Inverses Notes

* Inverse functions "undo" each other, just like subtracting 3 is inverse of adding 3.

To find inverses, switch x & y.

If given points, that's easy...

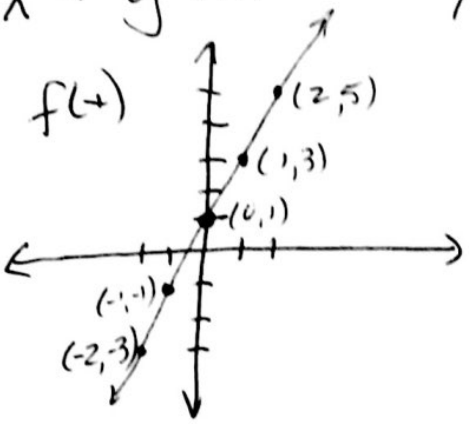
Ex. $\{(-2,3), (1,6), (2,1), (4,7)\}$

Inverse $= \{(3,-2), (6,1), (1,2), (7,4)\}$:)

* Notice if (x,y) on $f(x)$, then (y,x) on $f^{-1}(x)$ ← inverse.

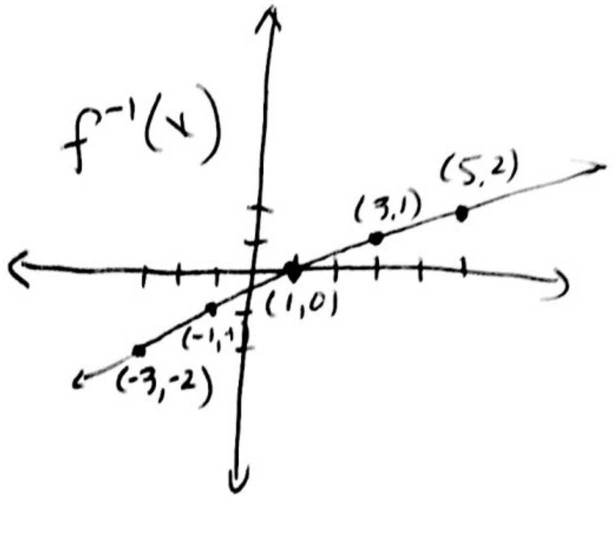
Notation: $f^{-1}(x)$ means "inverse of $f(x)$"

If given graph, you can find inverse graph by switching x & y values of points.



$f^{-1}(x) \Rightarrow$ Take pts from $f(x)$ & switch (x,y) to (y,x) & graph

$(2,5) \to (5,2)$



If given equation, find inverse by switching x and y and solving for y.

* Remember, $f(x) = y$

Ex1. $f(x) = 2x+1$

* This same as $y = 2x+1$

① Switch x & y

$x = 2y+1$

② Solve for y.

$x = 2y+1$
$-1 \qquad -1$

$\frac{x-1}{2} = \frac{2y}{2} \Rightarrow \boxed{\frac{1}{2}x - \frac{1}{2} = y}$

Write as $\boxed{f^{-1}(x) = \frac{1}{2}x - \frac{1}{2}}$.

* These are the functions graphed on previous page *

Ex.2 $f(x) = \sqrt{x-2}+4 \Longrightarrow y = \sqrt{x-2}+4$

$x = \sqrt{y-2}+4$
$-4 \qquad -4$

$(x-4)^2 = (\sqrt{y-2})^2$ ← Square both sides to "undo" $\sqrt{\ }$

$(x-4)^2 = y-2$
$+2 \qquad +2$

$(x-4)^2+2 = y$

$\boxed{f^{-1}(x) = (x-4)^2+2}$

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To verify inverses, show composition $= x$

$(f \circ f^{-1})(x) = x$ OR $(f^{-1} \circ f)(x) = x$

because they should "undo" each other.

Let's look @ last example.

$f(x) = \sqrt{x-2} + 4 \qquad f^{-1}(x) = (x-4)^2 + 2$

$$(f \circ f^{-1})(x) = \underbrace{\sqrt{(x-4)^2+2-2}}_{\text{Simplify}} + 4 = \underbrace{\sqrt{(x-4)^2}}_{\text{Simplify}} + 4 = \underbrace{x-4+4}_{\text{Simplify}} = x \checkmark$$

Another example: Are $f(x) = 2x-3$ and $g(x) = \frac{1}{2}x+3$ inverses?

$$f(g(x)) = \underbrace{2\left(\tfrac{1}{2}x+3\right)}_{\text{Simplify}} - 3 = x+6-3 = x+3$$

No

Only inverses if composition $= x$!

Another... Are $f(x) = \sqrt[3]{x-2}$ and $g(x) = x^3+2$ inverses?

$$f(g(x)) = \sqrt[3]{x^3+2-2} = \sqrt[3]{x^3} = x$$

Yes

Inverse Relations
Find the inverse for each relation.

1. $\{ (1, -3), (-2, 3), (5, 1), (6, 4) \}$
2. $\{ (-5, 7), (-6, -8), (1, -2), (10, 3) \}$

Finding Inverses
Find an equation for the inverse for each of the following relations.

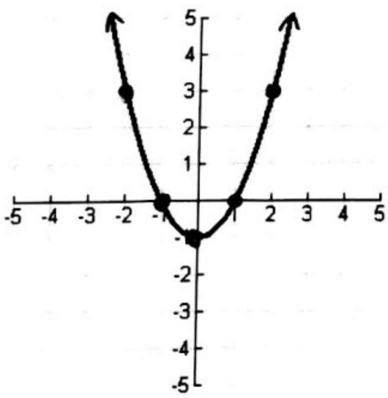
3. $y = 3x + 2$
4. $y = -5x - 7$
5. $y = 12x - 3$
6. $y = -8x + 16$
7. $y = \frac{2}{3}x - 5$
8. $y = -\frac{3}{4}x + 5$
9. $y = -\frac{5}{8}x + 10$
10. $y = \frac{1}{2}x + 8$
11. $y = x^2 + 5$
12. $y = x^2 - 4$
13. $y = (x + 3)^2$
14. $y = (x - 6)^2$
15. $y = \sqrt{x - 2},\ y \geq 0$
16. $y = \sqrt{x + 5},\ y \geq 0$
17. $y = \sqrt{x} + 8,\ y \geq 8$
18. $y = \sqrt{x} - 7,\ y \geq -7$

Verifying Inverses
Verify that *f* and *g* are inverse functions.

19. $f(x) = x + 6,\ g(x) = x - 6$
20. $f(x) = 5x + 2,\ g(x) = \frac{x - 2}{5}$
21. $f(x) = -3x - 9,\ g(x) = -\frac{1}{3}x - 3$
22. $f(x) = 2x - 7,\ g(x) = \frac{x + 7}{2}$
23. $f(x) = -4x + 8,\ g(x) = -\frac{1}{4}x + 2$
24. $f(x) = \frac{1}{2}x - 7,\ g(x) = 2x + 14$

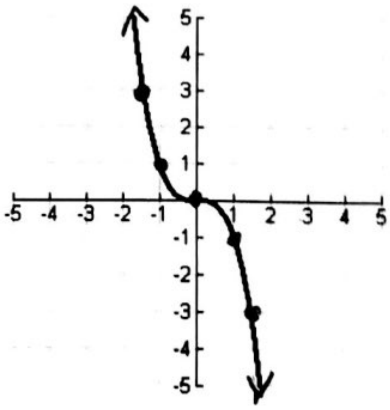
Graphing Inverses
Graph the inverse for each relation below (put your answer on the same graph).

25.



f(x) domain:
range:
f⁻¹(x) domain:
range:

26.



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