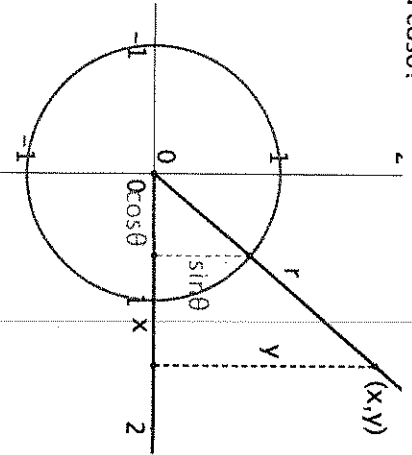


If you know a point on the terminal side of an angle θ , but that point does not lie on the unit circle, how can you find the $\sin\theta$ and $\cos\theta$?



For any angle θ in standard position, a point (x,y) on its terminal side, and $r = \sqrt{x^2 + y^2}$, the sine and cosine of θ are:

$$\sin\theta = \frac{y}{r} \quad \cos\theta = \frac{x}{r}$$

thus... $\tan\theta = \frac{y}{x}$

Note:
r is always positive

A **Pythagorean triple** consists of three positive integers a, b, and c, such that $a^2 + b^2 = c^2$. A triple is commonly written (a, b, c) .
Some Pythagorean triples:

- (3, 4, 5) (5, 12, 13) (8, 15, 17) (7, 24, 25) (20, 21, 29)

Practice

Find the values of the sine, cosine, and tangent functions of an angle θ in standard position if the given point lies on its terminal side.

1. (5, 12)

$x=5, y=12$
 $r = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$

$\sin\theta = \frac{12}{13}$ $\cos\theta = \frac{5}{13}$ $\tan\theta = \frac{12}{5}$

2. (-3, 4)

$r=5$
 $\sin\theta = \frac{4}{5}$ $\cos\theta = -\frac{3}{5}$ $\tan\theta = -\frac{4}{3}$
 $\csc\theta = \frac{5}{4}$ $\sec\theta = -\frac{5}{3}$ $\cot\theta = -\frac{3}{4}$

ORDERED PAIR	x	y	r	$\sin\theta$	$\cos\theta$	$\tan\theta$	HONORS		
							$\csc\theta$	$\sec\theta$	$\cot\theta$
(-8, -15)	-8	-15	17	$-\frac{15}{17}$	$-\frac{8}{17}$	$\frac{15}{8}$	$-\frac{17}{15}$	$-\frac{17}{8}$	$\frac{8}{15}$
(-24, 7)	-24	7	25	$\frac{7}{25}$	$-\frac{24}{25}$	$-\frac{7}{24}$	$\frac{25}{7}$	$-\frac{25}{24}$	$-\frac{24}{7}$
(2, -3)	2	-3	$\sqrt{13}$	$-\frac{3\sqrt{13}}{13}$	$\frac{2\sqrt{13}}{13}$	$-\frac{3}{2}$	$\frac{13}{3}$	$\frac{13}{2}$	$-\frac{2}{3}$

QUADRANT	x	y	r	$\sin\theta$	$\cos\theta$	$\tan\theta$	HONORS		
							$\csc\theta$	$\sec\theta$	$\cot\theta$
III	-12	-5	13	$-\frac{5}{13}$	$-\frac{12}{13}$	$\frac{5}{12}$	$\frac{13}{5}$	$\frac{13}{12}$	$\frac{12}{5}$
IV	24	-7	25	$-\frac{7}{25}$	$\frac{24}{25}$	$-\frac{7}{24}$	$\frac{25}{7}$	$\frac{25}{24}$	$\frac{24}{7}$
II	-4	3	5	$\frac{3}{5}$	$-\frac{4}{5}$	$-\frac{3}{4}$	$\frac{5}{3}$	$\frac{5}{4}$	$\frac{4}{3}$

$\tan\theta = \frac{y}{x} = \frac{-7}{24}$ Quadrant 3

$\sin\theta = -\frac{7}{25}$ Q4

x → neg
y → neg

$y = -5$ * Find r
 $x = -12$ $r = 13$

$y = -7$ x pos
 $r = 25$ y neg

$x^2 + (-7)^2 = 25^2$
 $x = 24$

Key

Point on Terminal Side Worksheet
Complete the charts.

