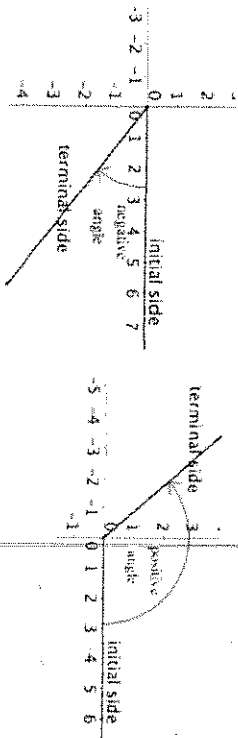


**Trigonometry:** the branch of mathematics that studies relationships involving lengths and angles of triangles.

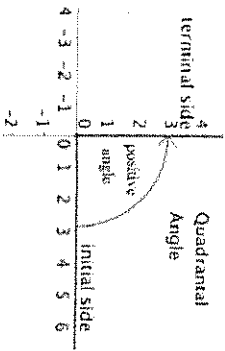
**Angle:** generated by the rotation of two rays that share a fixed endpoint

- > **Initial side:** fixed ray
- > **Terminal side:** ray that rotates away from the initial side
- Positive angle – counterclockwise rotation
- Negative angle – clockwise rotation

An angle with its vertex at the origin and its initial side along the positive x-axis is said to be in **standard position**.



If the terminal side of an angle in standard position coincides with one of the axes, the angle is called a **quadrantal angle**.



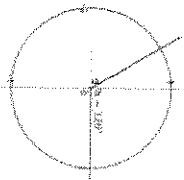
**Units for measuring angles:**

**Degrees:** A circle is divided into 360 equal degrees, so that a right angle is  $90^\circ$ .

**Radians:** The radian measure of an angle in standard position is defined as the length of the corresponding arc on the unit circle.

A **unit circle** is a circle of radius 1 whose center is at the origin. The equation of the unit circle is  $x^2 + y^2 = 1$ .

The circumference of a unit circle is  $2\pi$ , so the radian measure of an angle of one full revolution (or  $360^\circ$ ) is  $2\pi$ .



**Degree/Radian Conversion:**

>  $180^\circ = \pi$  radians

- > Radians are usually written in term of  $\pi$
- > the degree symbol ( $^\circ$ ) is always written, the word "radians" is usually omitted

One radian is the measure of the central angle of a circle that intercepts an arc equal in length to the radius of the circle.

**Examples:**

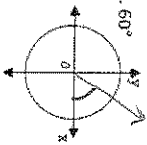
1. Change  $30^\circ$  to radians:  $30^\circ \cdot \frac{\pi}{180} = \frac{\pi}{6}$  radians

2. Change  $\frac{3\pi}{4}$  radians to degrees:  $\frac{3\pi}{4} \cdot \frac{180}{\pi} = 135^\circ$

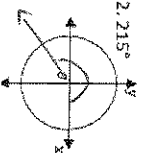
**Practice:**

Sketch each angle in standard position.

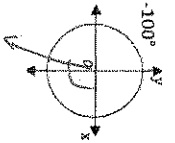
1.  $60^\circ$



2.  $215^\circ$



3.  $-100^\circ$



4.  $\frac{5\pi}{4}$



*Kay*

If each angle has the given measure and is in standard position, determine the quadrant in which its terminal side lies.

7.  $75^\circ$  Q1    8.  $-132^\circ$  (3)    9.  $248^\circ$  (3)    10.  $-305^\circ$  (1)    11.  $465^\circ$  (2)

12.  $\frac{7\pi}{8}$  (2)    13.  $-\frac{3\pi}{4}$  (3)    14.  $\frac{\pi}{5}$  (4)    15.  $\frac{13\pi}{6}$  (4)    16.  $\frac{5\pi}{14}$  (1)

Convert degrees to radians:

17.  $45^\circ$     18.  $150^\circ$     19.  $72^\circ$     20.  $270^\circ$     21.  $99^\circ$

Convert radians to degrees:

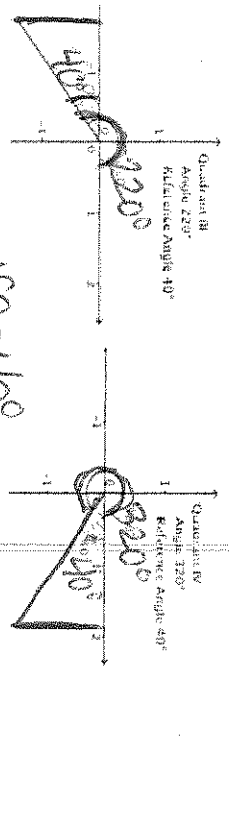
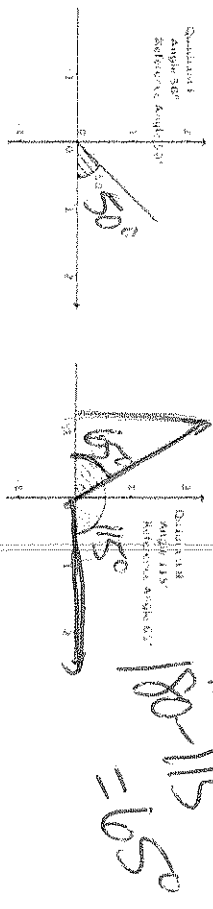
$\frac{\pi}{4}$      $5\pi/6$      $\frac{2\pi}{5}$      $\frac{3\pi}{2}$      $\frac{11\pi}{20}$

22. 2    23. 4    24.  $\frac{2\pi}{5}$     25.  $\frac{5\pi}{6}$     26.  $\frac{3\pi}{14}$     27.  $90^\circ$     28.  $135^\circ$     29.  $72^\circ$     30.  $150^\circ$     31.  $38.57^\circ$

Two angles in standard position are called **coterminal angles** if they have the same terminal side. Angles with the same terminal side differ only in the number of revolutions.

