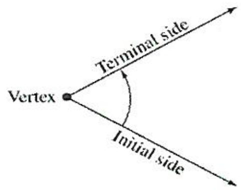
Unit 5: Geometry and Trigonometry **Name ______________________**
Lesson 4 – Radian and Degree Measure

Trigonometry (from Greek trigōnon, "triangle" and metron, "measure") is a branch of mathematics that studies relationships involving lengths and angles of triangles.

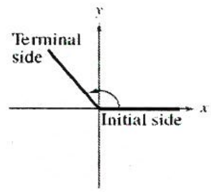
Angle: figure formed by 2 rays that share a common endpoint.

- *Initial side*: fixed ray
- *Terminal side:* ray that rotates away from the initial side

- An angle with its vertex at the origin and its initial side fixed along the positive x-axis is said to be in standard position.

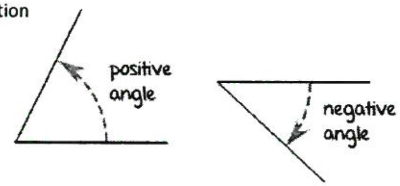


Angle
FIGURE 1



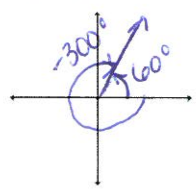
Angle in Standard Position
FIGURE 2

- *Positive angle* – counterclockwise rotation
- *Negative angle* – clockwise rotation

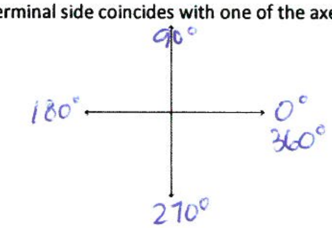


- In trigonometry, angles are often named with Greek letters such as α, β, θ (alpha, beta, theta).
- The *measure of an angle* is determined by the amount of rotation from the initial side to the terminal side.

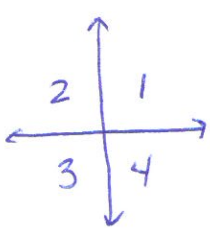
- Angles that have the same terminal side are said to be **coterminal**.
  Example: 60° angle is coterminal with −300° angle .



- **Quadrantal angle:** angle in standard position whose terminal side coincides with one of the axes (angle measure is a multiple of 90°)

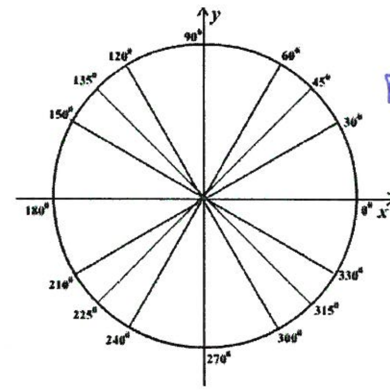


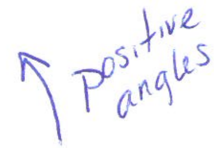
Quadrants:



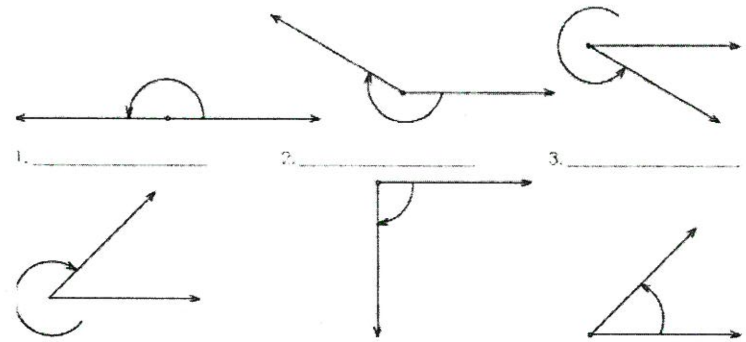
Units for measuring angles We use 2 different types of measurement to measure angles in this unit.

1. **Degree measure:** A measure of one degree (1°) is equivalent to a rotation of $\frac{1}{360}$ of a complete revolution.





Estimate the degree measure of each of the following angles.



1. ____________ 2. ____________ 3. ____________

4. ____________ 5. ____________ 6. ____________

What quadrant do the following angles' terminal side lie in?

A) 130° B) 350° C) −100° D) 200°

4.1 Angles and Radian Measure

This section will cover how angles are drawn and also arc length and rotations.

Angles are measured a couple of different ways. The first unit of measurement is a **degree** in which 360° (degrees) is equal to one revolution. Most likely the reason why we use 360 is from the Babylonians, whose year is based on 360 days.

