

Graphing Transformations of 1/x

Asymptote: a line that a curve approaches but never reaches

Parent Function $f(x) = \frac{1}{x}$

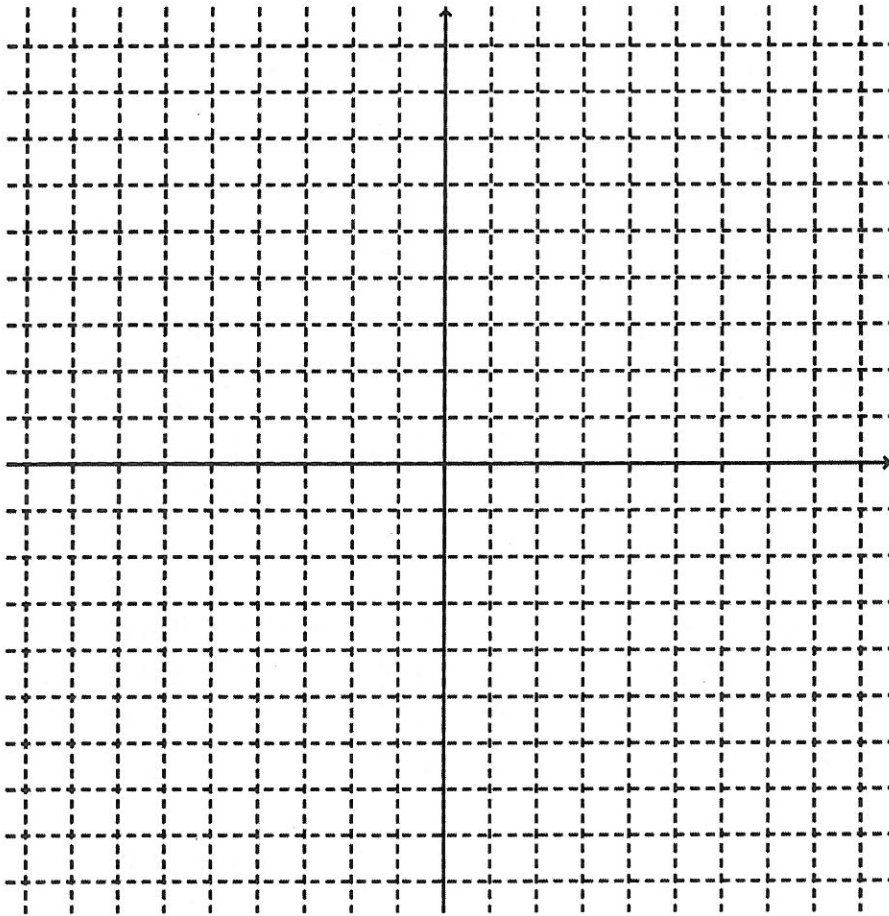
Horizontal Asymptote at $y=0$

Vertical Asymptote at $x=0$

Graph: $f(x) = \frac{1}{x}$

General Form: $f(x) = \frac{a}{x-h} + k$

- Horizontal Asymptote at $y = k$
- Vertical Asymptote at $x = h$
- (h,k) is the intersection of the asymptote
- "a" vertically stretches or shrinks, if a is negative the graph flips over the x axis

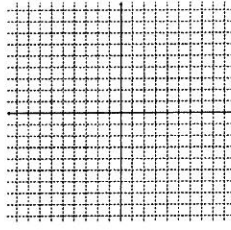


Examples: State the transformations/asymptotes, then graph.

1. $f(x) = \frac{1}{x-3}$

Transformations:

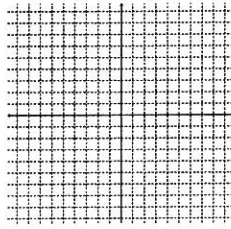
V.A.: _____
H.A.: _____



2. $f(x) = \frac{1}{x-4} + 2$

Transformations:

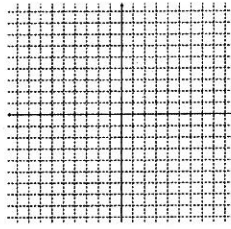
V.A.: _____
H.A.: _____



3. $f(x) = \frac{1}{x+3} - 4$

Transformations:

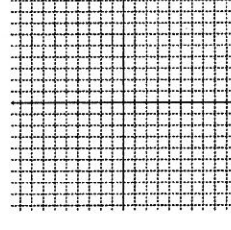
V.A.: _____
H.A.: _____



4. $f(x) = \frac{2}{x+3} + 1$

Transformations:

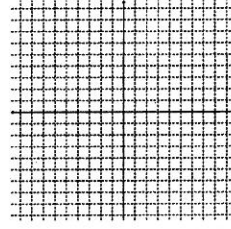
V.A.: _____
H.A.: _____



5. $f(x) = \frac{-3}{x-2} - 1$

Transformations:

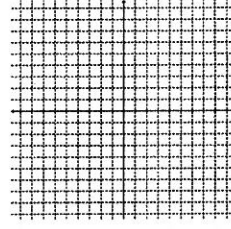
V.A.: _____
H.A.: _____



6. $f(x) = \frac{-1}{x+2}$

Transformations:

V.A.: _____
H.A.: _____

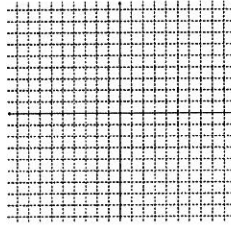


Graphing Worksheet 1/x - State the transformations, and asymptotes, then graph.

