

(NO CALCULATOR)

Determine the inverses of the following. Be sure to indicate domain restrictions if required.

1.  $f(x) = \frac{4}{3}x - 12$

$$\frac{3}{4}(x+12) = \left(\frac{4}{3}y\right)\frac{3}{4}$$

$$y = \frac{3}{4}x + 9$$

2.  $f(x) = \sqrt{x+3} - 8$

$$x = \sqrt{y+3} - 8$$

$$(x+8)^2 = \sqrt{y+3}^2$$

$$y = (x+8)^2 - 3$$

$$x \geq -8$$

3.  $f(x) = \frac{x^3}{27} - 4$

$$x+4 = \frac{y^3}{27}$$

$$27(x+4) = y^3$$

$$y = \sqrt[3]{27(x+4)}$$

OR  $y = \sqrt[3]{27x+108}$

OR  $y = 3\sqrt[3]{x+4}$

4.  $f(x) = x^2 - 6x + 5$

$$x-5 = y^2 - 6y + 5$$

$$\sqrt{x+4} = \sqrt{(y-3)^2}$$

$$\pm\sqrt{x+4} = y-3$$

$$y = \pm\sqrt{x+4} + 3$$

Solve the following:

6.  $3^{x-2} = 81^{x+1}$

$$3^{x-2} = 3^{4(x+1)}$$

$$x-2 = 4x+4$$

$$-6 = 3x$$

$$x = -2$$

7.  $(8)^x = \frac{1}{32}$

$$2^{3x} = 2^{-5}$$

$$x = \frac{-5}{3}$$

8.  $\sqrt[5]{5} = 25^{4x}$

$$5^{\frac{1}{5}} = 5^{2(4x)}$$

$$\frac{1}{5} = 8x$$

$$x = \frac{1}{40}$$

9.  $4(8^{x-2}) = 32^{2x-1}$

$$2^2(2^{3x-6}) = 2^{5(2x-1)}$$

$$2+3x-6 = 10x-5$$

$$3x-4 = 10x-5$$

$$1 = 7x$$

$$x = \frac{1}{7}$$

10.  $\log_5(x+6) = \log_5(3x-8)$

$$x+6 = 3x-8$$

$$14 = 2x$$

$$x = 7$$

11.  $\frac{3\log_2(3x-9)}{3} = \frac{9}{3}$

$$\log_2(3x-9) = 3$$

$$3x-9 = 2^3$$

$$3x-9 = 8$$

$$3x = 17$$

$$x = \frac{17}{3}$$

12.  $\ln(x^2 - x) = \ln 2$

$$x^2 - x = 2$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x = 2 \quad x = -1$$

13. Rewrite as a logarithm:

$$3^x = w \quad \log_3 w = x$$

14. Rewrite as exponential:  $\ln x = 6$

$$e^6 = x$$

Evaluate:

15.  $\log_{\frac{1}{3}} 1$  0

16.  $\ln(e^4)$  4

17.  $\log_4\left(\frac{1}{32}\right)$   $-\frac{5}{2}$

$$4^x = \frac{1}{32}$$

$$2^{2x} = 2^{-5}$$

Write an exponential model that describes the situation. Then find the value after the given years.

1. You bought a sculpture for \$380. Each year the value of the sculpture increases by 8%. What is the value after 20 years?

Model:  $380(1.08)^{20}$

Value/Answer:  $\$1771.16$

2. You buy a new car valued at \$19,000. It decreases at a rate of 14% each year. What is the value of the car in 3 years?

Model:  $19000(0.86)^3$   
 $\uparrow$   
 $1 - .14$

Value/Answer:  $\$12085.06$

Without graphing, give the initial value and determine whether each equation represents exponential growth or exponential decay. Then determine the percent increase or decrease.

3.  $y = 2(.6)^x$  Initial Value: 2  
Growth/Decay? Decay  
% increase/decrease? 40%

4.  $y = 3(1.184)^x$  Initial Value: 3  
Growth/Decay? Growth  
% increase/decrease? 18.4%

Solve each equation using inverse operations. You must show your work! Round your answer to <sup>3</sup> decimal places. Circle your final answer.

5.  $\log_4(x-2) = 3$

$4^3 = x-2$   
 $64 = x-2$   $x = 66$

7.  $\frac{2 \ln(x+3)}{2} = \frac{4}{2}$

$\ln(x+3) = 2$   
 $e^2 = x+3$   $x = e^2 - 3$   
 $= 4.389$

9.  $2 \cdot \frac{1}{2} e^{3x} = 62 \cdot 2$

$e^{3x} = 124$   $x = 1.607$   
 $\frac{\ln 124}{3} = \frac{3x}{3}$

11.  $e^{2x-1} - 7 = 12$

$e^{2x-1} = 19$   
 $2x-1 = \ln 19$   $x = 1.972$

13.  $3^{x+8} = 27^{x-1}$

$3^{x+8} = 3^{3(x-1)}$

$x+8 = 3x-3$

$11 = 2x$

$x = \frac{11}{2}$

6.  $2^{x-1} - 3 = 12$

$\uparrow +3$   $\uparrow +3$   $2^{x-1} = 15$   
 $(x-1) \log 2 = \log 15$   
 $\frac{(x-1) \log 2}{\log 2} = \frac{\log 15}{\log 2}$   
 $x = 4.907$

