

# Graphing Transformations of $1/x$

**Asymptote:** a line that a curve approaches but never reaches

**Parent Function**  $f(x) = \frac{1}{x}$

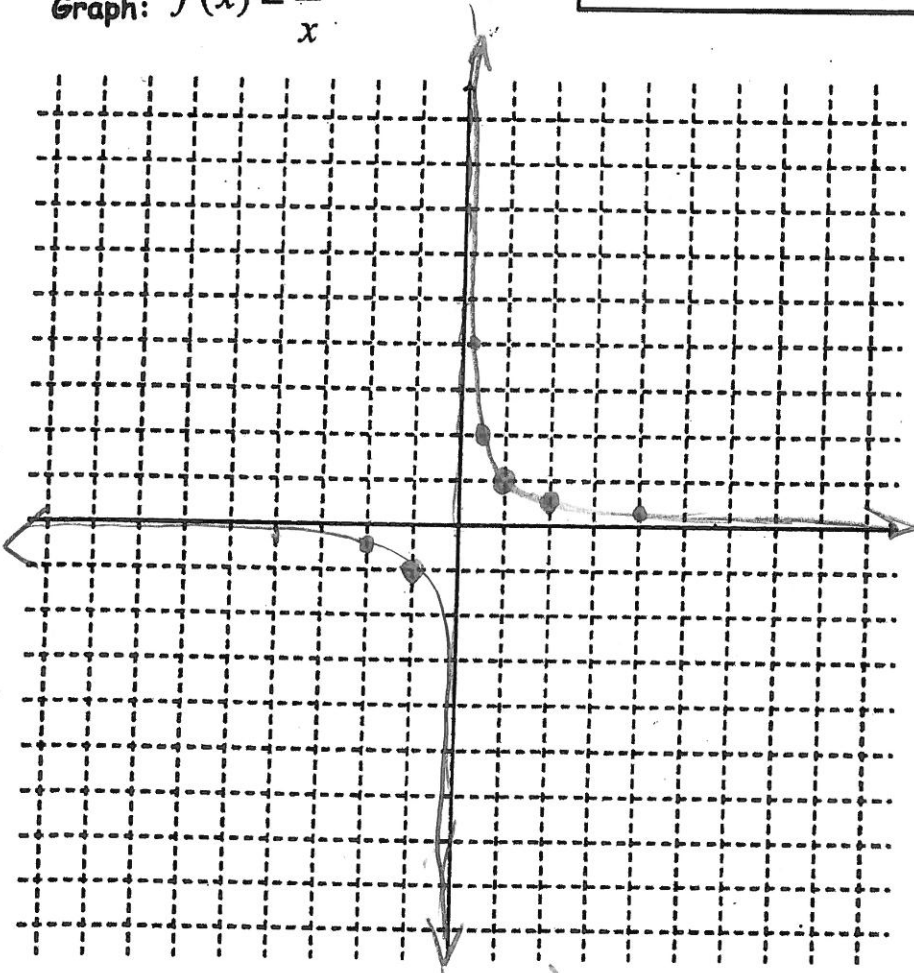
Horizontal Asymptote at  $y=0$

Vertical Asymptote at  $x=0$

**General Form:**  $f(x) = \frac{a}{x-h} + k$

- Horizontal Asymptote at  $y = k$
- Vertical Asymptote at  $x = h$
- $(h, k)$  is the intersection of the asymptote
- "a" vertically stretches or shrinks, if a is negative the graph flips over the x axis

**Graph:**  $f(x) = \frac{1}{x}$



x	y
1	1
-1	-1
2	1/2
4	1/4
1/2	2

$x \neq 0$

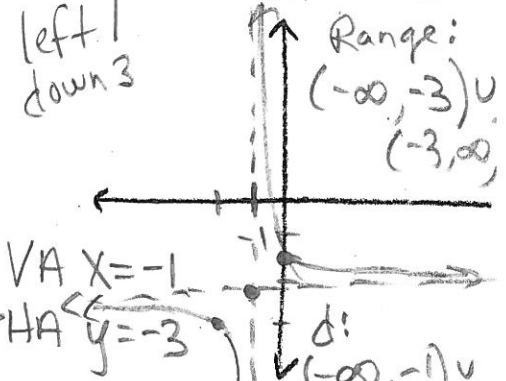
Domain:  $(-\infty, 0) \cup (0, \infty)$  H.A  $y=0$

Range:  $(-\infty, 0) \cup (0, \infty)$  V.A  $x=0$

**Graph:**

$f(x) = \frac{1}{x+1} - 3$

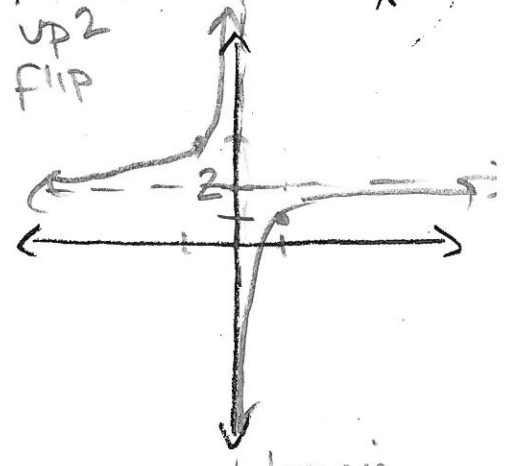
left 1  
down 3



VA  $x = -1$   
HA  $y = -3$

**Graph:**  $f(x) = \frac{-1}{x} + 2$

UP 2  
FLIP



VA:  $x=0$   
HA:  $y=2$   
domain:  $(-\infty, 0) \cup (0, \infty)$   
range:  $(-\infty, 2) \cup (2, \infty)$

