A <u>sequence</u> is a set of numbers whose domain is the set of positive integers.

Ex. $3, 6, 9, 12, \ldots, 3n$

Describe sequences based on their terms.
Here, 1^{st} term = 3, 2^{nd} term = 6 + so on.
Use subscripts with notation ⇒
$a_1 = 3 \quad a_2 = 6 \quad a_3 = 9$ etc. $a_n = 3n$

Where n = # of the term + a_n = actual <u>value</u> of the term.

The numbers in the sequence will follow a pattern, some are easier to see than others.

2 ways to give formulas to generate sequences - explicit + recursive.

A <u>recursive</u> formula describes what you do ~~#~~ to previous terms to get to the next term.
In the example, what do you do to get from one term to next? Add 3 ⇒ so...

$$\underset{n^{th}\text{ term}}{a_n} = \underset{\text{previous term}}{a_{n-1}} + 3$$

but this alone would not be enough to generate the sequence. If I just gave this formula, where do you start?? Must give starting value.

<u>So</u>, $a_n = a_{n-1} + 3$ + $a_1 = 3$. For recursive formula must give both!!

Recursive formulas are generally what you're doing in your head to write out a few terms of the sequence. They're useless if you want a random term, like the 100^{th} term, without wanting to write out the whole thing. So for these cases, we like explicit formul.

Explicit formulas allow you to find the n^{th} term without knowing any previous terms.

In the example, you were given $a_n = 3n \Rightarrow$ this is the explicit formula. So if you want a given term, like the 100^{th} term, all you do is substitute $n = 100$ into the formula. So, the 100^{th} term, $a_{100} = 3(100) = 300$.

We will mainly only focus on 2 types of sequences in here - arithmetic + geometric.

Arithmetic sequence is where each term is found by adding a constant (called the common difference) to the previous term.

- This 1st example was an arithmetic seq. The common difference was 3.

In general with arithmetic sequences, the recursive formula $\Rightarrow a_n = a_{n-1} + d \;;\; a_1 = a_1$

Explicit $\Rightarrow a_n = a_1 + (n-1)d$.

With that example $\Rightarrow 3, 6, 9, 12 \ldots$

$$a_n = \underset{\uparrow \atop 1^{st}\text{ term}}{3} + (n-1)\underset{\uparrow \atop \text{diff}}{(3)} = 3 + 3n - 3 = \underset{\uparrow \atop \text{what given for explicit!}}{3n}$$

TRY: (or show ⇒ let them think on their own first, then show)

Write recursive + explicit formulas:

1. $26, 21, 16, \ldots$

ans. rec ⇒ $a_1 = 26$ $a_n = a_{n-1} - 5$

exp: $a_n = 26 + (n-1)(-5) = 31 - 5n$

2. $5, 12, 19, \ldots$

rec: $a_1 = 5$ $a_n = a_{n-1} + 7$

exp: $a_n = 5 + (n-1)(7) = 7n - 2$

3. Write out the next 4 terms: $2, -3, -8, \ldots$ $\boxed{-13, -18, -23, -28}$

4. Find the 25^{th} term if $a_1 = -1$ & $d = -10$ $\boxed{-241}$

ans. $a_n = -1 + (n-1)(-10)$ ⇒ $a_{25} = -1 + (25-1)(-10)$

5. 124 is the ___th term of $-2, 5, 12, \ldots$

Want n! $a_n = 124$ $a_1 = -2$ $d = 7$

$124 = -2 + (n-1)(7)$ ⇒ $\boxed{n = 19}$

6. $-\frac{17}{4}$ is the ___th term of $2\frac{1}{4}, 2, 1\frac{3}{4}, \ldots$

$-\frac{17}{4} = 2\frac{1}{4} + (n-1)(-\frac{1}{4})$ $\boxed{27}$

* Remember, n will __always__ be a positive integer! If not, did something wrong ☺

7. Find the missing terms in the arithmetic sequence.

a) $55, \_, \_, \_, 115$

ans. Find d! $a_1 = 55$ $a_5 = 115$

$115 = 55 + (5-1)d$ $d = 15$

$\boxed{70, 85, 100}$

b) $2, \_, \_, \_, \_, \_, 20$

$\boxed{5, 8, 11, 14, 17}$

c) $\_, -6, \_, \_, 15, \_$

$\boxed{-13, 1, 8, 22}$

Here, I "pretend" $a_1 = -6$ + $a_4 = 15$ to find d.

