Test

End behavior ⇒ deg even or odd, L.C. pos or neg.

Real roots ⇒ x-intercepts ⇒ "zeros"
  ⇒ behavior with mult ⇒ bounce or cross

Sketch graphs using end behav. + zeros

Local max/mins ⇒ calculator
  "turning pt" ⇒ $\frac{\#}{TP}$ = one less than deg.

roots (incl. mult) = degree

long division
synthetic division
Give one factor, find all others (use division)
Give one zero, find all others
Find all roots → graph to find real
Given roots → write poly in standard form

Solve by factoring.

1. $x^3 + 4x^2 + 9x = -36$

2. $16x^4 - 81 = 0$

3. $2x^4 - 18x^2 - 20 = 0$

Divide using long division.

4. $\left(10x^2 - 11x - 6\right) \div \left(2x - 3\right)$

5. $\left(12x^4 - 5x^2 - 3\right) \div \left(3x^2 + 1\right)$

Divide using synthetic division.

6. $\left(3x^4 + 12x^3 - 5x^2 - 18x + 8\right) \div \left(x + 4\right)$

7. $\left(x^3 - 2x^2 - 9\right) \div \left(x - 3\right)$

Given one zero of the function, find the remaining zeros.

8. $f(x) = x^3 + 2x^2 + 16x + 32; \ -2$

9. $f(x) = x^3 - 5x^2 - 2x + 24; \ 4$

Use the synthetic substitution to evaluate the function at the given value.

10. $f(x) = 3x^3 - 7x^2 + 4x - 2$ at $x = -2$

11. $f(x) = x^3 - 5x^2 - 2x + 24$ at $x = 4$

Use the remainder theorem to determine if the given binomial is a factor of the polynomial.

12. $(x + 2); f(x) = x^4 - 3x^2 + 2x - 8$

13. $(x - 3); f(x) = x^4 - 14x^2 + 9x + 18$

Given one factor of a polynomial, find the rest of the factors.

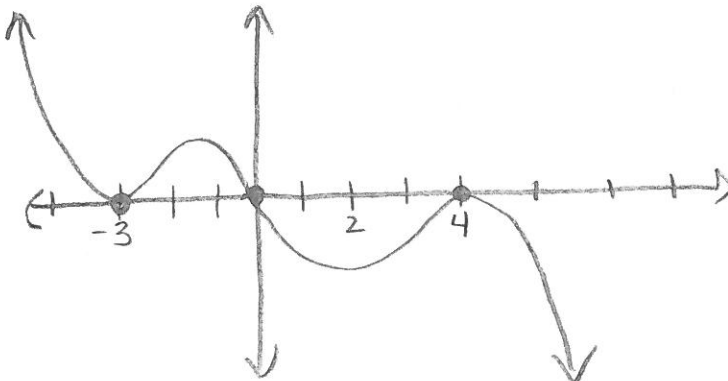14. $x - 4; x^3 + x^2 - 14x - 24$

15. $x + 5; x^3 + 6x^2 - x - 30$

Given the following zeros determine the least possible degree polynomial, then write a polynomial function in standard form with the given zeros.

16. $3i, -2$

17. $9, 0, -1, -i$

18. **For the given graph, give a possible polynomial function (in factored form):**

19. **Sketch (no calculator) the graphs of the polynomials considering end behavior and behavior at roots.**

a) $y = -2x(x + 4)^2(x - 3)(x - 6)^3$

b) $y = 3(x + 4)^2(x - 5)(x + 1)^4(x^2 + 9)$

Test Review

① $x^3+4x^2+9x+36=0$
$x^2(x+4)+9(x+4)=0$
$(x+4)(x^2+9)=0$
$\boxed{x=-4,\ \pm3i}$

② $16x^4-81=0$
$(4x^2+9)(4x^2-9)=0$
$x^2=\dfrac{-9}{4}\qquad x^2=\dfrac{9}{4}$
$\boxed{x=\pm\dfrac{3i}{2}\qquad x=\pm\dfrac{3}{2}}$

③ $2x^4-18x^2-20=0$
$2(x^2-10)(x^2+1)=0$
$\boxed{x=\pm\sqrt{10}\quad x=\pm i}$

④
$$2x-3\ \overline{\smash{\big)}\ 10x^2-11x-6}$$
$\qquad\quad 5x+2$
$\qquad\ -\underline{10x^2+15x}$
$\qquad\qquad\quad 4x-6$
$\qquad\qquad\quad \underline{4x-6}$
$\qquad\qquad\qquad\quad 0$
$\boxed{5x+2}$