# Graphing Transformations of $1/x^2$

Asymptote: a line that a curve approaches but never reaches

Parent Function  $f(x) = \frac{1}{x^2}$

Horizontal Asymptote at  $y=0$

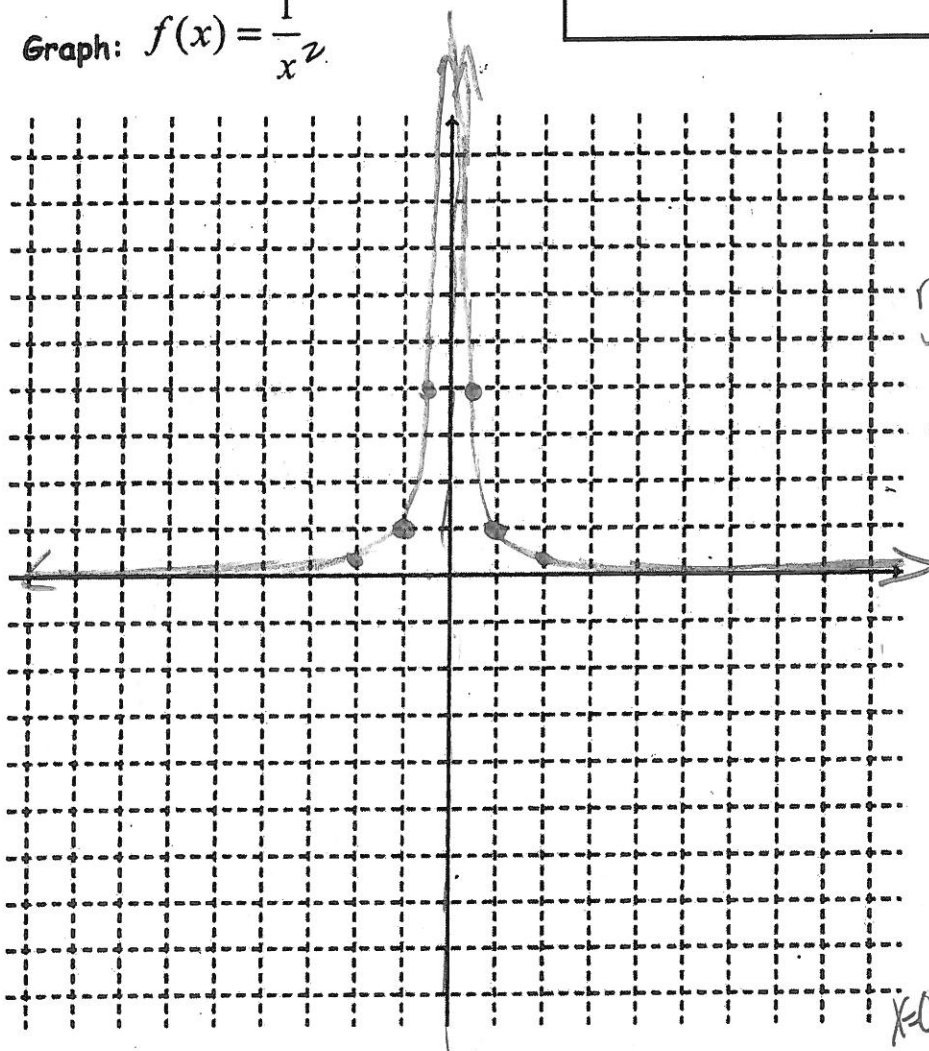
Vertical Asymptote at  $x=0$

General Form:  $f(x) = \frac{a}{(x-h)^2} + k$

- Horizontal Asymptote at  $y = k$
- Vertical Asymptote at  $x = h$
- $(h, k)$  is the intersection of the asymptote
- "a" vertically stretches or shrinks, if a is negative the graph flips over the x axis

Graph:  $f(x) = \frac{1}{x^2}$

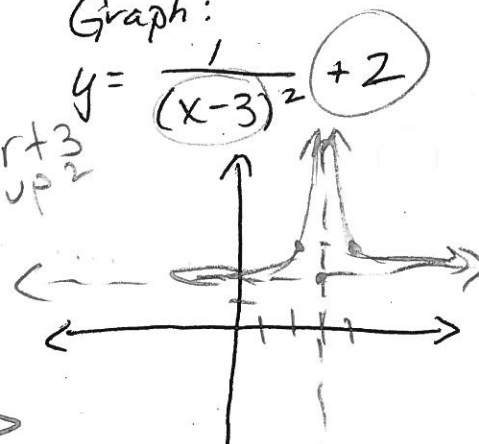
x	y
1	1
2	1/4
1/2	4
-1	1



Graph:

$y = \frac{1}{(x-3)^2} + 2$

r+3  
up 2



VA:  $x=3$

HA:  $y=2$

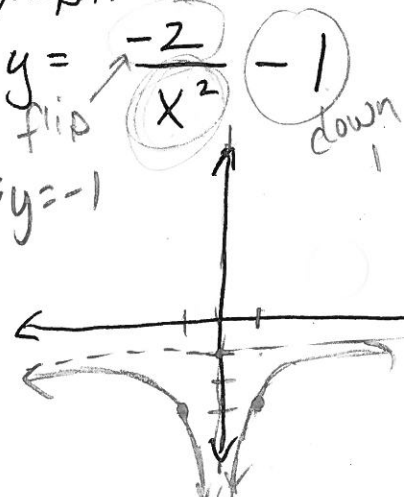
d:  $(-\infty, 3) \cup (3, \infty)$

r:  $(2, \infty)$

Graph:

$y = \frac{-2}{x^2} - 1$

x=0 VA  
flip  
HA:  $y=-1$



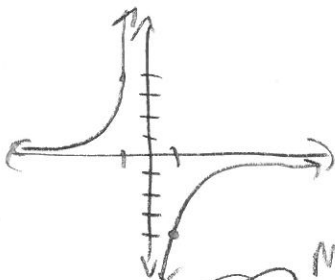
Domain:  $(-\infty, 0) \cup (0, \infty)$  H.A.  $y=0$   
 Range:  $(0, \infty)$  V.A.  $x=0$

d:  
r:

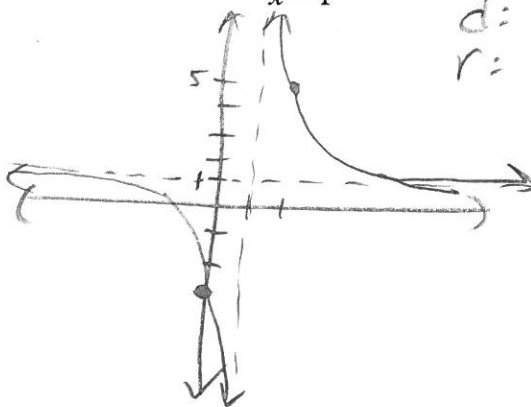
# Graphing Simple Rational Functions

Identify the vertical asymptotes, horizontal asymptote, domain, and range of each. Sketch.

