Limits using L'Hopital's Rule

* Indeterminate forms $\frac{0}{0}$ and $\frac{\pm\infty}{\pm\infty}$ ← could be $\pm\infty$

If possible, simplify:

Ex. $\lim\limits_{x\to 3} \frac{x^2-9}{x-3}$ (this is an old one) if plug in 3, get $\frac{0}{0}$

$$= \lim_{x\to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}} = \boxed{6}$$

$\lim\limits_{x\to\infty} \frac{3x^3+7x-4}{5x^3+6}$ ⇒ if plug in ∞, get $\frac{\infty}{\infty}$. Use rule!

same degree top + bott!

$$= \lim_{x\to\infty} \frac{3x^3+7x-4}{5x^3+6} = \boxed{\frac{3}{5}}$$

Sometimes though, indeterminate + doesn't simplify.

Ex $\lim\limits_{x\to 0} \frac{\sin x}{x}$ ⇒ Could graph or use table feature + see answer, but if plug in get $\frac{0}{0}$ + does <u>not</u> simplify.

<u>L'Hopital's Rule</u>

Suppose f and g are differentiable on (a,b) containing c. If $\frac{f(x)}{g(x)}$ has indeterminate form $\frac{0}{0}$ or $\frac{\infty}{\infty}$ <u>at</u> $x=c$, then $\lim\limits_{x\to c} \frac{f(x)}{g(x)} = \lim\limits_{x\to c} \frac{f'(x)}{g'(x)}$.

In other words, take deriv. of top + deriv. of bottom + <u>then</u> take the limit.

plug in 0

Ex. $\lim\limits_{x\to 0} \frac{\overset{0}{\sin x}}{\underset{0}{x}} = \lim\limits_{x\to 0} \frac{\cos x}{1} = 1$

* This is <u>only</u> true if $\frac{0}{0}$ or $\frac{\infty}{\infty}$!!

Ex. 2 $\lim_{x\to 0} \frac{\cos x + 2x - 1}{3x} = \lim_{x\to 0} \frac{-\sin x + 2}{3} = \boxed{\frac{2}{3}}$

(plug in 0 + get $\frac{0}{0}$) (plug in 0 for x)

Ex. 3 $\lim_{x\to 0} \frac{e^x + e^{-x} - 2}{1 - \cos 2x} = \lim_{x\to 0} \frac{e^x - e^{-x}}{2\sin 2x} = \lim_{x\to 0} \frac{e^x + e^{-x}}{4\cos 2x} = \frac{2}{4}$

$= \boxed{\frac{1}{2}}$

(plug in $0 \Rightarrow \frac{0}{0}$) (plug in $0 \Rightarrow$ still $\frac{0}{0}$! Do it again!! ☺)

woohoo!

Ex. 4. $\lim_{x\to \frac{\pi}{2}^-} \frac{4\tan x}{1 + \sec x} = \lim_{x\to \frac{\pi}{2}^-} \frac{4\sec^{\cancel{2}} x}{\cancel{\sec x}\tan x} = \lim_{x\to \frac{\pi}{2}^-} \frac{4\sec x}{\tan x}$

(plug in $\frac{\pi}{2}$ + get $\frac{\infty}{\infty}$) (simplify) (Still $\frac{\infty}{\infty}$ + won't get better $\Rightarrow$ use identities)

$= \lim_{x\to \frac{\pi}{2}^-} \frac{\frac{4}{\cos x}}{\frac{\sin x}{\cos x}} = \lim_{x\to \frac{\pi}{2}^-} \frac{4}{\sin x} = \frac{4}{1} = \boxed{4}$

(Now can plug in)

Ex. 5. $\lim_{x\to 0} \frac{e^x + e^{-x}}{x^2} = \frac{2}{0} = \boxed{+\infty}$

(plug in 0 + get $\frac{2}{0} \Rightarrow$ <u>not</u> indeterminate!)

Ex. 6 $\lim_{x\to \infty} \frac{3x^3 - 2x + 6}{6x^3 + x^2 - 1} \Rightarrow$ we already know this $= \frac{1}{2}$ but <u>could</u> use L'Hopitals

$\lim_{x\to \infty} \frac{3x^3 - 2x + 6}{6x^3 + x^2 - 1} = \lim_{x\to \infty} \frac{9x^2 - 2}{18x^2 + 2x} = \lim_{x\to \infty} \frac{18x}{36x + 2} = \lim_{x\to \infty} \frac{18}{36}$

$= \boxed{\frac{1}{2}}$