⑤
$$3x^2+1\ \overline{\smash{\big)}\ 12x^4+0x^3-5x^2+0x-3}$$
$3x^2+0x+1$
$\qquad\qquad\quad 4x^2-3$
$\qquad\qquad\ \underline{12x^4+0x^3+4x^2}$
$\qquad\qquad\qquad\qquad -9x^2+0x-3$
$\qquad\qquad\qquad\qquad \underline{-9x^2+0x-3}$
$\qquad\qquad\qquad\qquad\qquad\quad 0$
$\boxed{4x^2-3}$

⑥
$$-4\ \big|\ 3\quad 12\quad -5\quad -18\quad 8$$
$$\underline{\qquad -12\quad 0\quad 20\quad -8}$$
$$\quad 3\quad 0\quad -5\quad 2\quad \boxed{0}$$
$\boxed{3x^3-5x+2}$

⑦
$$3\ \big|\ 1\quad -2\quad 0\quad -9$$
$$\underline{\qquad 3\quad 3\quad 9}$$
$$\quad 1\quad 1\quad 3\quad \boxed{0}$$
$\boxed{x^2+x+3}$

⑧
$$-2\ \big|\ 1\quad 2\quad 16\quad 32$$
$$\underline{\qquad -2\quad 0\quad -32}$$
$$\quad 1\quad 0\quad 16\quad \boxed{0}$$
$x^2+16=0$
$x=\pm\sqrt{-16}$
$\boxed{x=\pm4i}$

⑨
$$4\ \big|\ 1\quad -5\quad -2\quad 24$$
$$\underline{\qquad 4\quad -4\quad -24}$$
$$\quad 1\quad -1\quad -6\quad \boxed{0}$$
$x^2-x-6=0$
$(x-3)(x+2)=0$
$\boxed{x=3\quad x=-2}$

⑩
$$-2\ \big|\ 3\quad -7\quad 4\quad -2$$
$$\underline{\qquad -6\quad 26\quad -60}$$
$$\quad 3\quad -13\quad 30\quad \boxed{-62}$$
Ans: $\boxed{-62}$

⑪
$$4\ \big|\ 1\quad -5\quad -2\quad 24$$
$$\underline{\qquad 4\quad -4\quad -24}$$
$$\quad 1\quad -1\quad -6\quad \boxed{0}$$
Ans: $\boxed{0}$

(12).

$$-2 \,\lfloor\; 1 \quad 0 \quad -3 \quad 2 \quad -8$$
$$\;\; -2 \quad 4 \quad -2 \quad 0$$
$$\overline{\;1 \quad -2 \quad 1 \quad 0 \quad \boxed{-8}}$$

$\underline{No}$ Remainder $\neq 0$

(13)

$$3 \,\lfloor\; 1 \quad 0 \quad -14 \quad 9 \quad 18$$
$$\;\; 3 \quad 9 \quad -15 \quad -18$$
$$\overline{\;1 \quad 3 \quad -5 \quad -6 \quad \boxed{0}}$$

$\underline{yes}$ rem $= 0$

(14)

$$4 \,\lfloor\; 1 \quad 1 \quad -14 \quad -24$$
$$\;\; 4 \quad 20 \quad 24$$
$$\overline{\;1 \quad 5 \quad 6 \quad \boxed{0}}$$

$x^2 + 5x + 6$

$(x+2)(x+3)$

"Factor completely"

$\boxed{(x-4)(x+2)(x+3)}$

(15)

$$-5 \,\lfloor\; 1 \quad 6 \quad -1 \quad -30$$
$$\;\; -5 \quad -5 \quad 30$$
$$\overline{\;1 \quad 1 \quad -6 \quad \boxed{0}}$$

$x^2 + x - 6$

$(x+3)(x-2)$

"Factor completely"

$\boxed{(x+5)(x+3)(x-2)}$

(16) $(x-3i)(x+3i)(x+2)$

$(x^2 - 3ix + 3ix - 9i^2)(x+2)$

$(x^2 + 9)(x+2)$

$\boxed{x^3 + 2x^2 + 9x + 18}$

(17) $(x-9)(x-0)(x+1)\underbrace{(x+i)(x-i)}$

$\; x^2 + ix - ix - i^2$

$x(x-9)(x+1)(x^2+1)$

$(x^2 - 9x)(x+1)(x^2+1)$

$(x^3 - 8x^2 - 9x)(x^2+1)$

$x^5 - 8x^4 - 9x^3 + x^3 - 8x^2 - 9x$

$\boxed{= x^5 - 8x^4 - 8x^3 - 8x^2 - 9x}$

(18) $\boxed{-x(x+3)^2(x-4)^2}$

(19)

(20)