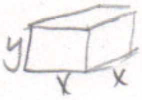
Optimization: In-class examples

Ex.1 A manufacturer wants to design an open box having a square base and a surface area of 108 in^2. What dimensions will produce a box with maximum volume?



$V = x^2 y \Rightarrow \max V.$

$V = x^2\left(\frac{108-x^2}{4x}\right) = 27x - \frac{1}{4}x^3$

$V' = 27 - \frac{3}{4}x^2 = 0$

$x = 6$ $+ \mid - \; V'$ 6 max

$108 = x^2 + 4xy$

$y = \frac{108-x^2}{4x} = \frac{27}{x} - \frac{x}{4}$

$\boxed{6\text{in} \times 6\text{in} \times 3\text{in}}$

Ex. 2 A circular cylindrical metal container, open at the top, is to have a capacity of 24π in^3. The cost of the material used for the bottom of the container is 15 cents per in^2, and that of the material used for the curved part is 5 cents per in^2. If there is no waste of material, find the dimensions that will minimize the cost of the material.

$V = 24\pi = \pi r^2 h \Rightarrow h = \frac{24\pi}{\pi r^2} = \frac{24}{r^2}$

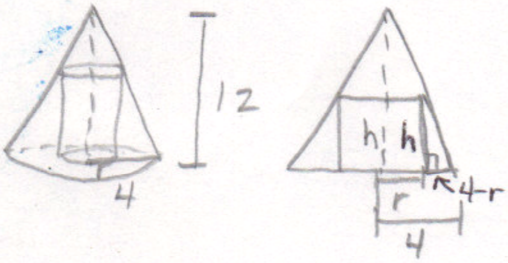
min $C = .15(\pi r^2) + .05(2\pi r h)$

$C = .15\pi r^2 + .05\left(2\pi r \cdot \frac{24}{r^2}\right) = .15\pi r^2 + \frac{2.4\pi}{r}$

$C' = .3\pi r - \frac{2.4\pi}{r^2} = \frac{.3\pi r^3 - 2.4\pi}{r^2} = \frac{.3\pi(r^3-8)}{r^2} = 0$ $- \mid + \; C'$ 2 min

$\boxed{\text{Min cost @ } r = 2\text{in} \;\; h = 6\text{in}}$

Ex. 3 Find the maximum volume of a right circular cylinder that can be inscribed in a cone of altitude 12 cm, and base radius 4 cm, if the axes of the cylinder and cone coincide.



*Similar Δs

$\frac{h}{4-r} = \frac{12}{4} \Rightarrow h = 12 - 3r$

max $V = \pi r^2 h$

$V = \pi r^2(12 - 3r)$

$V = 12\pi r^2 - 3\pi r^3$

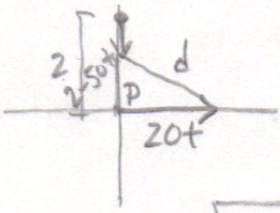
$V' = 24\pi r - 9\pi r^2 = 3\pi r(8 - 3r)$

$r = 0 \quad r = 8/3$ $0 \; + \; 8/3 \; - \; 4$ max

$\boxed{\max V = \frac{256\pi}{9} = 89.361\,cm^3 \text{ @ } r = 8/3\text{ cm } \& \; h = 4\text{cm}}$

Ex.4 A North-South highway intersects an East-West highway at point P. An automobile crosses P at 10 am, traveling east at a constant speed of 20 mph. At that same instant another automobile is 2 miles north of P, traveling south at 50 mph. Find the time at which they are closest to each other and approximate the minimum distance between the automobiles.

* To min √ can min quantity under √



min $d = \sqrt{(20t)^2 + (2-50t)^2}$

$D = 400t^2 + (2 - 50t)^2$

$D' = 800t + 2(-50)(2 - 50t) = 5800t - 200 = 0$ $- \mid + $ $\frac{1}{29}$ min

$t = \frac{1}{29}$ hr.

$\boxed{\text{min dist} = .743 \text{ mi at } t = \frac{1}{29}\text{hr}}$

Ex 5 A wire 60 inches long is to be cut into two pieces. One of the pieces will be bent into the shape of a circle and the other into the shape of an equilateral triangle. Where should the wire be cut so that the sum of the areas of the circle and the triangle is minimized? Maximized? — only 1 crit #, so @ endpt

x | 60-x

Circle $A = \pi r^2$

$x = 2\pi r \quad r = \frac{x}{2\pi}$

max $A = \pi r^2 + \sqrt{3}/4\, s^2 = \pi\left(\frac{x}{2\pi}\right)^2 + \frac{\sqrt{3}}{4}\left(\frac{60-x}{3}\right)^2 = \frac{x^2}{4\pi} + \frac{\sqrt{3}}{4}\left(20 - \frac{1}{3}x\right)^2$

$A' = \frac{1}{2\pi}x + \frac{\sqrt{3}}{4}\left(-\frac{1}{3}\right)2\left(20 - \frac{1}{3}x\right) = \left(\frac{1}{2\pi} + \frac{\sqrt{3}}{18}\right)x - \frac{10\sqrt{3}}{3} = 0$

$x = 22.607498$

$0 \; - \; 22.607498 \; + \; 60 \;\; A'$ min

$\boxed{\text{Min @ } x = 22.607 \;\; \text{max @ all circle so } x = 0}$