Another unit of measurement for angles is **radians**. In radians, 2π is equal to one revolution. So a conversion between radians and degrees is $2\pi = 360^\circ$, or $\pi = 180^\circ$.

When converting from degrees to radians:

Multiply your degrees by $\frac{\pi}{180}$

When converting from radians to degrees:

Multiply your radians by $\frac{180}{\pi}$

EXAMPLE: Convert 60° to radians.

We will take 60 and multiply it by $\frac{\pi}{180}$ and you will get: $60 \cdot \frac{\pi}{180}$. This reduces to $\frac{\pi}{3}$.

EXAMPLE: Convert -405° to radians.

We will take -405 and multiply it by $\frac{\pi}{180}$ and you will get: $-450 \cdot \frac{\pi}{180}$. This reduces to $-\frac{5\pi}{2}$.

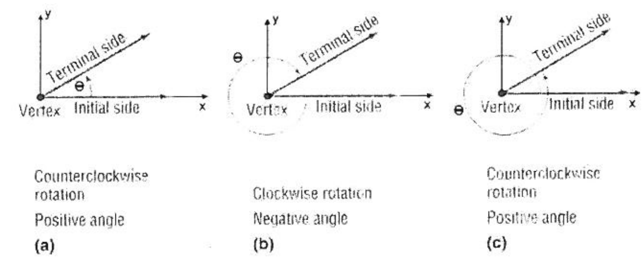
EXAMPLE: Convert $\frac{4\pi}{3}$ into degrees.

We will take $\frac{4\pi}{3}$ and multiply it by $\frac{180}{\pi}$ and you will get: $\frac{4\pi}{3} \cdot \frac{180}{\pi}$. This reduces to 240°.

EXAMPLE: Convert $-\frac{3\pi}{2}$ into degrees.

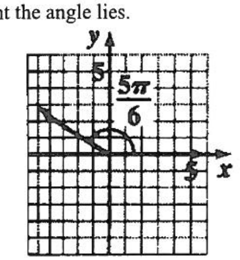
We will take $-\frac{3\pi}{2}$ and multiply it by $\frac{180}{\pi}$ and you will get: $-\frac{3\pi}{2} \cdot \frac{180}{\pi}$. This reduces to -270°

We will use θ (theta) to represent an angle's measurement. In the figure below it describes how you know if an angle is positive or negative. The vertex of the angle is at the origin of a rectangular coordinate system. The positive x axis is always where an angle is measure from, and this is called the initial side. An angle drawn this way is said to be in **standard form**. An angle that goes counterclockwise is always positive, and clockwise angles are negative.



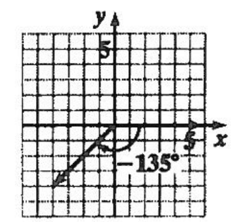
EXAMPLE: Draw each angle in standard position. Indicate which quadrant the angle lies.

a.) $\frac{5\pi}{6}$



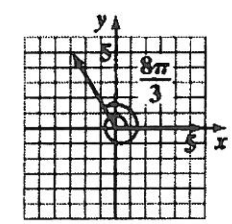
If you are not sure where to draw this angle, first convert it into degrees: $\frac{5\pi}{6} \cdot \frac{180}{\pi} = 150^\circ$. Our angle is measured from the positive x-axis. Since the angle is positive we need to go in the counterclockwise direction. We see that this ends up in Quadrant II.

b.) -135°



Our angle is measured from the positive x-axis. Since we have a negative angle, we need to go in the clockwise direction. We see that we end up in Quadrant III.

c.) $\frac{8\pi}{3}$



If you are not sure where to draw this angle, first convert it into degrees: $\frac{8\pi}{3} \cdot \frac{180}{\pi} = 480^\circ$. Our angle is measured from the positive x-axis. Since the angle is positive we need to go in the counterclockwise direction. Since this angle is more than 360 degrees, we need to subtract 360 from 480. We will get 120 degrees. So we need to got around 360 degrees and then go an extra 120 degrees. We see that this ends up in Quadrant II.

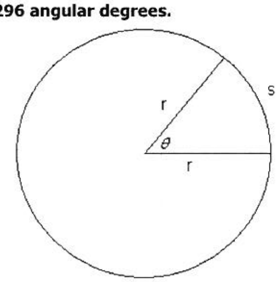
Coterminal Angles

Coterminal Angles are angles who share the same initial side and terminal sides. Finding coterminal angles is as simple as adding or subtracting $360°$ or 2π to each angle, depending on whether the given angle is in degrees or radians. There are an infinite number of coterminal angles that can be found. Following this procedure, all coterminal angles can be found. This is the basis for solving trigonometric equations which will be done in the future.

