

CIRCLES

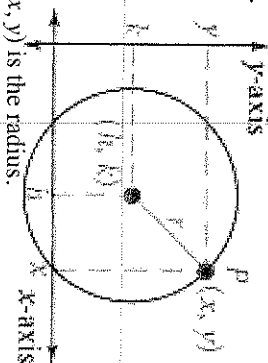
A circle is the set of all points in a plane that are a distance r from a given point, called the center. The distance r is the radius of the circle.

You can use the distance formula to find an equation of a circle with a radius r and a center at the point (h, k) .

Let (x, y) be any point on the circle.

Let (h, k) be the point at the center of the circle.

Let r be the radius.



The distance from the center (h, k) to (x, y) is the radius.

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = d \quad \text{Distance Formula}$$

$$\sqrt{(x - h)^2 + (y - k)^2} = r$$

Distance from (h, k) to $(x, y) = r$

$$(x - h)^2 + (y - k)^2 = r^2$$

Square each side to get the equation of a circle.

STANDARD FORM of the equation of a circle:

$$(x - h)^2 + (y - k)^2 = r^2$$

GENERAL FORM of the equation of a circle:

$$x^2 + y^2 + Dx + Ey + F = 0$$

- > The equation of the circle with a center at the origin is $x^2 + y^2 = r^2$
- > To convert from standard to general form – multiply it out.
- > To convert from general to standard form – complete the square.
- > All circles are similar.

Example 1: Write an equation of a circle with center $(-4, 3)$ and radius 4.

$$(x + 4)^2 + (y - 3)^2 = 16$$

Key

Example 2: Find the center and radius of the circle with equation.

$$(x - 16)^2 + (y + 9)^2 = 144$$

Center $(16, -9)$ $(x - h)^2 + (y - k)^2 = r^2$

$$r = \sqrt{144} = 12$$

Example 3: Convert from standard form to general form by multiplying it out.

$$(x + 6)^2 + (y - 2)^2 = 4$$

Standard Form

Multiply it out

Combine and move terms to look

$$\text{like: } x^2 + y^2 + Dx + Ey + F = 0$$

General Form

$$x^2 + y^2 + 12x - 4y + 36 = 0$$

Example 4: Convert from general form to standard form by completing the square. Then find the center and radius.

$$x^2 + y^2 + 8y - 10x + 16 = 0 \quad \text{General Form}$$

Group the x terms, group the y terms, move constants to the other side.

$$x^2 - 10x + 25 + y^2 + 8y + 16 = -16 \quad \text{Complete the square for } x \text{ and } y.$$

+25 Simplify.

$$(x - 5)^2 + (y + 4)^2 = 25$$

Standard form

Center = $(5, -4)$

radius = 5

Practice:

HONORS

1. Write an equation of a circle with a radius of 8 and a center of (4, 3).

$$(x-4)^2 + (y-3)^2 = 64$$

2. Write an equation of a circle with a radius of 6 and a center of (-3, -8).

$$(x+3)^2 + (y+8)^2 = 36$$

3. Write the equation of a circle given the center is (2, -5) with a radius of 7.

$$(x-2)^2 + (y+5)^2 = 49$$

4. Find the center and radius. $(x-1)^2 + (y+4)^2 = 81$

$$(1, -4) \quad r=9$$

5. Find the radius and center of: $(x-5)^2 + (y-2)^2 = 20$

$$C(5, 2) \quad r=2\sqrt{5}$$

6. Convert to general form. $(x-2)^2 + (y+3)^2 = 9$

$$x^2 - 4x + 4 + y^2 + 6y + 9 = 9 \quad \boxed{x^2 + y^2 - 4x + 6y + 4 = 0}$$

7. Convert to general form. $(x+4)^2 + (y-2)^2 = 1$

$$x^2 + 8x + 16 + y^2 - 4y + 4 = 1$$

$$\boxed{x^2 + y^2 + 8x - 4y + 19 = 0}$$

8. Convert to standard form, then find the center and radius. $x^2 - 8x + y^2 + 11 = 0$

$$x^2 - 8x + 16 + y^2 = -11 + 16$$

$$(x-4)^2 + (y-0)^2 = 5$$

Center (4, 0)
rad = $\sqrt{5}$

9. Convert to standard form, then find the center and radius. $x^2 + y^2 + 4x - 6y = -4$

$$x^2 + 4x + 4 + y^2 - 6y + 9 = -4 + 4 + 9$$

$$(x+2)^2 + (y-3)^2 = 9$$

Center (-2, 3)
rad = 3

10. Convert to standard form, then find the center and radius. $3x^2 = 9 - 3y^2 - 6y$

$$\frac{3}{3}$$

$$x^2 = 3 - y^2 - 2y$$

$$x^2 + y^2 + 2y + 1 = 3 + 1$$

$$x^2 + (y+1)^2 = 4$$

$$C = (0, -1) \quad r = 2$$

Example 5: Write an equation of a circle whose diameter has endpoints (3, 5) and (6, 1).

