AP Calculus: Review Area, Volume, Distance, Arc Length

I. Find the area bounded by the following:

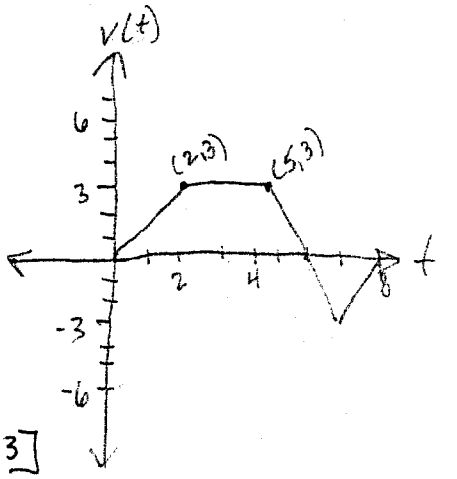
1. $y = 2x$ and $y = x^2-4x$
2. $x = y^3-y$ and $x = 1-y^4$
3. $y = x+5$, $y^2 = x$, $y = -1$, and $y = 2$
4. $y = x^2$ and $y = 2/(x^2+1)$
5. $y = x^2$ and $y = 2\cos(x)$

II. Find the volume of the solid obtained by rotating the region bounded by the given curves about the given axis or line or by using cross-sections. Sketch the regions.

6. $y^2 = x$, $x = 2y$; about the x-axis
7. $y = x^4$, $y = \sin(\pi x/2)$; about $y = -1$
8. $x+y = 1$, $y = x+1$, and $x = 2$; about the y-axis
9. $x = 4-y^2$, $x = 8-2y^2$; about $x = 9$
10. $y = \sec x$, $y = 1$, $x = -1$, $x = 1$; about the x-axis
11. $x-y = 1$, $y = (x-4)^2+1$; about $y = 7$
12. The base of a solid is the region bounded by the graph of $y = 1-x^2$ and the x-axis. The cross sections perpendicular to the x-axis are squares. Find the volume.
13. Find the volume of the solid whose base is the region bounded by the graphs of $y=x^3$, $x=1$, and the x-axis, and whose cross sections perpendicular to the x-axis are semicircles.
14. The base of a solid is the upper semicircle bounded by the x-axis and the graph of $x^2+y^2 = 1$. The cross sections perpendicular to the y-axis are isosceles right triangles with one side (not the hypotenuse) as the base. Find the volume.

III.

15. The velocity of a particle over time is given by $v(t) = x^3 - 4x^2 + 2x + 1$ meters per second. Determine the total distance traveled from $t = 0$ to $t = 5$ seconds.
16. Find the value of *a* such that the line $x = a$ bisects the area under the curve $y = 1/x^3$ when $1 \le x \le 4$.
17. Given graph of v(t) and initial position $x(0) = 3$ meters, determine:



a) x(5)
b) Displacement from $t = 0$ to $t = 8$ sec.
c) When moving right?
d) When moving left?
e) When at a maximum distance from the origin?

IV Find the arc length:

18. $y = 3\cos x - 2^x$ $[-2,4]$
19. $y = 3x^2 - 2x/x-1/+3$ $[0,3]$
20. $(x+3)^2 = 8(y-1)^3$ from $(-2, 3/2)$ to $(5,3)$

KEY

AP Calculus: Review Area, Volume, Distance

I. Find the area bounded by the following:

1. $y = 2x$ and $y = x^2-4x$ $\int_0^6 (2x-(x^2-4x))\,dx = 36$

2. $x = y^3-y$ and $x = 1-y^4$ $\int_{-1}^{1} [(1-y^4)-(y^3-y)]\,dy = 1.6$

3. $y = x+5$, $y^2 = x$, $y = -1$, and $y = 2$ $\int_{-1}^{2} (y^2-(y-5))\,dy = 16.5$

4. $y = x^2$ and $y = 2/(x^2+1)$ $\int_{-1}^{1} \left(\frac{2}{x^2+1} - x^2\right)dx = 2.475$

5. $y = x^2$ and $y = 2\cos(x)$ $\int_{-1.02169}^{1.02169} [2\cos x - x^2]\,dx = 2.70097$

II. Find the volume of the solid obtained by rotating the region bounded by the given curves about the given axis or line or by using cross-sections. Sketch the regions.