1)  $f(x) = -\frac{4}{x}$  VA:  $x=0$   
 HA:  $y=0$   
 d:  $(-\infty, 0) \cup (0, \infty)$   
 r:  $(-\infty, 0) \cup (0, \infty)$

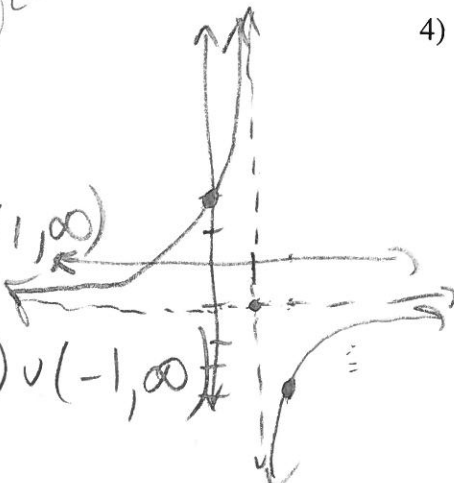


2)  $f(x) = \frac{4}{x-1} + 1$  VA:  $x=1$   
 HA:  $y=1$   
 d:  $(-\infty, 1) \cup (1, \infty)$   
 r:  $(-\infty, 1) \cup (1, \infty)$

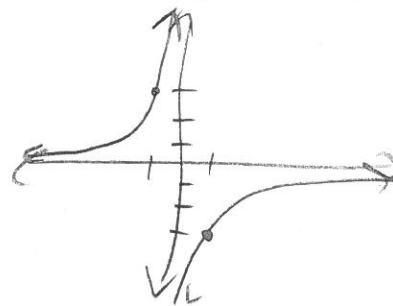


3)  $f(x) = \frac{3}{x-1} - 1$  (circled) (circled) (circled)  
 VA:  $x=1$   
 HA:  $y=-1$   
 dom:  $(-\infty, 1) \cup (1, \infty)$   
 range:  $(-\infty, -1) \cup (-1, \infty)$

mult by 3  
down 1

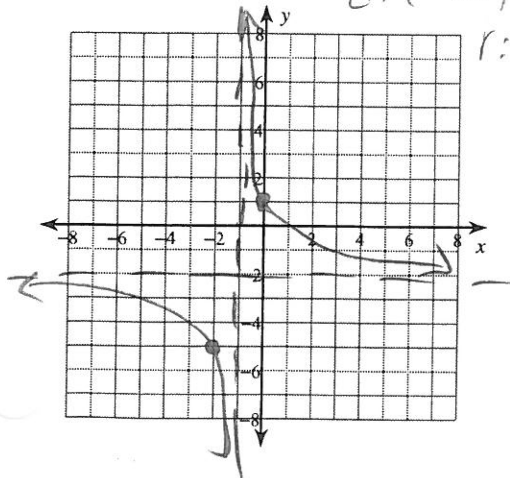


4)  $f(x) = -\frac{3}{x}$  VA:  $x=0$   
 HA:  $y=0$   
 d:  $(-\infty, 0) \cup (0, \infty)$   
 r:  $(-\infty, 0) \cup (0, \infty)$

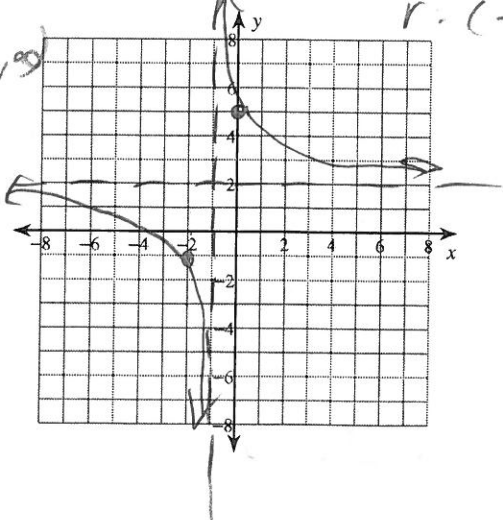


Identify the vertical asymptotes, horizontal asymptote, domain, and range of each. Then sketch the graph.

5)  $f(x) = \frac{3}{x+1} - 2$  VA:  $x=-1$   
 HA:  $y=-2$   
 d:  $(-\infty, -1) \cup (-1, \infty)$   
 r:  $(-\infty, -2) \cup (-2, \infty)$



6)  $f(x) = \frac{3}{x+1} + 2$  VA:  $x=-1$   
 HA:  $y=2$   
 d:  $(-\infty, -1) \cup (-1, \infty)$   
 r:  $(-\infty, 2) \cup (2, \infty)$



