Integration Technique Review – Chapter 7

1. $\int \frac{5t}{5+2t^2}\,dt$ 2. $\int \frac{\cot\sqrt{y}\csc\sqrt{y}}{\sqrt{y}}\,dy$ 3. $\int (1+\sin\theta)^5\cos\theta\,d\theta$

4. $\int \frac{\sin 2x}{4+\cos 2x}\,dx$ 5. $\int e^{-\cot x}\csc^2 x\,dx$ 6. $\int \frac{t}{\sqrt{1-9t^2}}\,dt$

7. $\int \frac{1}{\sqrt{1-9t^2}}\,dt$ 8. $\int \frac{e^{2x}}{1+e^{4x}}\,dx$ 9. $\int \frac{\cos\theta}{1+\sin^2\theta}\,d\theta$

10. $\int \frac{x^3-x+3}{x^2+x-2}\,dx$ 11. $\int \frac{x+2}{x^2-4x}\,dx$ 12. $\int \frac{x^2+x+3}{x^4+6x^2+9}\,dx$

13. $\int x^3 \ln x\,dx$ 14. $\int x\sec x\tan x\,dx$ 15. $\int \frac{x}{e^x}\,dx$

16. $\int \frac{2x-3}{(x-1)^2}$ 17. $\int \frac{1}{(1+x^2)\arctan x}\,dx$ 18. $\int \frac{1}{\sqrt{e^{2x}-1}}\,dx$

19. $\int x\sec^2 x\,dx$ 20. $\int \frac{\ln 2x}{x^2}\,dx$ 21. $\int \frac{e^{\sqrt{x+4}}}{\sqrt{x+4}}\,dx$

22 $\int \tan^4 3x\sec^2 3x\,dx$ 23. $\int x^2\cos x\,dx$

28. $\int \cos^2 x\,dx$ 29. $\int \cos^3 x\sin^4 x\,dx$

Find the area between $y = (x-8)^{\frac{-2}{3}}$ and y = 0 for [0, 8)

Find the area of the region to the right of x = 1 between $y = \frac{2}{4x^2-1}$ and the x-axis.

Evaluate the following integrals. Determine if the integral converges or diverges.

(1) $\int_0^1 \frac{1}{x}\,dx$ (2) $\int_1^\infty \frac{1}{x}\,dx$ (3) $\int_0^\infty xe^{-x}\,dx$

(4) $\int_0^\infty \frac{1}{1+x^2}\,dx$ (5) $\int_5^\infty \frac{1}{\sqrt{x-1}}\,dx$ (6) $\int_0^1 \frac{1}{1-x}\,dx$

(7) $\int_1^\infty \ln x\,dx$ (14) $\int_{-\infty}^\infty xe^{-x^2}\,dx$ (6) $\int_0^2 \frac{x}{1-x}\,dx$ (12) $\int_1^4 \frac{1}{(x-2)^{\frac{2}{3}}}\,dx$

(22) $\int_0^2 \frac{1}{\sqrt{4-x^2}}\,dx$ (23) $\int_0^4 \frac{x}{\sqrt{16-x^2}}\,dx$ (27) $\int_0^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$

ANSWERS:

1. $\frac{5}{4}\ln|5+2t^2|+c$ 2. $-2\csc\sqrt{y}+c$ 3. $\frac{1}{6}(1+\sin\theta)^6+c$

4. $-\frac{1}{2}\ln|4+\cos 2x|+c$ 5. $e^{-\cot x}+c$ 6. $-\frac{1}{9}(1-9t^2)^{1/2}+c$

7. $\frac{1}{3}\sin^{-1}(3t)+c$ 8. $\frac{1}{2}\tan^{-1}(e^{2x})+c$ 9. $\tan^{-1}(\sin\theta)+c$

10. $\frac{1}{2}x^2-x+\ln|x+2|+\ln|x-1|+c$ 11. $-\frac{1}{2}\ln|x|+\frac{3}{2}\ln|x-4|+c$

12. $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)-\frac{1}{2}(x^2+3)^{-1}+c$ 13. $\frac{1}{4}x^4\ln x-\frac{1}{16}x^4+c$

14. $x\sec x-\ln|\sec x+\tan x|+c$ 15. $-xe^{-x}-e^{-x}+c$

16. $2\ln|x-1|+(x-1)^{-1}+c$ 17. $\ln|\arctan x|+c$ 18. $\operatorname{arcsec}(e^x)+c$

19. $x\tan x+\ln|\cos x|+c$ 20. $-\frac{\ln 2x}{x}-\frac{1}{x}+c$

21. $2e^{\sqrt{x+4}}+c$ 22. $\frac{1}{15}\tan^5 3x+c$ 23. $x^2\sin x+2x\cos x-2\sin x+c$

28. $\frac{1}{2}x+\frac{1}{4}\sin 2x+c$

29. $\frac{1}{5}\sin^5 x-\frac{1}{7}\sin^7 x+c$

24. Find the area between $y=(x-8)^{\frac{2}{3}}$ and $y=0$ for $[0,8)$ (6)

25. Find the area of the region to the right of $x=1$ between $y=\frac{2}{4x^2-1}$ and the x-axis.

$$=\lim_{b\to\infty}\left[\frac{1}{2}\ln|2x-1|-\frac{1}{2}\ln|2x+1|\right]_1^b$$

$$=\lim_{b\to\infty}\left[\frac{1}{2}\ln\left|\frac{2b-1}{2b+1}\right|-\frac{1}{2}\ln\frac{1}{3}\right]$$

$\frac{1}{2}\ln 3$

OR

$-\frac{1}{2}\ln\frac{1}{3}$

Evaluate the following integrals. Determine if the integral converges or diverges.

(1) $\int_0^1 \frac{1}{x}\,dx$ $= \infty$ diverges (2) $\int_1^\infty \frac{1}{x}\,dx$ $= \infty$ diverges (3) $\int_0^\infty xe^{-x}\,dx$ $= 1$ conv.

(4) $\int_0^\infty \frac{1}{1+x^2}\,dx$ $= \pi/2$ conv. (5) $\int_5^\infty \frac{1}{\sqrt{x-1}}\,dx$ $= \infty$ diverges (6) $\int_0^1 \frac{1}{1-x}\,dx$ $= \infty$ diverges

(7) $\int_1^\infty \ln x\,dx$ $= \infty$ div. (14) $\int_{-\infty}^\infty xe^{-x^2}\,dx$ $= 0$ conv. (6) $\int_0^2 \frac{x}{1-x}\,dx$ → split @ 1 $= \infty$ div. (12) $\int_1^4 \frac{1}{(x-2)^{\frac{2}{3}}}\,dx$ → split @ 2 $= 3 + 3\sqrt[3]{2}$ conv.

(22) $\int_0^2 \frac{1}{\sqrt{4-x^2}}\,dx$ $= \pi/2$ conv. (23) $\int_0^4 \frac{x}{\sqrt{16-x^2}}\,dx$ $= 4$ conv. (27) $\int_0^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$ → split! Problems @ both ends! $= 2$ conv.