ORDERED PAIR	x	y	r	sin θ	cos θ	tan θ	HONORS		
							csc θ	sec θ	cot θ
(7, -24)	7	-24	25	$-\frac{24}{25}$	$\frac{7}{25}$	$-\frac{24}{7}$	$-\frac{25}{24}$	$\frac{25}{7}$	$\frac{7}{24}$
(-20, -21)	-20	-21	29	$-\frac{21}{29}$	$-\frac{20}{29}$	$\frac{21}{20}$	$-\frac{29}{21}$	$-\frac{29}{20}$	$\frac{20}{21}$
(-3, 4)	-3	4	5	$\frac{4}{5}$	$-\frac{3}{5}$	$\frac{4}{3}$	$\frac{5}{4}$	$-\frac{5}{3}$	$-\frac{3}{4}$
(3, 2)	3	2	$\sqrt{13}$	$\frac{2\sqrt{13}}{13}$	$\frac{3\sqrt{13}}{13}$	$\frac{2}{3}$	$\frac{\sqrt{13}}{2}$	$\frac{\sqrt{13}}{3}$	$\frac{3}{2}$

QUADRANT	x	y	r	sin θ	cos θ	tan θ	HONORS		
							csc θ	sec θ	cot θ
III	-15	-8	17	$-\frac{8}{17}$	$-\frac{15}{17}$	$\frac{8}{15}$	$-\frac{17}{8}$	$-\frac{17}{15}$	$\frac{15}{8}$
IV	3	$-\sqrt{5}$	8	$-\frac{\sqrt{5}}{8}$	$\frac{3}{8}$	$-\frac{\sqrt{5}}{3}$	$-\frac{8\sqrt{5}}{3}$	$\frac{8}{3}$	$-\frac{3\sqrt{5}}{8}$
I	3	4	5	$\frac{4}{5}$	$\frac{3}{5}$	$\frac{4}{3}$	$\frac{5}{4}$	$\frac{5}{3}$	$\frac{3}{4}$
II	5	12	13	$\frac{12}{13}$	$\frac{5}{13}$	$\frac{12}{5}$	$\frac{13}{12}$	$\frac{13}{5}$	$\frac{5}{12}$

1. Find $\sin \theta$ if $\cos \theta = \frac{8}{17}$, Quad I
 $\sin \theta = \frac{15}{17}$

2. Find $\cos \theta$ if $\sin \theta = -\frac{4}{5}$, Quad III
 $\cos \theta = -\frac{3}{5}$

More Practice Worksheet

In which quadrant does the terminal side lie if the angle is in standard position?

1. -280° (1)
2. 425° (1)
3. $-\frac{3\pi}{4}$ (3)
4. $\frac{17\pi}{6}$ (2)

Determine the coterminal angle between 0° & 360° for the given angle.

5. -225° 135°
6. 520° 160°
7. $-\frac{7\pi}{8}$ $\frac{9\pi}{8}$
8. $\frac{19\pi}{6}$ $\frac{7\pi}{6}$

Express each angle in radians. Leave in terms of π .

9. 468° $\frac{13\pi}{5}$
10. -210° $-\frac{7\pi}{6}$

Express each angle in degrees.

11. $\frac{5\pi}{6}$ 150°
12. $-\frac{5\pi}{4}$ -225°

Find the reference angle for each angle. Your answer should be in the same units as the given angle.

13. -60° 60°
14. 210° 30°
15. $\frac{5\pi}{6}$ $\frac{\pi}{6}$
16. $\frac{5\pi}{4}$ $\frac{3\pi}{4}$
17. $\frac{-7\pi}{8}$ $\frac{7\pi}{8}$

If θ is an angle in standard position whose terminal side is in the given Quadrant, find the values of the following:

18. $\sin \theta = \frac{12}{13}$, Quad II $x = -5$ $\cos \theta = -\frac{5}{13}$ $\tan \theta = -\frac{12}{5}$

Honors: $\sec \theta = -\frac{13}{5}$ $\csc \theta = \frac{13}{12}$ $\cot \theta = -\frac{5}{12}$

19. $\cos \theta = \frac{\sqrt{3}}{2}$, Quad I $y = 1$ $\sin \theta = \frac{1}{2}$ $\tan \theta = \frac{\sqrt{3}}{3}$

Honors: $\sec \theta = \frac{2}{\sqrt{3}}$ $\csc \theta = 2$ $\cot \theta = \sqrt{3}$

$\theta = 30^\circ = \frac{\pi}{6}$