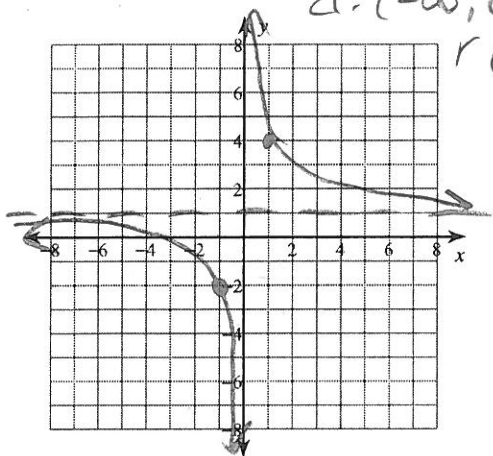
$$7) f(x) = \frac{3}{x} + 1$$

$$VA: x=0$$

$$HA: y=1$$

$$d: (-\infty, 0) \cup (0, \infty)$$

$$r: (-\infty, 1) \cup (1, \infty)$$



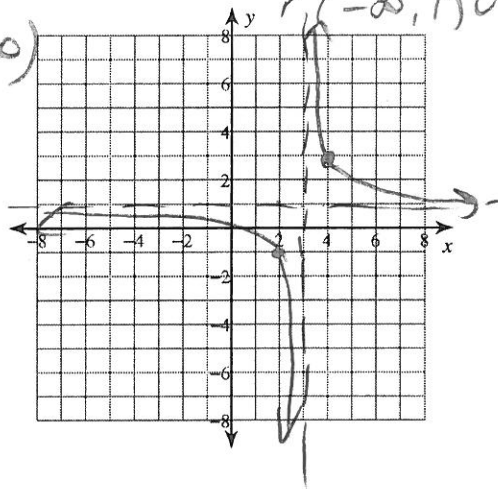
$$8) f(x) = \frac{2}{x-3} + 1$$

$$VA: x=3$$

$$HA: y=1$$

$$d: (-\infty, 3) \cup (3, \infty)$$

$$r: (-\infty, 1) \cup (1, \infty)$$



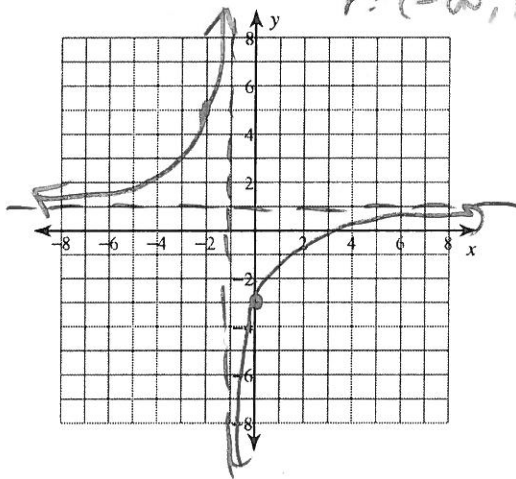
$$9) f(x) = -\frac{4}{x+1} + 1$$

$$VA: x=-1$$

$$HA: y=1$$

$$d: (-\infty, -1) \cup (-1, \infty)$$

$$r: (-\infty, 1) \cup (1, \infty)$$



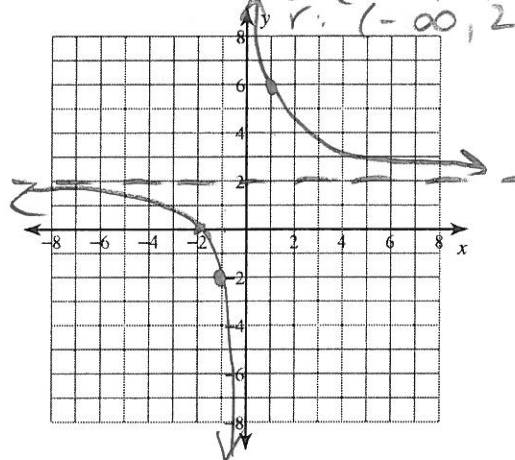
$$10) f(x) = \frac{4}{x} + 2$$

$$VA: x=0$$

$$HA: y=2$$

$$d: (-\infty, 0) \cup (0, \infty)$$

$$r: (-\infty, 2) \cup (2, \infty)$$



### Critical thinking question:

11) Write a function of the form  $f(x) = \frac{a}{x-h} + k$  with a vertical asymptote at  $x=25$ , horizontal asymptote at  $y=3$  and passing through  $(24, 5)$

$$y = \frac{a}{x-25} + 3$$

$$5 = \frac{a}{24-25} + 3$$

$$2 = \frac{a}{-1} \quad a = -2$$

$$y = \frac{-2}{x-25} + 3$$

Name Key

Date \_\_\_\_\_

Period \_\_\_\_\_

## Graphing Rational Functions Worksheet 2

Find the VA and HA of the following:

1.  $\frac{x^2 + 4x - 5}{x^2 + 9x + 20} = \frac{\cancel{(x+5)}(x-1)}{(x+4)\cancel{(x+5)}}$

VA  $x = -4$

HA  $y = 1$

hole:  $(-5, 6)$

2.  $\frac{x^2 - 9}{x + 3} = \frac{\cancel{(x+3)}(x-3)}{\cancel{x+3}}$

VA none

HA none

hole:  $(-3, -6)$

3.  $\frac{x+6}{2x^2 + 9x - 18} = \frac{x+6}{(2x-3)(x+6)}$

VA  $x = 3/2$

HA  $y = 0$

hole:  $(-6, -1/15)$

Graph each equation and fill in all the blanks.

4.  $y = \frac{3}{x+2}$

Domain  $(-\infty, -2) \cup (-2, \infty)$

VA  $x = -2$

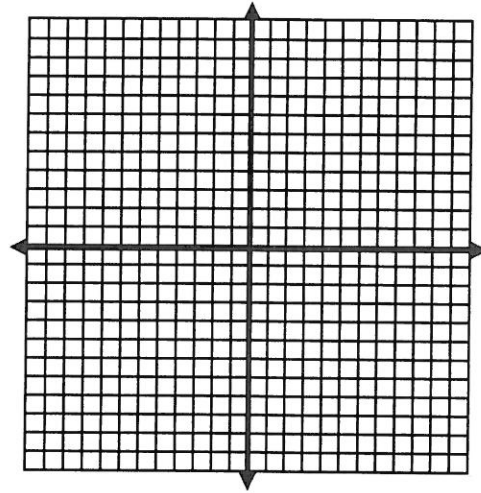
Holes none

-int none

y-int  $(0, 3/2)$

HA  $y = 0$

Continuous/Discontinuous infinite



5.  $y = \frac{x^2 - 9}{x - 3} = \frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}}$

Domain  $(-\infty, 3) \cup (3, \infty)$

VA none

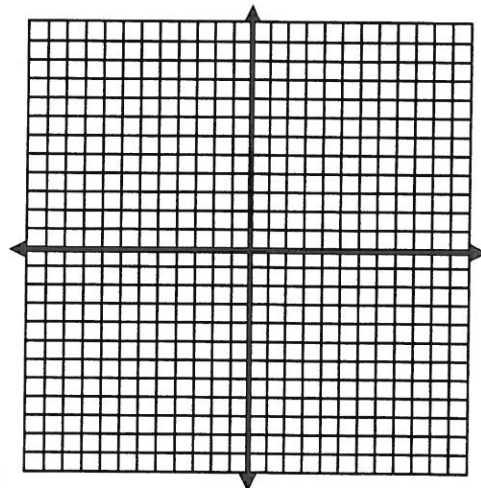
Holes  $(3, 6)$

x-int  $(-3, 0)$

y-int  $(0, 3)$

A none

Continuous/Discontinuous hole (removable)