1) $f(x) = \frac{4}{x-5}$

Transformations:

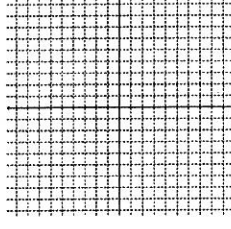
V.A.: _____ H.A.: _____



2) $f(x) = \frac{1}{x} - 1$

Transformations:

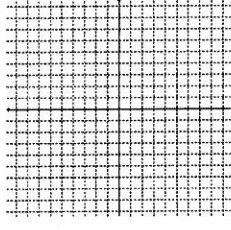
V.A.: _____ H.A.: _____



3) $f(x) = \frac{2}{x+1} - 3$

Transformations:

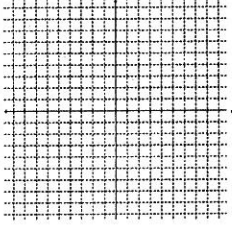
V.A.: _____ H.A.: _____



4) $f(x) = \frac{-1}{x-4}$

Transformations:

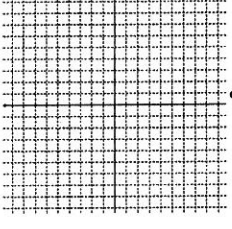
V.A.: _____ H.A.: _____



5) $f(x) = \frac{1}{x-2} + 2$

Transformations:

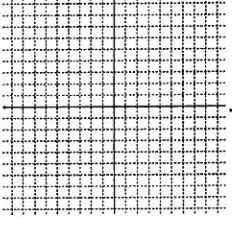
V.A.: _____ H.A.: _____



6) $f(x) = \frac{-1}{x+2} - 3$

Transformations:

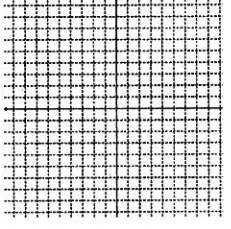
V.A.: _____ H.A.: _____



7) $f(x) = \frac{1}{x+1} - 1$

Transformations:

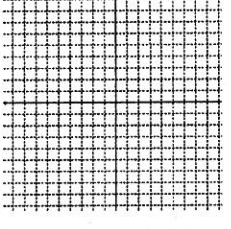
V.A.: _____ H.A.: _____



8) $f(x) = \frac{-2}{x-3}$

Transformations:

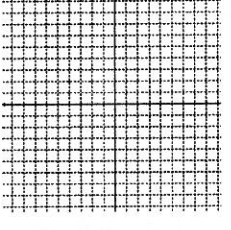
V.A.: _____ H.A.: _____



9) $f(x) = \frac{1}{x} + 2$

Transformations:

V.A.: _____ H.A.: _____



HONORS

Rational Functions: A rational function is the quotient of 2 polynomials. It has the form:

$$f(x) = \frac{P(x)}{Q(x)} \text{ where } Q(x) \neq 0$$

Graphs of rational functions will either be continuous (no jumps, breaks, or holes) or not continuous. Asymptote: A line that a curve approaches but never reaches.

Vertical Asymptotes and Holes

Points of discontinuity exist for each real zero of Q(x). Each real zero will either be a vertical asymptote or a hole in the graph.

- Common real zeros of P(x) and Q(x) are holes.
- All other real zeros of Q(x) are vertical asymptotes.

Examples:

$$y = \frac{x}{x+2} \quad \text{V.A. } x=-2$$

$$y = \frac{(x+5)(x-2)}{(x-2)} \quad \text{hole at } x=2$$

$$y = \frac{(x-4)(x-2)}{x(x-3)(x-4)} \quad \text{hole at } x=4, \text{ V.A. } x=0, x=3$$

- Remember, vertical lines start with "x="
- Vertical Asymptotes and Holes are the values of x that make the denominator 0 (what x can't be).

Horizontal Asymptotes

The graph of a rational function has at most one horizontal asymptote.

- $y=0$ if degree in denominator > degree in numerator
- $y=a/b$ if degree in denominator = degree in numerator
 - a = coefficient of term in numerator with highest degree
 - b = coefficient of term in denominator with highest degree
- none if degree in denominator < degree in numerator

Examples:

$$y = \frac{x}{x^2+2} \quad \text{H.A. is } y=0 \quad (\text{degree in den. } > \text{ degree in num.})$$

$$y = \frac{2x^2+1}{3x^2+2} \quad \text{H.A. is } y=2/3 \quad (\text{degree in den.} = \text{degree in num.})$$

$$y = \frac{2x^3+1}{x+2} \quad \text{no H.A. (degree in den. } < \text{ degree in num.)}$$

- Remember, horizontal lines start with "y="
- Horizontal asymptotes are the values that y approaches as $x \rightarrow \infty$ or $x \rightarrow -\infty$
- Is there any easier way to find them?