$15 = -6 + (4-1)d$

$d = 7$ + then add or subtract 7 as needed to get terms!

8. How many multiples of 11 are between 13 + 384?

* Want n. $a_1 = 22$ $a_n = 374$

$d = 11$

$374 = 22 + (n-1)(11)$

$\boxed{n = 33}$

Geometric Sequences

Geometric Sequences is where to find the next term you multiply a constant (called common ratio) to the previous term.

Ex! $6, 18, 54, \dots$ or Ex. 2 $28, 14, 7, \frac{7}{2}, \dots$

$r = 3$ $r = \frac{1}{2}$

recursive formula $\Rightarrow a_n = r a_{n-1}$ $a_1 = a_1$

explicit $\Rightarrow a_n = a_1 r^{n-1}$

So for ex 1. $\Rightarrow$ recursive $a_n = 3a_{n-1}$ $a_1 = 6$; explicit $a_n = 6(3)^{n-1}$

ex 2 $\Rightarrow a_n = \frac{1}{2}a_{n-1}$ $a_1 = 28$; $a_n = 28\left(\frac{1}{2}\right)^{n-1}$

* Notice for geometric, the explicit formula is exponential. To ever solve for n, will need logarithms!!

Try:

1. Find the 1st 4 terms if $a_1 = -6$ & $r = -\frac{2}{3}$

ans. $-6, 4, -\frac{8}{3}, \frac{16}{9}$

2. Find the 6th term if $a_1 = -4$ & $r = -2$

$a_6 = (-4)(-2)^{6-1} = (-4)(-2)^5 = \boxed{128}$

3. Find the 12th term if $a_6 = 8$ & $r = \frac{1}{2}$

* Can do couple of ways $\Rightarrow$ find a_1 first $\Rightarrow$ $8 = a_1\left(\frac{1}{2}\right)^{6-1} \Rightarrow a_1 = 256$

$a_{12} = 256\left(\frac{1}{2}\right)^{11} \Rightarrow \boxed{\frac{1}{8}}$

OR, "pretend" $a_6 = a_1 = 8$ so want $a_{12} = \boxed{a_7} = 8\left(\frac{1}{2}\right)^{7-1} = \boxed{\frac{1}{8}}$

either way is fine!!

4. Find missing geometric means:

a) $7, \underline{\pm 14}, \underline{28}, \underline{\pm 56}, 112$

b) $\underline{\pm 3}, \underline{6}, \underline{\pm 12}, 24, \underline{\pm 48}, \underline{96}, \underline{\pm}, 384$

(24: pretend a_1; 384: a_5)

* Find r! a) $112 = 7r^{5-1}$ b) $384 = 24r^4$

$16 = r^4$ $16 = r^4$

5. A vacuum pump removes $\frac{1}{5}$ of the air from a sealed container on each stroke of its piston. What % of air remains after 5 strokes?

* Any time dealing w/ percents, always start w/ 10

So, 0 strokes = 100 = a_1

5 strokes = a_6

$\frac{1}{5}$ removed $\Rightarrow$ $\frac{4}{5}$ remains, so $r = \frac{4}{5}$.

$a_6 = 100\left(\frac{4}{5}\right)^{6-1} \Rightarrow \boxed{32.8\%}$

Series

A series is the sum of the given number of terms of a sequence. S_n = sum of 1^{st} n terms.

So if the sequence is $3, 6, 9, 12, \ldots$

$$S_4 = 3+6+9+12 = \boxed{30}$$

If finding sum of terms in arithmetic sequence, then arithmetic series. Likewise for geometric.

In general for arithmetic series $\Rightarrow$ $S_n = \frac{n}{2}(a_1 + a_n)$

Ex. Find sum of first 10 odd integers.

*Want S_{10}. So $n=10$ $a_1 = 1$ (1^{st} odd) to find a_{10} can use explicit form: $a_n = a_1 + (n-1)d$

$$a_{10} = 1 + (10-1)(2) = 19$$

b/c odd # diff = 2

So $S_{10} = \frac{10}{2}(1+19) = \boxed{100}$

* If ever don't know n, a_1, or a_n use $a_n = a_1 + (n-1)d$

Ex. A supermarket display has 6 rows of stacked boxes. The top row has 35 boxes + each row has 3 less than the one below it. How many boxes in display?