$$6. y = \frac{x^2 - 2x - 3}{x - 2} = \frac{(x-3)(x+1)}{x-2}$$

Domain  $(-\infty, 2) \cup (2, \infty)$

VA  $x = 2$

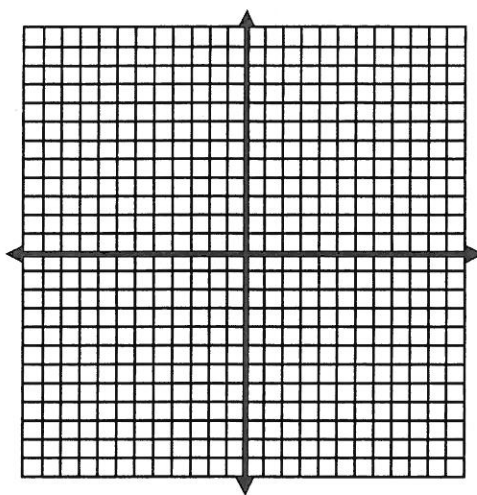
Holes none

x-int  $(3, 0)$   $(-1, 0)$

y-int  $(0, 3/2)$

HA none

Continuous/Discontinuous Infinite



$$7. y = \frac{x+1}{(x-3)^2}$$

Domain  $(-\infty, 3) \cup (3, \infty)$

VA  $x = 3$

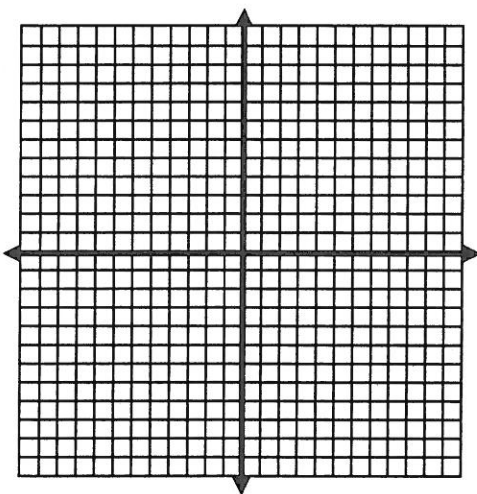
Holes none

x-int  $(-1, 0)$

y-int  $(0, 1/9)$

HA  $y = 0$

Continuous/Discontinuous Infinite @ 3



$$8. y = \frac{x-4}{-4x-16} = \frac{x-4}{-4(x+4)}$$

Domain  $(-\infty, -4) \cup (-4, \infty)$

VA  $x = -4$

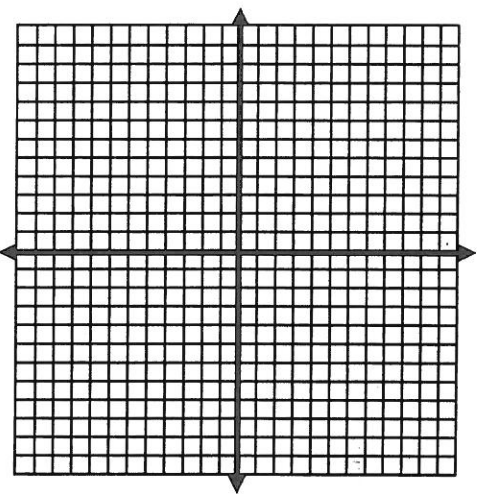
Holes none

x-int  $(4, 0)$

y-int  $(0, 1/4)$

HA  $y = -1/4$

Continuous/Discontinuous Infinite





# Graphing Rational Functions

Identify the points of discontinuity, holes, vertical asymptotes, x-intercepts, and horizontal asymptote of each.

1)  $f(x) = \frac{1}{3x^2 + 3x - 18}$  Discontinuities: -3, 2  
 Vertical Asym.:  $x = -3, x = 2$   
 Holes: None  
 Horz. Asym.:  $y = 0$   
 X-intercepts: None

2)  $f(x) = \frac{x-2}{x-4}$  Discontinuities: 4  
 Vertical Asym.:  $x = 4$   
 Holes: None  
 Horz. Asym.:  $y = 1$   
 X-intercepts: 2 (2, 0)

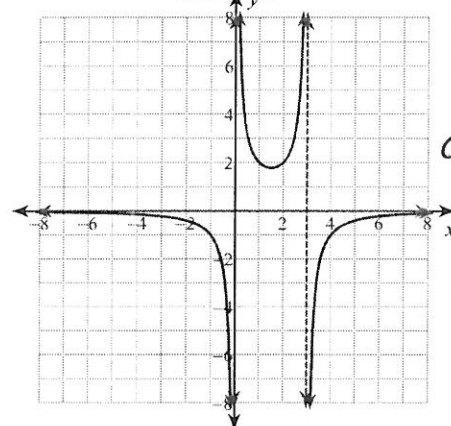
3)  $f(x) = \frac{x^3 - x^2 - 6x}{-3x^2 - 3x + 18}$  Discontinuities: 2, -3  
 Vertical Asym.:  $x = 2, x = -3$   
 Holes: None  
 Horz. Asym.: None  
 X-intercepts: 0, -2, 3

4)  $f(x) = \frac{x^2 + x - 6}{-4x^2 - 16x - 12}$  Discontinuities: -1, -3  
 Vertical Asym.:  $x = -1$   
 Holes:  $x = -3$   $(-3, -\frac{5}{8})$   
 Horz. Asym.:  $y = -\frac{1}{4}$   
 X-intercepts: 2 (2, 0)

*Handwritten work:*  
 $-4(x^2 + 4x + 3) = -4(x+3)(x+1)$   
 $x^2 + x - 6 = (x+3)(x-2)$   
 $\frac{(x+3)(x-2)}{-4(x+3)(x+1)} = \frac{x-2}{-4(x+1)}$

Identify the points of discontinuity, holes, vertical asymptotes, and horizontal asymptote of each. Then sketch the graph.

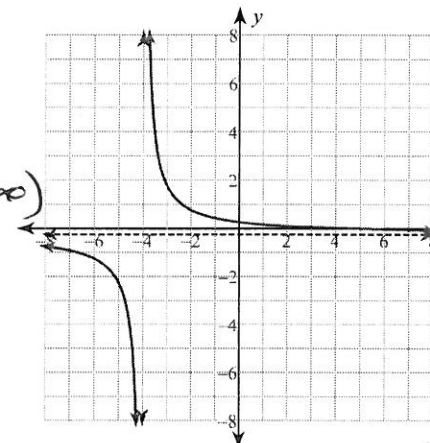
5)  $f(x) = -\frac{4}{x^2 - 3x}$   
 $x(x-3)$



Discontinuities: 0, 3  
 Vertical Asym.:  $x = 0, x = 3$   
 Holes: None  
 Horz. Asym.:  $y = 0$

domain:  $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$   
 x int none  
 y int none

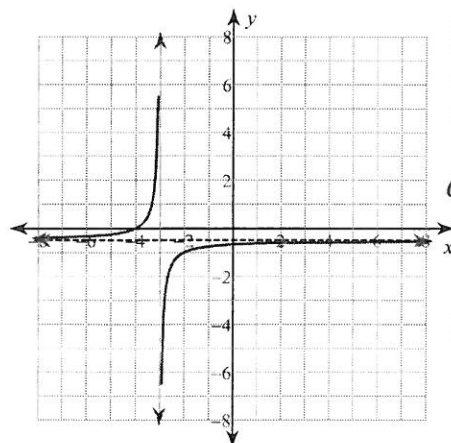
6)  $f(x) = \frac{x-4}{-4x-16}$



Discontinuities: -4  
 Vertical Asym.:  $x = -4$   
 Holes: None  
 Horz. Asym.:  $y = -\frac{1}{4}$

dom:  $(-\infty, -4) \cup (-4, \infty)$   
 x int: (4, 0)  
 y int:  $(0, \frac{1}{4})$

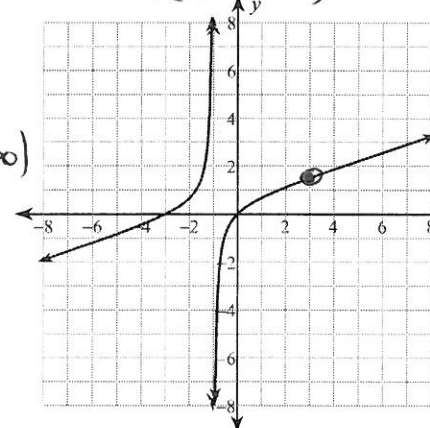
7)  $f(x) = \frac{x+4}{-2x-6}$



Discontinuities: -3  
 Vertical Asym.:  $x = -3$   
 Holes: None  
 Horz. Asym.:  $y = -\frac{1}{2}$

dom  $(-\infty, -3) \cup (-3, \infty)$   
 x int (-4, 0)  
 y int  $(0, -\frac{2}{3})$