8.  $\log_6 x + \log_6(x-5) = 2$

$\log_6(x^2 - 5x) = 2$   $x^2 - 5x = 36$   
 $x^2 - 5x - 36 = 0$   
 $(x-9)(x+4) = 0$

10.  $4^x + 2 = 8$

$4^x = 6$   $x = 9$   ~~$x = 4$~~   
 $x \log 4 = \log 6$   $\frac{x \log 6}{\log 4} = 1.292$   
 $x = 1.292$

12.  $\log(x^2) = \log(8x+20)$

$x^2 = 8x+20$

$x^2 - 8x - 20 = 0$

$(x-10)(x+2) = 0$

$x = 10$   
 $x = -2$

14.  $\ln 3x - \ln 7 = -2$

$\ln\left(\frac{3x}{7}\right) = -2$

$e^{-2} = \frac{3x}{7}$

$x = .3158$

$$3000 = 1000 \left(1 + \frac{0.085}{4}\right)^{4t}$$

$$3 = (1.02125)^{4t}$$

$$\log 3 = 4t \log(1.02125)$$

$$t = 13.062 \text{ yrs}$$

16. Suppose you deposit \$3000 in an account that pays 4% interest compounded semi-annually. What is the balance in the account after 12 years?

$$3000 \left(1 + \frac{0.04}{2}\right)^{2(12)} = \$4825.31$$

17. Suppose \$250 is deposited in a savings account. The interest rate is 5% compounded monthly. How long will it be until the account has \$600?

$$600 = 250 \left(1 + \frac{0.05}{12}\right)^{12t}$$

$$2.4 = (1.041\bar{6})^{12t}$$

$$t = 17.546 \text{ yrs}$$

18. The half-life of plutonium is 24,000 years. How much of a 2 gram sample of plutonium will remain 4,500 years?

$$2 \left(\frac{1}{2}\right)^{4500/24000} = 1.756 \text{ g}$$

19. Carbon-14 has a half-life of 5730 years. A sample of fossilized wood initially contained 24g of C-14 when alive. How many years will it take the sample to decay to 1.5g.

$$1.5 = 24 \left(\frac{1}{2}\right)^{t/5730}$$

$$0.625 = \left(\frac{1}{2}\right)^{t/5730}$$

$$t = 22920 \text{ years}$$

20. A \$40,000 car purchased in 2012 depreciates at a constant rate of 12% per year. What was the value of the car in 2016. (Let  $t = 0$  represent 2012).

$$40000 (.88)^4 = \$23987.81$$

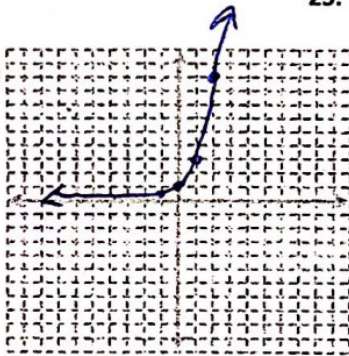
21. In a swamp, the number of bugs increases at a rate of 6% each hour. If there are currently 3500 bugs, how many bugs will there be in 8 hours?

$$3500 (1.06)^8 = 5578 \text{ bugs}$$

Graph. Give the domain and the equations of any asymptotes.

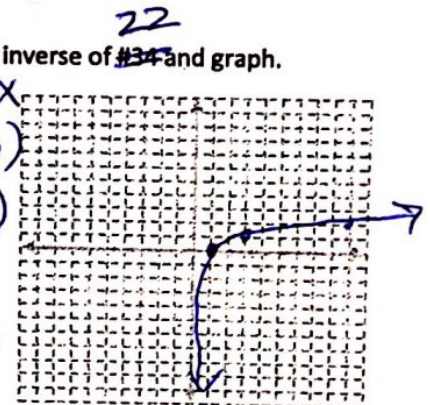
22.  $y = 3^x$

Domain:  $(-\infty, \infty)$   
 Range:  $(0, \infty)$   
 Asymptote:  $y = 0$   
 y-intercept:  $(0, 1)$



23. Find the equation of the inverse of #22 and graph.

Equation:  $y = \log_3 x$   
 Domain:  $(0, \infty)$   
 Range:  $(-\infty, \infty)$   
 Asymptote:  $x = 0$   
 x-intercept:  $(1, 0)$



x	y
-1	1/3
0	1
1	3
2	9

$$x = 3^y$$

$$y = \log_3 x$$

→ switch x & y

\* remember for inverses, x & y switch so domain & range switch too.