*Want sum $\Rightarrow$ S_6. Know $a_6 = 35$ $d = -3$ $n = 6$

Need a_1 $\Rightarrow$ $35 = a_1 + (6-1)(-3)$ $\Rightarrow$ $a_1 = 50$

$S_6 = \frac{6}{2}(50+35) = 3(85) = \boxed{255 \text{ boxes}}$

Infinite sum

Geometric series: $S_n = \dfrac{a_1(1-r^n)}{1-r}$ $\qquad S_\infty = \dfrac{a_1}{1-r}$ if $|r|<1$

Again, if ever need a_1, r, or n use explicit $a_n = a_1 r^{n-1}$

Ex. Find sum of 1st n terms if $a_1 = 48$, $a_n = 3$, & $r = -\frac{1}{2}$.

* Need n! $a_n = a_1 r^{n-1} \Rightarrow 3 = 48\left(-\frac{1}{2}\right)^{n-1} \Rightarrow \frac{1}{16} = \left(-\frac{1}{2}\right)^{n-1}$

$\left(-\frac{1}{2}\right)^4 = \left(-\frac{1}{2}\right)^{n-1}$

$n = 5$

$$S_5 = \frac{48\left(1-\left(-\frac{1}{2}\right)^5\right)}{1 - -\frac{1}{2}} = \boxed{33}$$

Ex. Find the 1st term if $S_7 = 3279$ & $r = 3$.

$$S_7 = \frac{a_1(1-r^n)}{1-r} \Rightarrow 3279 = \frac{a_1(1-3^7)}{1-3} \Rightarrow \boxed{a_1 = 3}$$

Ex. Find sum of $30 + 10 + \frac{10}{3} + \cdots$ * Infinite sum. $r = \frac{1}{3}$ $a_1 = 30$ $S_\infty = \dfrac{30}{1-\frac{1}{3}} = \boxed{45}$

A convenient way to indicate a sum is using sigma notation

$$S_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \ldots + a_n$$

The bottom # tells you what term to start w/
The top # is the stopping term
a_i = explicit formula.
Plug in the values from bottom to top & add results.

Ex. $\displaystyle\sum_{n=3}^{6}(2n-4) = 2 + 4 + 6 + 8 = \boxed{20}$

Plugged in 3 plugged in 4 etc.

But if want to total a lot of #s, best if recognize as arithmetic or geometric & use formula.

In example, arithmetic!! Can think of "1st term" as when plug in 3 $\Rightarrow$ 2 (bottom #)

"nth term" when plug in 6 $\Rightarrow$ 8 (top #)

$S_4 = \frac{4}{2}(2+8) = \boxed{20}$ ← 4 terms added

Ex. $\sum_{n=1}^{20} 3(2)^{n-1}$ ⇒ geometric!! $a_1 = 3$ $r = 2$ $n = 20$

1 to 20 = 20 terms

$$= \frac{3(1-2^{20})}{1-2} =$$

Ex. $\sum_{n=2}^{\infty} 4\left(\frac{1}{3}\right)^n$ ⇒ infinite geo. sum $a_1 = \frac{4}{9}$ (plug in $n=2$)

$r = \frac{1}{3}$

$$S_\infty = \frac{\frac{4}{9}}{1-\frac{1}{3}} = \boxed{\frac{2}{3}}$$

Ex. $\sum_{n=10}^{50} (3n-1)$ ⇒ Arithmetic!! Write out couple terms ⇒ $29 + 32 + \ldots$ 149

$a_1 = 29$ $a_n = 149$ $n = 41$ terms b/t 10 + 50

$$S_n = \frac{41}{2}(29+149) = \boxed{3649}$$

Be careful w/ how written!! Parentheses matter!!

$$\sum_{n=1}^{10}(4n-3) \neq \sum_{n=1}^{10} 4n - 3$$

Here use formula $4n-3$ to generate series

$a_1 = 1$ $a_{10} = 37$

$$S_{10} = \frac{10}{2}(37+1)$$
$$= (38)(5) = \boxed{190}$$

Here use $4n$ + subtract 3 after finding sum.

$a_1 = 4$ $a_{10} = 40$

$$S_{10} = \frac{10}{2}(40+4) = 5(44) = 220$$

Now subt. 3 ⇒ $220 - 3 = \boxed{217}$