midpoint of diameter is the center of the circle; diameter = 2 • radius,

Use midpoint formula to find the center:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Center: $(\frac{9}{2}, 3)$

Use distance formula to find the diameter: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Diameter: 5

Radius is $\frac{1}{2}$ the diameter: $\frac{5}{2}$

Use the center and radius to write the equation in standard form:

$$\boxed{(x - \frac{9}{2})^2 + (y - 3)^2 = \frac{25}{4}}$$

Practice:

11. Write an equation of a circle whose diameter has endpoints (-2, 8) and (2, -2).

$$\boxed{(x-2)^2 + (y-3)^2 = 25}$$

$$d = \sqrt{(12-7)^2 + (9-7)^2} = \sqrt{25+4}$$

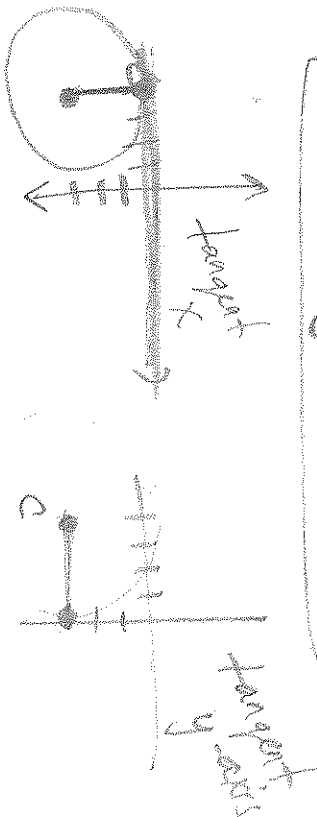
$$r = \sqrt{29} / 2$$

12. Write an equation for a circle with center (7, 2) that passes through (12, 9)

$$\boxed{(x-7)^2 + (y-7)^2 = 29}$$

13. Write an equation for a circle with center (-4, -3) that is tangent to the x-axis.

$$\boxed{(x+4)^2 + (y+3)^2 = 9}$$



odds

Key

I. Write the equation of the circle that satisfies each set of conditions.

1. Center (0, 3), radius is 7 units

$$x^2 + (y-3)^2 = 49$$

2. Center (-8, 7), radius is $\frac{1}{2}$ units

$$(x+8)^2 + (y-7)^2 = \frac{1}{4}$$

3. Center (-1, -5), radius is 2 units

$$(x+1)^2 + (y+5)^2 = 4$$

II. Find the center and radius of the circle with the given equation.

4. $x^2 + (y+2)^2 = 4$

$(0, -2)$
 $r = 2$

5. $x^2 + y^2 = 144$

$(0, 0)$
 $r = 12$

6. $(x-3)^2 + (y-1)^2 = 25$

$(3, 1)$ $r = 5$

7. $(x+3)^2 + (y+7)^2 = 81$

$(-3, -7)$ $r = 9$

8. $(x-3)^2 + y^2 = 16$

$(3, 0)$ $r = 4$

9. $x^2 + y^2 + 6y = -50 - 14x$

$x^2 + 14x + 49 + y^2 + 6y + 9 = -50 + 49 + 9$

10. $x^2 + y^2 + 2x - 10 = 0$

$x^2 + 2x + 1 + y^2 = 10 + 1$
 $(x+1)^2 + y^2 = 11$

$(x+7)^2 + (y+3)^2 = 8$

C: $(-7, -3)$ $r = 2\sqrt{2}$

C: $(-1, 0)$ $r = \sqrt{11}$

11. $x^2 + y^2 - 18x - 18y + 53 = 0$

$x^2 - 18x + 81 + y^2 - 18y + 81 = -53 + 81 + 81$

12. $x^2 + y^2 + 2x + 4y = 9$

$x^2 + 2x + 1 + y^2 + 4y + 4 = 9 + 1 + 4$
 $(x+1)^2 + (y+2)^2 = 14$

$(x-9)^2 + (y-9)^2 = 109$

$(9, 9)$ $r = \sqrt{109}$

$(-1, -2)$ $r = \sqrt{14}$

Equations of Circles Worksheet 2

Honors

Write the equation of the circle that satisfies each set of conditions.

1. Endpoints of a diameter at (-5, 2) and (3, 6)

$C = (-1, 4)$

$(x+1)^2 + (y-4)^2 = 20$

2. Endpoints of a diameter at (11, 18) and (-13, -19)

$C = (-1, -1/2)$ $(x+1)^2 + (y+1/2)^2 = \frac{1945}{4}$

3. Center at (8, -9), passes through (21, 22)

$(x-8)^2 + (y+9)^2 = 1130$

4. Center at (-8, -7), tangent to y-axis

$(x+8)^2 + (y+7)^2 = 64$

5. Center at (4, 2), tangent to the x-axis

$(x-4)^2 + (y-2)^2 = 4$