- Angles differing in radian measure by multiples of  $2\pi$  are equivalent.
- Angles differing in degree measure by  $360^\circ$  are equivalent.
- Every angle has infinitely many coterminal angles.

**Reference Angle:** the acute angle ( $< 90^\circ$ ) formed by the terminal side of the given angle and the x-axis. Reference angles are always positive and always less than  $90^\circ$ .



**Practice:**

Find one positive angle and one negative angle that are coterminal with each angle.

1.  $110^\circ$     2.  $-424^\circ$     3.  $\frac{7\pi}{8}$     4.  $-\frac{9\pi}{5}$   
 5.  $470^\circ$     6.  $-64^\circ$     7.  $\frac{23\pi}{8}$     8.  $\frac{\pi}{5}$     9.  $-\frac{19\pi}{5}$   
 10.  $-250^\circ$     11.  $296^\circ$     12.  $-\frac{9\pi}{8}$

Find the reference angle for each angle with the given measure.

1.  $-35^\circ$     2.  $65^\circ$     3.  $30^\circ$     4.  $85^\circ$   
 5.  $35^\circ$     6.  $65^\circ$     7.  $30^\circ$     8.  $85^\circ$

9.  $\frac{7\pi}{4}$     10.  $-\frac{8\pi}{5}$     11.  $\frac{7\pi}{6}$     12.  $\frac{4\pi}{9}$   
 13.  $\frac{\pi}{4}$     14.  $\frac{\pi}{5}$     15.  $\frac{\pi}{6}$     16.  $\frac{\pi}{9}$

- Find a coterminal angle between  $0^\circ$  and  $360^\circ$ .
- 13)  $-330^\circ$     14)  $-435^\circ$     15)  $640^\circ$     16)  $-442^\circ$   
 17)  $30^\circ$     18)  $285^\circ$     19)  $280^\circ$     20)  $278^\circ$

- Find a coterminal angle between  $0$  and  $2\pi$  for each given angle.
- 17)  $\frac{11\pi}{3}$     18)  $-\frac{35\pi}{18}$     19)  $\frac{15\pi}{4}$     20)  $-\frac{19\pi}{12}$   
 21)  $\frac{5\pi}{4}$     22)  $\frac{25\pi}{36}$     23)  $\frac{7\pi}{4}$     24)  $\frac{5\pi}{12}$

- Find a positive and a negative coterminal angle for each given angle.
- 21)  $\frac{5\pi}{4}$     22)  $\frac{25\pi}{36}$     23)  $\frac{13\pi}{4}$     24)  $-\frac{47\pi}{36}$   
 25)  $-\frac{9\pi}{4}$     26)  $\frac{97\pi}{36}$     27)  $\frac{47\pi}{4}$     28)  $\frac{36}{36}$

Practice:

Key

For questions 1-9, change the given angle to radians AND determine the quadrant.

1)  $315^\circ$  (3)

$\frac{7\pi}{4}$

3)  $60^\circ$  (4)

$-\frac{\pi}{3}$

4)  $212^\circ$  (3)

$\frac{53\pi}{45}$

5)  $-168^\circ$  (3)

$-\frac{14\pi}{15}$

6)  $125^\circ$  (1)

$\frac{5\pi}{72}$

7)  $-310^\circ$  (1)

$-\frac{31\pi}{18}$

8)  $600^\circ$  (3)

$\frac{10\pi}{3}$

9)  $-720^\circ$  (1) ~~16\pi~~

$-4\pi$

For questions 10-17, change the given angle to degrees AND determine the quadrant.

10)  $\frac{3\pi}{4}$  (2)  $135^\circ$

11)  $-\frac{9\pi}{5}$  (1)  $-324^\circ$

12)  $\frac{15\pi}{8}$  (4)  $337.5^\circ$

13)  $-\frac{\pi}{10}$  (4)  $-18^\circ$

14)  $\frac{7\pi}{10}$  (2)  $126^\circ$

15)  $-\frac{16\pi}{15}$  (2)  $-192^\circ$

16)  $\frac{88\pi}{9}$

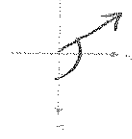
(Q4)  $1760^\circ$

17)  $-\frac{29\pi}{12}$

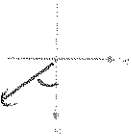
(Q4)  $-435^\circ$

Exercises  
Sketch each of the following angles in standard position. (Do not use a protractor; just draw a brief sketch.)

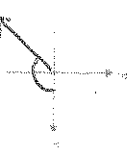
1.  $\theta = 120^\circ$



2.  $\theta = -45^\circ$



3.  $\theta = -130^\circ$



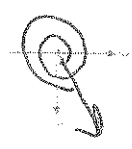
4.  $\theta = 270^\circ$



5.  $\theta = -90^\circ$



6.  $\theta = 750^\circ$



Determine the reference angle for the following (leave degrees in degrees and radians in radians)

7)  $-20^\circ$

8)  $160^\circ$

9)  $300^\circ$

10)  $\frac{10\pi}{3}$

11)  $\frac{7\pi}{4}$

12)  $-\frac{\pi}{6}$

$20^\circ$

$20^\circ$

$60^\circ$

$\frac{\pi}{3}$

$\frac{\pi}{4}$

$\frac{\pi}{6}$