8)  $f(x) = \frac{x^3 - 9x}{3x^2 - 6x - 9} = \frac{x(x-3)(x+3)}{3(x-3)(x+1)}$



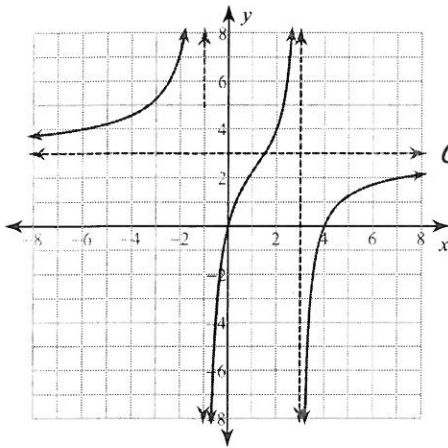
Discontinuities: -1, 3  
 Vertical Asym.:  $x = -1$   
 Holes:  $x = 3$   $(3, \frac{2}{3})$   
 Horz. Asym.: None

dom:  $(-\infty, -1) \cup (-1, 3) \cup (3, \infty)$   
 x int & y int (0, 0)





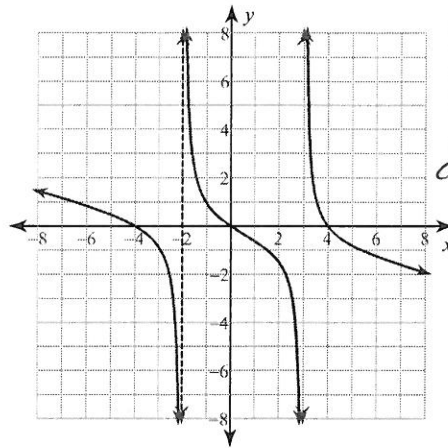
$$9) f(x) = \frac{3x^2 - 12x}{x^2 - 2x - 3}$$



Discontinuities: -1, 3  
Vertical Asym.:  $x = -1, x = 3$   
Holes: None  
Horz. Asym.:  $y = 3$

dom:  $(-\infty, -1) \cup (-1, 3) \cup (3, \infty)$   
 $x$  int:  $(0, 0)$   
 $x$  int:  $(4, 0)$

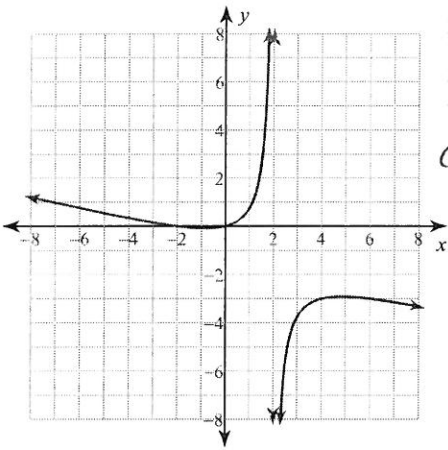
$$10) f(x) = \frac{x^3 - 16x}{-4x^2 + 4x + 24}$$



Discontinuities: 3, -2  
Vertical Asym.:  $x = 3, x = -2$   
Holes: None  
Horz. Asym.: None

dom:  $(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$   
 $x$  int:  $(0, 0)$ ,  $(4, 0)$ ,  $(-4, 0)$   
 $y$  int:  $(0, 0)$

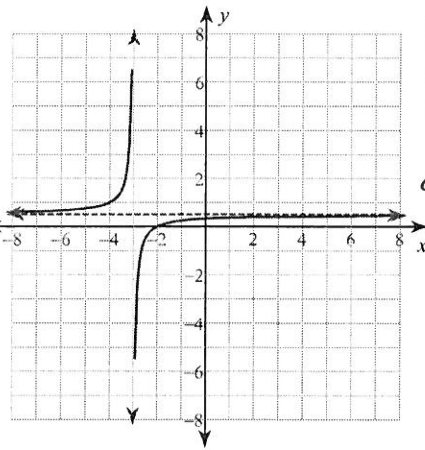
$$11) f(x) = \frac{x^2 + 2x}{-4x + 8} = \frac{x(x+2)}{-4(x-2)}$$



Discontinuities: 2  
Vertical Asym.:  $x = 2$   
Holes: None  
Horz. Asym.: None

dom:  $(-\infty, 2) \cup (2, \infty)$   
 $x$  int:  $(0, 0)$ ,  $(-2, 0)$   
 $y$  int:  $(0, 0)$

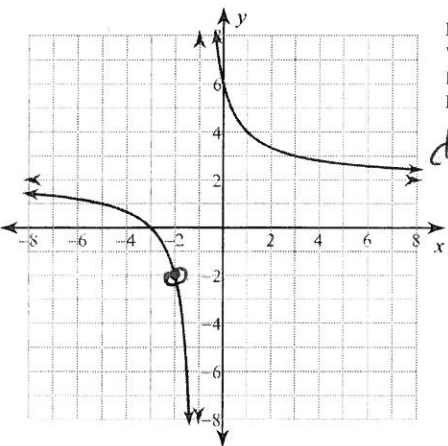
$$12) f(x) = \frac{x+2}{2x+6}$$



Discontinuities: -3  
Vertical Asym.:  $x = -3$   
Holes: None  
Horz. Asym.:  $y = \frac{1}{2}$

dom:  $(-\infty, -3) \cup (-3, \infty)$   
 $x$  int:  $(-2, 0)$   
 $y$  int:  $(0, \frac{1}{3})$

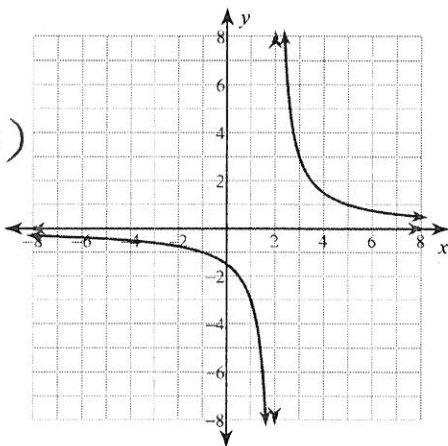
$$13) f(x) = \frac{2(x^2 + 5x + 6)}{2x^2 + 10x + 12} = \frac{2(x+3)(x+2)}{(x+2)(x+1)}$$



Discontinuities: -1, -2  
Vertical Asym.:  $x = -1$   
Holes:  $x = -2$   $(-2, 2)$   
Horz. Asym.:  $y = 2$

dom:  $(-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$   
 $x$  int:  $(-3, 0)$   
 $y$  int:  $(0, 6)$

$$14) f(x) = \frac{3}{x-2}$$



Discontinuities: 2  
Vertical Asym.:  $x = 2$   
Holes: None  
Horz. Asym.:  $y = 0$

dom:  $(-\infty, 2) \cup (2, \infty)$   
 $x$  int: none  
 $y$  int:  $(0, -\frac{3}{2})$



