

Analyzing and Solving Polynomial Equations

1) $x^4 - 5x^2 - 36 = 0$

~~2) $x^3 + 3x^2 - 14x - 20 = 0$~~

3) $x^3 - 2x^2 + 3x - 6 = 0$

4) $x^4 - 14x^2 + 45 = 0$

5) $x^4 + 6x^2 + 8 = 0$

6) $x^4 + 3x^2 - 18 = 0$

7) $x^3 - 1 = 0$

8) $x^3 + 3x^2 - x - 3 = 0$

$$9) x^3 - 2x^2 - 3x + 6 = 0$$

$$10) x^6 - 2x^4 - 4x^2 + 8 = 0$$

$$11) x^5 + 2x^4 + 11x^3 + 22x^2 + 24x + 48 = 0$$

$$12) x^6 + 5x^4 - 4x^2 - 20 = 0$$

$$13) x^6 - x^4 - x^2 + 1 = 0$$

$$14) x^8 - 26x^4 + 25 = 0$$

Analyzing and Solving Polynomial Equations

State the number of complex roots, the possible number of real and imaginary roots, the possible number of positive and negative roots, and the possible rational roots for each equation. Then find all roots.

1) $x^4 - 5x^2 - 36 = 0$

$$(x^2 - 9)(x^2 + 4) = 0$$

$$x = \pm 3 \quad x = \pm 2i$$

~~2) $x^3 + 3x^2 - 14x - 20 = 0$~~

3) $x^3 - 2x^2 + 3x - 6 = 0$

$$x^2(x-2) + 3(x-2)$$

$$(x^2 + 3)(x-2) = 0$$

$$x = \pm\sqrt{3} \quad x = 2$$

4) $x^4 - 14x^2 + 45 = 0$

$$(x^2 - 9)(x^2 - 5) = 0$$

$$x = \pm 3 \quad x = \pm\sqrt{5}$$

5) $x^4 + 6x^2 + 8 = 0$

$$(x^2 + 4)(x^2 + 2) = 0$$

$$x = \pm 2i \quad x = \pm i\sqrt{2}$$

6) $x^4 + 3x^2 - 18 = 0$

$$(x^2 + 6)(x^2 - 3) = 0$$

$$x = \pm i\sqrt{6} \quad x = \pm\sqrt{3}$$

7) $x^3 - 1 = 0$

$$(x-1)(x^2+x+1) = 0$$

$$x = \frac{-1 \pm \sqrt{1-4(1)(1)}}{2}$$

$$= \frac{-1 \pm \sqrt{-3}}{2}$$

$$= \frac{-1 \pm i\sqrt{3}}{2}$$

$$x=1$$

8) $x^3 + 3x^2 - x - 3 = 0$

$$x^2(x+3) - 1(x+3) = 0$$

$$(x^2 - 1)(x+3) = 0$$

$$x = \pm 1 \quad x = -3$$

$$9) x^3 - 2x^2 - 3x + 6 = 0$$

$$x^2(x-2) - 3(x-2)$$

$$(x^2 - 3)(x-2) = 0$$

$$x = \pm\sqrt{3} \quad x = 2$$

$$10) x^6 - 2x^4 - 4x^2 + 8 = 0$$

$$x^4(x^2-2) - 4(x^2-2) = 0$$

$$(x^4 - 4)(x^2 - 2)$$

$$(x^2 - 2)(x^2 + 2)(x^2 - 2)$$

$$x = \pm\sqrt{2} \rightarrow \text{both m 2}$$

$$x = \pm i\sqrt{2}$$

$$11) x^5 + 2x^4 + 11x^3 + 22x^2 + 24x + 48 = 0$$

$$x^4(x+2) + 11x^2(x+2) + 24(x+2) = 0$$

$$(x+2)(x^4 + 11x^2 + 24)$$

$$(x+2)(x^2+8)(x^2+3) = 0$$

$$x = -2 \quad x = \pm 2i\sqrt{2} \quad x = \pm i\sqrt{3}$$

$$12) x^6 + 5x^4 - 4x^2 - 20 = 0$$

$$x^4(x^2+5) - 4(x^2+5) = 0$$

$$(x^4 - 4)(x^2 + 5)$$

$$(x^2 - 2)(x^2 + 2)(x^2 + 5) = 0$$

$$x = \pm\sqrt{2} \quad x = \pm i\sqrt{2} \quad x = \pm i\sqrt{5}$$

$$13) x^6 - x^4 - x^2 + 1 = 0$$

$$x^4(x^2-1) - 1(x^2-1) = 0$$

$$(x^4 - 1)(x^2 - 1)$$

$$(x^2 - 1)(x^2 + 1)(x^2 - 1) = 0$$

$$x = \pm 1 \Rightarrow \text{both m 2}$$

$$x = \pm i$$

$$14) x^8 - 26x^4 + 25 = 0$$

$$(x^4 - 25)(x^4 - 1) = 0$$

$$(x^2 - 5)(x^2 + 5)(x^2 - 1)(x^2 + 1) = 0$$

$$x = \pm\sqrt{5} \quad x = \pm i\sqrt{5} \quad x = \pm 1 \quad x = \pm i$$