Radians are often used in trigonometry to represent angle measures. Radian measures are very common in calculus, so it is important to have an understanding of what a radian is.

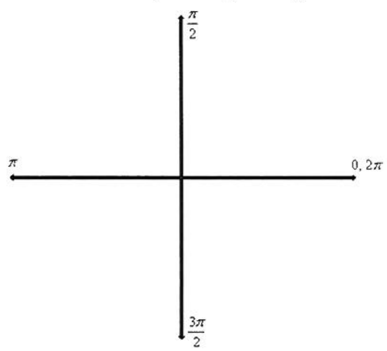
Definition of a Radian

A radian is the measure of a central angle θ that intercepts an arc s equal in length to the radius r of the circle. There are 2π, or approximately 6.28318, radians in a complete circle. Thus, one radian is about 57.296 angular degrees.



In other words, if we were to take the length of the radius of a circle, and lay in on the edge of a circle, that length would be one radian.

The number π is often used when describing radian measure. The approximate value of π is 3.14159... A plane, in trigonometry, can not only be divided into quadrants using degree measures, but radian as well. Observe the following moving in a counterclockwise direction.



When studying trigonometry, angles are usually measured in radians.

In relation to degrees, 180° is π radians. This means 2π radians is 360°. Since the approximate value of π is 3.14159..., it follows that 360° is approximately 6.28318...radians. When evaluating angles in trigonometry or calculus, always be aware of whether the question is given in terms of degrees or radians. If no degree symbol is given, the problem is in radians.

Examples of finding coterminal angles

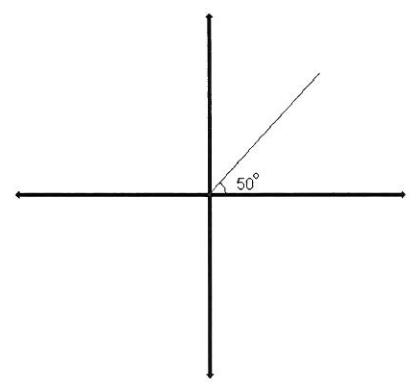
Find one positive angle that is coterminal to 50°.

Since the terminal side of a 50° angle resides in quadrant I, the terminal side of its coterminal angle must share that side. This means the new angle would make one complete revolution before having its terminal side come to rest at the same place.

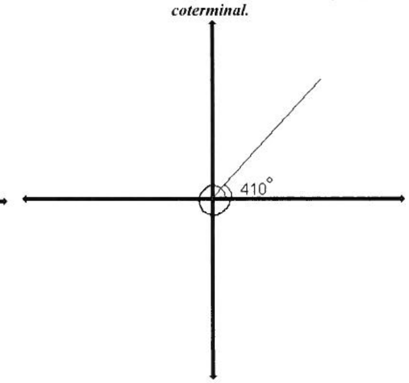
Therefore, to find the coterminal angle to a 50° angle, just add 360°.

$$50° + 360° = 410°$$

Below is the graphical representation of a 50° angle. Since its coterminal angle must share the same terminal side, it is reasonable to create a new angle that makes one complete revolution and ends up in the same place.



To find the coterminal angle of a 50° angle, add 360°. It would follow that 50°+360°=410°. A 410° angle is illustrated below. From the graphical representation of the angle, we can conclude that these two angle do indeed share the same terminal side, meaning they are coterminal.



Find one positive angle that is coterminal to 110°.

$$110° + 360° = 470°$$

Find two positive angles that are coterminal to -30°.

$$-30° + 360° = 330°$$
$$330° + 360° = 690°$$

In this case, the two positive coterminal angles to -30° are 330° and 690°.

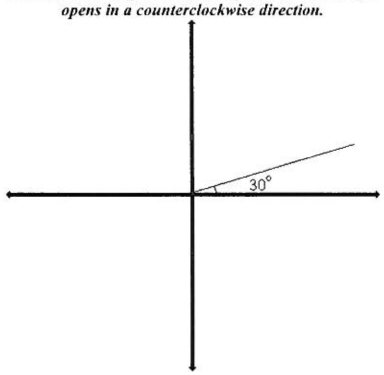
If more than one positive coterminal angle needs to be found, simply add another 360°. This would essentially make the new angle complete two full revolutions before its terminal side comes to rest.

Find one negative angle that is coterminal to 30°.