## Worksheet 4.2 day 1: Graphing Rational Functions

Find and ~~graph~~ <sup>classify</sup> all asymptotes and discontinuities.

$$1. y = \frac{(x-2)(x+1)(2x-3)}{(x-1)(x-2)}$$

$$VA: x=1$$

HA: none

hole: (2, 3)

$$\frac{(3)(1)}{1}$$

$$2. y = \frac{3x(x+1)}{x^2-9}$$

$$(x-3)(x+3)$$

$$VA: x=3 \quad x=-3$$

$$HA: y=3$$

hole: none

$$3. y = \frac{2x+1}{6x^2-x-2} = \frac{2x+1}{(2x+1)(3x-2)}$$

$$VA: x = 2/3$$

$$HA: y=0$$

hole:  $(-1/2, -2/5)$

$$4. y = \frac{3(x-2)}{(x-3)(x-2)}$$

$$VA: x=3$$

$$HA: y=0$$

hole: (2, -3)

$$5. y = \frac{x^2-9x-12}{x-1} = \frac{(x)(x)}{x-1}$$

$$VA: x=1$$

HA: none

hole: none

$$6. y = \frac{x+2}{x^2-1} = \frac{x+2}{(x-1)(x+1)}$$

$$VA: x=1 \quad x=-1$$

$$HA: y=0$$

hole: none

$$7. y = \frac{x^2 - 6x + 9}{x^2 - x - 6} = \frac{\cancel{(x-3)}(x-3)}{\cancel{(x-3)}(x+2)}$$

VA  $x = -2$

HA  $y = 1$

hole  $(3, 0)$

$$8. y = \frac{x-1}{2x^2 - 9x - 5} = \frac{x-1}{(2x+1)(x-5)}$$

VA  $x = -\frac{1}{2}$   $x = 5$

HA  $y = 0$

hole: none

$$9. y = \frac{x^2 - 4}{x - 2} = \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}}$$