HONORS

Slant Asymptotes

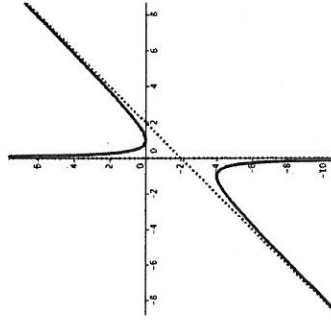
Slant asymptotes occur when the degree of the numerator is exactly one greater than the degree of the denominator.

Example: $y = \frac{x^2 - 2x + 1}{x}$

- 1) Use division to rewrite as a quotient $y = x - 2 + \frac{1}{x}$
- 2) As $x \rightarrow \infty$ or $x \rightarrow -\infty$, $\frac{1}{x} \rightarrow 0$ and $x - 2 + \frac{1}{x} \rightarrow x - 2$.

The line $y = x - 2$ is the slant asymptote.

V.A. $x=0$
H.A. none



Examples:

Find all asymptotes and holes, state the domain, and find the x and y intercepts.

1. $y = \frac{x}{x^2 - 4}$

2. $y = \frac{x^2 - 9}{x + 3}$

3. $y = \frac{2x^2 - 10}{x^2 + 4}$

4. $y = \frac{x^2 + 3x - 10}{x^2 + 2x - 8}$

5. $y = \frac{x^3 + x}{x^2 - 4}$

6. $y = \frac{3x + 1}{2x - 5}$

MORE....

7. $f(x) = \frac{5x}{x - 4}$

8. $f(x) = \frac{-2x + 1}{3x + 5}$

9. $f(x) = \frac{3}{x^2 + 4x}$

10. $h(x) = \frac{x^2 - 1}{x^2 - 6x - 7}$

11. $g(x) = \frac{x^2 - 6x + 9}{x^2 - x - 6}$

12. $m(x) = \frac{x^2 + 6x + 8}{x + 4}$

13. $f(x) = \frac{12x}{3x^2 + 1}$

14. $f(x) = \frac{2x^3}{x^2 + 1}$

15. $f(x) = \frac{x^2 - 9}{x + 2}$

To graph rational functions:

- 1) find asymptotes and holes
- 2) plot points (pick x values on either side of V.A.s)

Examples:

1. $y = \frac{x}{x^2 - 4}$

2. $y = \frac{x^2 - 9}{x + 3}$

3. $y = \frac{x^2 + 3x - 10}{x^2 + 2x - 8}$

4. $y = \frac{x^3 + x}{x^2 - 4}$

5. $y = \frac{3}{(x + 1)(x - 1)}$

Graphing Rational Functions Worksheet 1 - HONORS

Create a function of the form $y = f(x)$ that satisfies each set of conditions.

- Vertical asymptotes at $x = 4$, hole at $x = 0$

$$3. g(x) = \frac{x^2-2}{x^2+3x+3}$$

- Vertical asymptotes at $x = -5$ and $x = 1$, hole at $x = -1$

- Holes at $x = 3$ and $x = -7$, resembles $y = x$

$$6. g(x) = \frac{x^2+3x-3}{x+4}$$

$$4. h(x) = \frac{x^2}{x^2+1}$$

$$5. f(x) = \frac{(x+1)^2}{x^2-1}$$

Graph each function. (Use your own graph paper) First find the asymptotes, x-intercepts, y-intercepts, and holes.

$$4. y = \frac{3}{x+2}$$

$$5. y = \frac{x-5}{x+1}$$

$$26. y = \frac{(x+2)(x-2)}{x-2}$$

$$7. f(x) = \frac{x^3+3x-4}{x}$$

$$8. y = \frac{2x}{2x-6}$$

$$9. h(x) = \frac{x^2-9}{x-5}$$

$$7. y = \frac{x}{x-5}$$

$$8. y = \frac{-2}{(x-3)^2}$$

$$9. y = \frac{x^2-x}{x}$$

$$10. y = \frac{2}{x-4}$$

$$11. g(x) = \frac{x^2-6x+9}{x^2-x-6}$$

$$12. f(x) = \frac{(x-2)^2(x+1)^2}{(x-2)(x-1)}$$

$$10. y = \frac{-5}{(x-3)(x+1)}$$

$$11. y = \frac{x^2+3x-4}{x}$$

$$12. y = \frac{x}{1-x^2}$$

Rational Functions Worksheet 1 - HONORS

List all the asymptotes (i.e. horizontal, vertical, slant), holes, state the domain, find the x-intercept(s), and find the y-intercept.