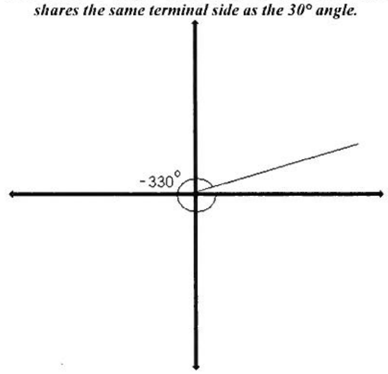
A negative angle moves in a clockwise direction. In this case, to find the negative coterminal angle, subtract 360° from 30°.

$$30° - 360° = -330°$$

Below is a 30° angle in standard position. This angle opens in a counterclockwise direction.



Here is a -330° angle. As the angle opens clockwise, it shares the same terminal side as the 30° angle.



Find one negative angle that is coterminal to 150°.

$$150° - 360° = -210°$$

Find one negative angle that is coterminal to 415°.

$$415° - 360° = 55°$$

Although 55° is a coterminal angle to 415°, this is not a solution to the problem. The problem specifically asked for a negative angle, so the process needs to take place one more time.

$$55° - 360° = -305°$$

These were all examples of finding coterminal angles. If the initial angle is given in the form or radians, add or subtract 2π instead of 360°.

Find a positive and negative angle that is coterminal to an angle that is $\frac{\pi}{6}$ radians.

$$\frac{\pi}{6} + 2\pi$$
$$\frac{\pi}{6} + \frac{12\pi}{6}$$
$$\frac{13\pi}{6}$$

Adding 2π to the original angle yields the positive coterminal angle.

$$\frac{\pi}{6} - 2\pi$$
$$\frac{\pi}{6} - \frac{12\pi}{6}$$
$$-\frac{11\pi}{6}$$

By subtracting 2π from the original angle, the negative coterminal angle has been found.

Find two positive angles that are coterminal to an angle that is $\frac{11\pi}{2}$ radians.

$$\frac{11\pi}{2} - 2\pi$$
$$\frac{11\pi}{2} - \frac{4\pi}{2}$$
$$\frac{7\pi}{2}$$

$$\frac{7\pi}{2} - 2\pi$$
$$\frac{7\pi}{2} - \frac{4\pi}{2}$$
$$\frac{3\pi}{2}$$

Since $\frac{11\pi}{2}$ is more than one complete revolution, 2π was subtracted from the initial angle yielding a coterminal angle of $\frac{7\pi}{2}$. This is still at least one full revolution, so 2π was subtracted yet again. This process resulted in the two positive coterminal angles of $\frac{7\pi}{2}$ and $\frac{3\pi}{2}$.

1. Mike mentions to Jeff that he always gets confused which ratio he should use when converting from degrees to radians, is it $\frac{\pi}{180°}$ or $\frac{180°}{\pi}$? Which one is the correct ratio? Explain a way to make sense of which one to use.

2. Convert the degree measures into radians. Leave answers as exact values in most reduced form.

a) 90° b) 30° c) 300° d) 270°

_________ radians _________ radians _________ radians _________ radians

3. Convert the following radian measures into degrees. Give the quadrant the angle is located in.

a) $\frac{5\pi}{3}$ b) $\frac{9\pi}{20}$ c) $-\frac{4\pi}{15}$ d) $\frac{7\pi}{6}$

_________ _________ _________ _________

e) $\frac{6\pi}{5}$ f) $-\frac{11\pi}{12}$ g) $\frac{3\pi}{10}$ h) $\frac{4\pi}{2}$

_________ _________ _________ _________

4. You are told that 15° is $\frac{\pi}{12}$ radians. How could you use that to determine what the radian value is for 45°?

5. Convert the degree measures into radians. Leave answers as exact values in most reduced form. Give the quadrant.

a) 315° b) - 135° c) -36° d) 333°

_________ radians _________ radians _________ radians _________ radians

UNIT 6 WORKSHEET 2
FINDING COTERMINAL ANGLES

Find one positive and one negative coterminal angle of each of the following. **There is no need to graph the angles.** If in degrees, give answers in degrees. If radians, give radians

A) 30° **B)** -40° **C)** 150° **D)** 220°

E) -330° **F)** $\frac{\pi}{3}$ **G)** $\frac{5\pi}{2}$ **H)** $-\frac{2\pi}{3}$

I) $-\frac{5\pi}{6}$ **J)** $\frac{5\pi}{3}$ **K)** $-\frac{4\pi}{3}$ **L)** 300°

M) 700° **N)** $-\frac{17\pi}{6}$ **O)** $\frac{7\pi}{3}$ **P)** −410°

Q) 1000° **R)** $\frac{31\pi}{6}$ **S)** $-\frac{15\pi}{4}$ **T)** $\frac{5\pi}{6}$