6. $y^2 = x$, $x = 2y$; about the x-axis $8\pi/3 = 8.37758$

7. $y = x^4$, $y = \sin(\pi x/2)$; about $y = -1$ $\pi\int_0^1 \left(\left(\sin\frac{\pi x}{2}+1\right)^2 - (x^4-1)^2\right)dx = 3.965$

8. $x+y = 1$, $y = x+1$, and $x = 2$; about the y-axis $\frac{32\pi}{3} = 33.51$

9. $x = 4-y^2$, $x = 8-2y^2$; about $x = 9$ $\pi\int_{-2}^{2} (9-(4-y^2))^2 - (9-(8-2y^2))^2\,dy = 89.6\pi = 281.487$

10. $y = \sec x$, $y = 1$, $x = -1$, $x = 1$; about the x-axis 3.5023

11. $x-y = 1$, $y = (x-4)^2+1$; about $y = 7$ $\pi\int_3^6 [6-(x-4)^2]^2 - (8-x)^2]\,dx = 39.6\pi = 124.40$

12. The base of a solid is the region bounded by the graph of $y = 1-x^2$ and the x-axis. The cross sections perpendicular to the x-axis are squares. Find the volume. $\int_{-1}^{1} (1-x^2)^2\,dx = 1.066$

13. Find the volume of the solid whose base is the region bounded by the graphs of $y=x^3$, $x=1$, and the x-axis, and whose cross sections perpendicular to the x-axis are semicircles. $\frac{1}{2}\pi\int_0^1 \left(\frac{x^3}{2}\right)^2 dx = .056699$

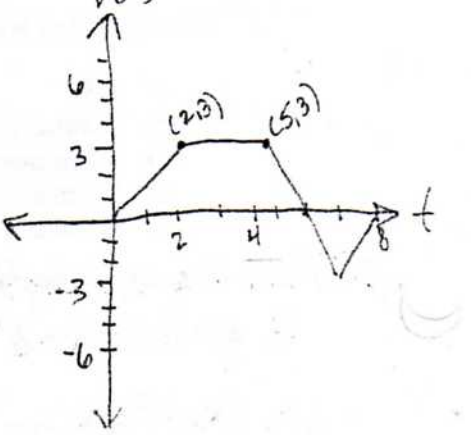
14. The base of a solid is the upper semicircle bounded by the x-axis and the graph of $x^2+y^2 = 1$. The cross sections perpendicular to the y-axis are isosceles right triangles with one side (not the hypotenuse) as the base. Find the volume. $\frac{1}{2}\int_0^1 (2\sqrt{1-y^2})^2\,dy = \frac{4}{3}$

III.

15. The velocity of a particle over time is given by $v(t) = x^3 - 4x^2 + 2x + 1$ meters per second. Determine the total distance traveled from $t = 0$ to $t = 5$ seconds. $\int_0^5 |x^3-4x^2+2x+1|\,dx = 29.5725$ m

16. Find the value of *a* such that the line $x = a$ bisects the area under the curve $y = 1/x^3$ when $1 \le x \le 4$. $\int_1^a \frac{1}{x^3}\,dx = \frac{15}{64}$ $\quad -\frac{1}{2}x^{-2}\Big|_1^a = -\frac{1}{2a^2}+\frac{1}{2} = \frac{15}{64}$ $\quad a = 1.372$

17. Given graph of v(t) and initial position $x(0) = 3$ meters, determine:



a) x(5) 15 m
b) Displacement from t = 0 to t = 8 sec. 10.5 m
c) When moving right? (0, 6)
d) When moving left? (6, 8)
e) When at a maximum distance from the origin? $t = 6$ sec

IV

18. 24.7699
19. 15.8314
20. 7.164