# Implicit Differentiation

**Implicit** → x & y's are mixed together

**Explicit** → solved for y.

Ex. $x^2+y^2=4$

$2x+y'\cdot 2y=0$

$y'\cdot 2y=-2x$

$y'=\frac{-2x}{2y}=\frac{-x}{y}$

$\boxed{y'=\frac{-x}{y}}$

Find eqn. tangent @ $(1,\sqrt{3})$

$y'=\frac{-1}{\sqrt{3}}$

$\boxed{y-\sqrt{3}=-\frac{1}{\sqrt{3}}(x-1)}$

Find $y''$. $\Rightarrow$ $y'=\frac{-x}{y}$ ($x^2+y^2=4$)

$$y''=\frac{-1y-(-x)(y')}{y^2}=\frac{-y-(-x)\left(\frac{-x}{y}\right)}{y^2}=\frac{-y-\frac{x^2}{y}}{y^2}\cdot\frac{y}{y}$$

$$=\frac{-y^2-x^2}{y^3}=\frac{-(x^2+y^2)}{y^3}=\boxed{\frac{-4}{y^3}}$$ ← from start, $x^2+y^2=4$

Ex. $y=\pm\sqrt{4-x^2}$

$y'=\pm(-2x)\frac{1}{2}(4-x^2)^{-1/2}$

$y'=\pm\left(\frac{-x}{\sqrt{4-x^2}}\right)$

Find equation of tangent @ $(1,\sqrt{3})$ → top half so $y=\sqrt{4-x^2}$

$y'=\frac{-x}{\sqrt{4-x^2}}$

$y'(1)=\frac{-1}{\sqrt{3}}$

$\boxed{y-\sqrt{3}=-\frac{1}{\sqrt{3}}(x-1)}$ same

Ex. $x^2y - 3y^3 + \sin x = 2x - y$

$2xy + x^2y' - 9y^2 \cdot y' + \cos x = 2 - y'$

$x^2y' - 9y^2y' + y' = 2 - 2xy - \cos x$

$y'(x^2 - 9y^2 + 1) = 2 - 2xy - \cos x$

$$y' = \frac{2 - 2xy - \cos x}{x^2 - 9y^2 + 1} = \frac{2xy + \cos x - 2}{9y^2 - x^2 - 1}$$

x. $\sqrt{xy} = \sin y$

$(y + xy')\frac{1}{2}(xy)^{-1/2} = y' \cdot \cos y$

$\frac{1}{2}y(xy)^{-1/2} + \frac{1}{2}xy'(xy)^{-1/2} = y' \cos y$

$\frac{1}{2}y(xy)^{-1/2} = y'\cos y - \frac{1}{2}xy'(xy)^{-1/2}$

$\frac{1}{2}y(xy)^{-1/2} = y'\left(\cos y - \frac{1}{2}x(xy)^{-1/2}\right)$

$$y' = \frac{\frac{1}{2}y(xy)^{-1/2}}{\cos y - \frac{1}{2}x(xy)^{-1/2}} = \frac{\frac{y}{2\sqrt{xy}}}{\cos y - \frac{x}{2\sqrt{xy}}} \cdot \frac{2\sqrt{xy}}{2\sqrt{xy}}$$

$$= \frac{y}{2\sqrt{xy}\cos y - x}$$