Solving Rational Equations

Because division by zero is not allowed, there may need to be restrictions placed on the variable in a rational expression.

Examples: $\dfrac{z^2-1}{z^2+5}$ *no restrictions on z*

$\dfrac{m^4+18m+1}{m^2-m-6}$ $m \neq 3,-2$

To Solve Rational Equations:

- Name the restrictions on the variable.
- If the rational equation is written as a proportion (a statement that two ratios are equal):
  - If the denominators are the same, the numerators must be equal.
  - If the denominators are different, cross multiply to solve .
- If the rational equation is not written as a proportion:
  1. Find the LCD of all terms.
  2. Multiply both sides of the equation by the LCD of all terms to eliminate fractions
  3. Solve for the variable.
- Always check solutions to be sure they work.
  - **Solutions cannot be restricted values.**

set num = since denom =

Ex 1: $\dfrac{7}{m-3}=\dfrac{m+2}{m-3}$ $m \neq 3$

$7=m+2$

$m=5$

Ex 1: The equation is a proportion and the denominators are the same so the numerators must be equal.

Ex 2: $\dfrac{5}{x-2}=\dfrac{x+3}{x-2}$ $x \neq 2$

$5=x+3$

$x=\cancel{2}$

No solution.

Ex 2: Since the restriction on x says x cannot be 2, there is no solution.

Cross multiply!

Ex 3: $\dfrac{7}{x+2}=\dfrac{6}{x-5}$ $x \neq -2,5$

$7(x-5)=6(x+2)$

$7x-35=6x+12$

$x=47$

Ex 3: The equation is a proportion with different denominators, so cross multiply to solve.

mult by common denom

Ex 4: $\dfrac{1}{2x}-\dfrac{2}{5x}=\dfrac{1}{2}$ $x \neq 0$

Ex 4: The equation is not a proportion so…
1. Find the LCD of all terms
2. Multiply both sides of the equation by the LCD to eliminate fractions
3. Solve for the variable.

Find the LCD:

2x = 2 * x
5x = 5 * x
2 = 2

- 2 is a factor, the greatest number of 2's is 1
- x is a factor, the greatest number of x's is 1
- 5 is a factor, the greatest number of 5's is 1

LCD is 2*x*5 = 10x

$$10x\left(\frac{1}{2x}-\frac{2}{5x}\right)=10x\left(\frac{1}{2}\right)$$

$$\frac{10x}{2x}-\frac{20x}{5x}=\frac{10x}{2}$$

$$5-4=5x$$

$$\frac{1}{5}=x$$

Ex. 5 $\dfrac{10x}{x+2}-\dfrac{2x-3}{x-2}=\dfrac{2x^2-3}{x^2-4}$

*mult by $(x+2)(x-2)$ Distribute!

$(x-2)(x+2)\left[\dfrac{10x}{x+2}-\dfrac{2x-3}{x-2}=\dfrac{2x^2-3}{(x-2)(x+2)}\right](x-2)(x+2)$

$10x(x-2)-(2x-3)(x+2)=2x^2-3$

$10x^2-20x-(2x^2+x-6)=2x^2-3$

$8x^2-21x+6=2x^2-3$

$6x^2-21x+9=0$

$3(2x^2-7x+3)=0$

$3(2x-1)(x-3)=0$

$\boxed{\tfrac{1}{2} \quad x=3}$

x+=0 solve