VA: none

HA none

hole  $(2, 4)$

$$10. y = \frac{x+2}{x^2 - 4} = \frac{\cancel{x+2}}{(x-2)\cancel{(x+2)}}$$

VA:  $x = 2$

HA:  $y = 0$

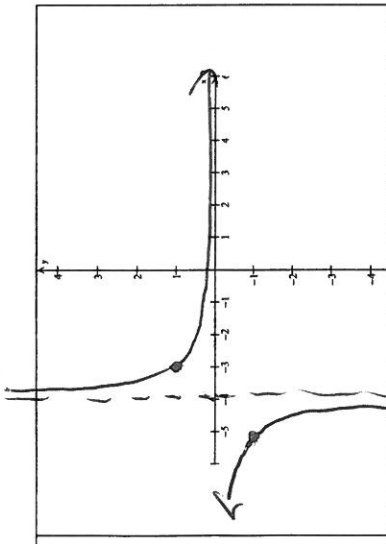
hole:  $(-2, -\frac{1}{4})$

1)  $f(x) = \frac{1}{x+4}$

Hole(s): *None*

Vertical Asymptote(s):  $x = -4$

Horizontal Asymptote:  $y = 0$

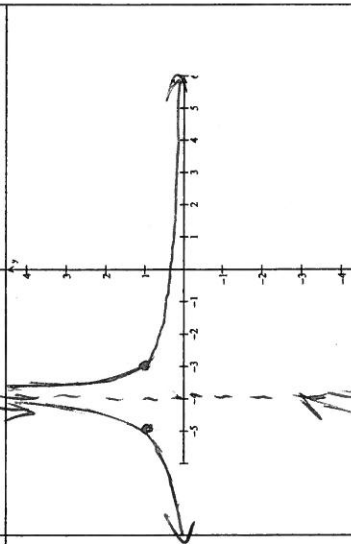


2)  $f(x) = \frac{1}{(x+4)^2}$

Hole(s): *None*

Vertical Asymptote(s):  $x = -4$

Horizontal Asymptote:  $y = 0$

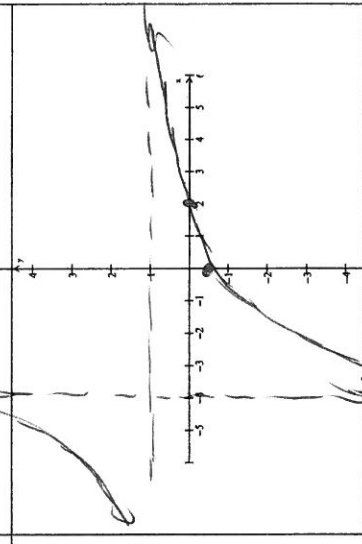


3)  $f(x) = \frac{x-2}{x+4}$

Hole(s): *None*

Vertical Asymptote(s):  $x = -4$

Horizontal Asymptote:  $y = 1$

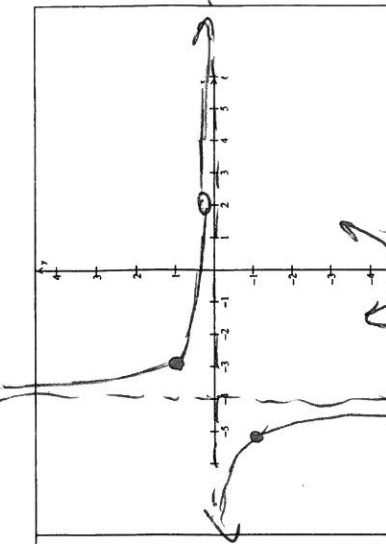


4)  $f(x) = \frac{x-2}{(x+4)(x-2)} = \frac{1}{x+4}$

Hole(s):  $(2, \frac{1}{6})$

Vertical Asymptote(s):  $x = -4$

Horizontal Asymptote:  $y = 0$

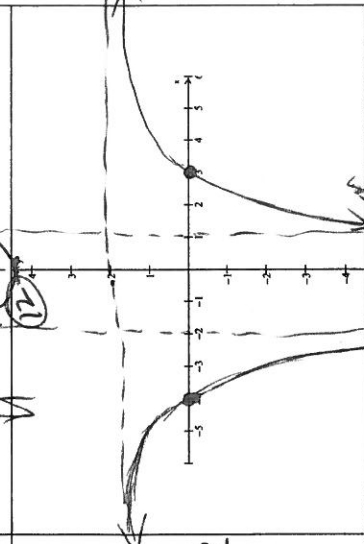


5)  $f(x) = \frac{(2x-6)(x+4)}{(x-1)(x+2)}$

Hole(s): *None*

Vertical Asymptote(s):  $x = 1 \text{ \& } x = -2$

Horizontal Asymptote:  $y = 2$

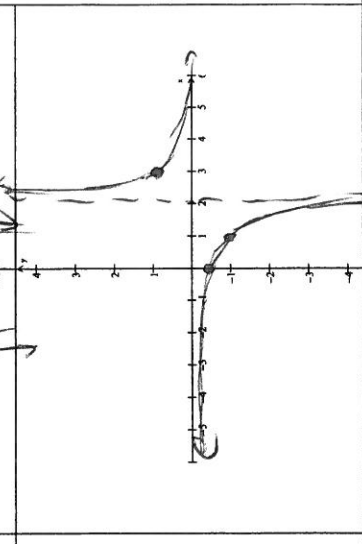


6)  $f(x) = \frac{1}{x-2}$

Hole(s): *None*

Vertical Asymptote(s):  $x = 2$

Horizontal Asymptote:  $y = 0$



$$9) f(x) = \frac{3x^2 - 12x}{x^2 - 2x - 3}$$

$$10) f(x) = \frac{x^3 - 16x}{-4x^2 + 4x + 24}$$

$$11) f(x) = \frac{x^2 + 2x}{-4x + 8}$$

$$12) f(x) = \frac{x + 2}{2x + 6}$$

$$13) f(x) = \frac{2x^2 + 10x + 12}{x^2 + 3x + 2}$$

$$14) f(x) = \frac{3}{x - 2}$$

<p>7) <math>f(x) = \frac{1}{(x-2)^2}</math></p> <p>Hole(s): None</p> <p>Vertical Asymptote(s): <math>x=2</math></p> <p>Horizontal Asymptote: <math>y=0</math></p>	
<p>8) <math>f(x) = \frac{x+1}{x-2}</math></p> <p>Hole(s): None</p> <p>Vertical Asymptote(s): <math>x=2</math></p> <p>Horizontal Asymptote: <math>y=1</math></p>	
<p>9) <math>f(x) = \frac{x-1}{(x+3)(x-5)} = \frac{1}{x+3}</math></p> <p>Hole(s): <math>(1, \frac{1}{4})</math></p> <p>Vertical Asymptote(s): <math>x=-3</math></p> <p>Horizontal Asymptote: <math>y=0</math></p>	

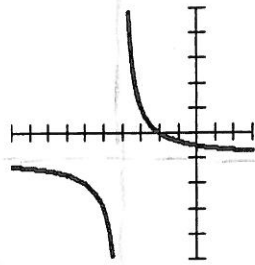
<p>10) <math>f(x) = \frac{(x+1)(x-2)}{(3x-6)(x-5)}</math></p> <p>Hole(s): <math>(2, -\frac{1}{3})</math></p> <p>Vertical Asymptote(s): <math>x=5</math></p>	
<p>11) <math>f(x) = \frac{(-3x+6)(x+4)}{(x-2)(x-1)}</math></p> <p>Hole(s): <math>(2, -18)</math></p> <p>Vertical Asymptote(s): <math>x=1</math></p> <p>Horizontal Asymptote: <math>y=-3</math></p>	<p style="text-align: center;">Dyng</p>
<p>12) <math>f(x) = \frac{(x-2)}{(x-2)(x-5)}</math></p> <p>Hole(s): <math>(2, \frac{1}{9})</math></p> <p>Vertical Asymptote(s): <math>x=5</math></p> <p>Horizontal Asymptote: <math>y=0</math></p>	



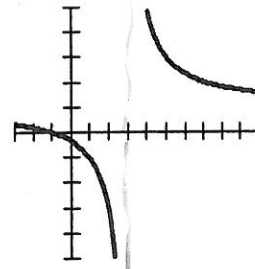
Based on the asymptotes, match each equation with its graph. (Don't use a calculator!)

F 13.  $f(x) = \frac{1}{x-1}$

A)

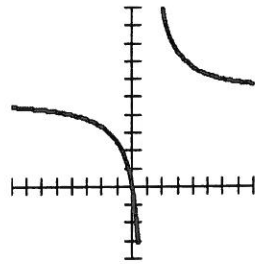


B)

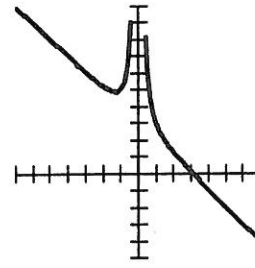


C 14.  $f(x) = \frac{5x}{x-1}$

C)



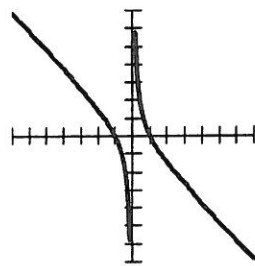
D)



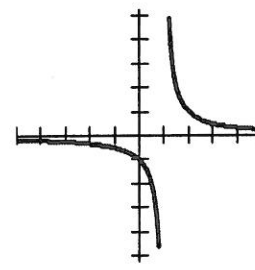
H 15.  $f(x) = \frac{3x^2}{x^2-1}$

VA  $x=1, x=-1$   
HA  $y=3$

E)



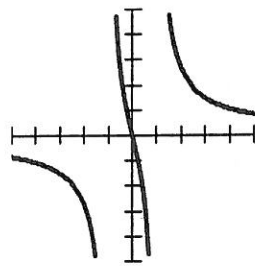
F)



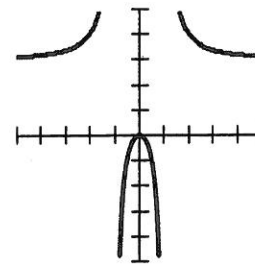
A 17.  $f(x) = \frac{x+2}{x+4}$

HA  $y=-1$

G)

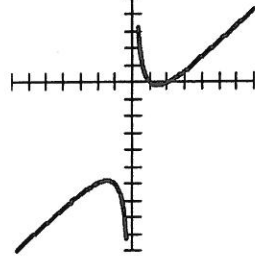


H)

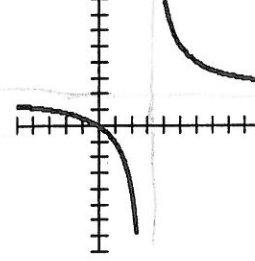


K 18.  $f(x) = \frac{x-1}{x-4}$

I)

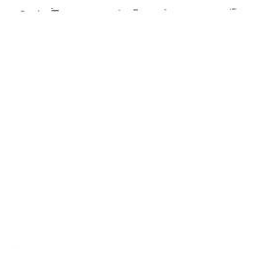


J)

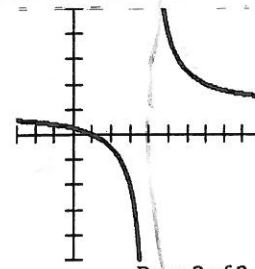


B 19.  $f(x) = \frac{x+1}{x-3}$

J)



K)



J 20.  $f(x) = \frac{2x}{x-3}$

E 21.  $f(x) = \frac{1-x^2}{x}$

I 22.  $f(x) = \frac{x^2-3x+2}{x}$

D 23.  $f(x) = \frac{1+3x^2-x^3}{x^2}$