)



We will graph the angle measure (the x value) in radians.

To graph by hand we will find 5 key points. These points are the maximum, the minimum, and the x-intercepts. We will usually graph only 1 cycle.

For $y = a \cos bx$ with $a \neq 0$, $b > 0$ and x in radians:

- $|a|$ is the amplitude of the function
- if a is negative the graph flips over the x-axis
- b is the number of cycles in the interval 0 to 2π
- $\frac{2\pi}{b}$ is the period of the function

Example: Sketch one cycle of $y = 1.5 \cos 2x$

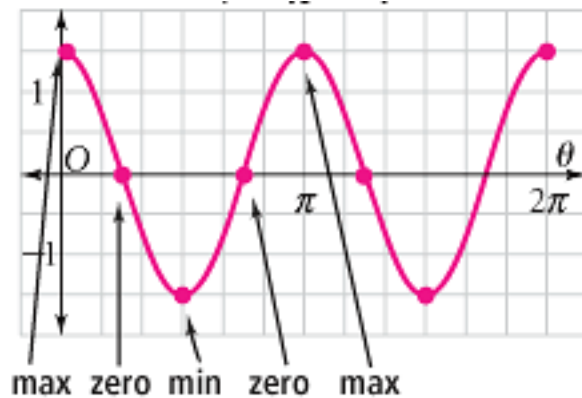
$|a| = 1.5$, so the amplitude is 1.5

$b = 2$ so there are 2 cycles from 0 to 2π

$\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$ so the period is π

Divide the period into fourths.

Using the values of the amplitude and period plot the pattern zero-max-zero-min-zero.



Translating Sine and Cosine Functions

Phase Shift: a horizontal translation of a periodic function.

For $y = a \sin b(x-h) + k$ or $y = a \cos b(x-h) + k$

- $|a|$ is the amplitude of the function
- if a is negative the graph flips over the x-axis
- b is the number of cycles in the interval 0 to 2π
- $\frac{2\pi}{b}$ is the period of the function
- h is the phase shift (horizontal shift)
- k is the vertical shift

Example: Sketch the graph of $y = \sin 2\left(x - \frac{\pi}{3}\right) - \frac{3}{2}$

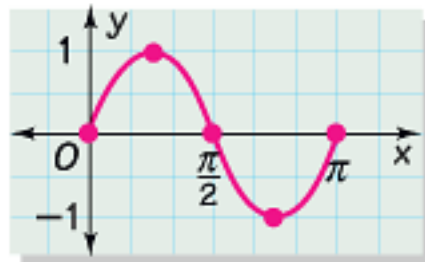
$|a| = 1$, so the amplitude is 1

$b = 2$ so there are 2 cycles from 0 to 2π

$\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$ so the period is π

Sketch one cycle of $y = \sin 2x$

Use the 5 key points.



Since $h = \frac{\pi}{3}$ and $k = -\frac{3}{2}$ translate

the graph $\frac{\pi}{3}$ units to the right and

$\frac{3}{2}$ units down. Sketch the graph.

