Other Indeterminate Forms

① $0 \cdot \infty$ $\lim_{x \to a} [f(x)g(x)]$ where $\lim_{x \to a} f(x) = 0$ and

$\lim_{x \to a} g(x) = \pm\infty$,

Rewrite as $\lim_{x \to a} \left[\frac{f(x)}{\frac{1}{g(x)}} \right]$ or $\lim_{x \to a} \left[\frac{g(x)}{\frac{1}{f(x)}} \right]$

+ use L'Hop Rule.

$$\text{Ex. } \lim_{x \to 0^+} \left(\overset{0 \cdot -\infty}{x^2 \cdot \ln x}\right) = \lim_{x \to 0^+} \frac{\ln x}{x^{-2}} = \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{2}{x^3}} = \lim_{x \to 0^+} \frac{x^2}{-2} = \boxed{0}$$

$$\frac{1}{x} \cdot \frac{x^3}{-2}$$

$$\text{Ex. } \lim_{x \to \frac{\pi}{2}^-} (\overset{0}{2x - \pi}) \overset{\infty}{\sec x} = \lim_{x \to \frac{\pi}{2}^-} \frac{\overset{0}{2x - \pi}}{\underset{0}{\cos x}} = \lim_{x \to \frac{\pi}{2}^-} \frac{2}{-\sin x} = -2$$

② $\infty - \infty = -\infty + \infty$

+ Get common denom + use L'Hop
+ $\ln(\quad) - \ln(\quad) \Rightarrow$ Use log prop.

Write as one fra

$$\text{Ex. } \lim_{x \to \infty} [\ln(3x-2) - \ln(5x+1)] = \lim_{x \to \infty} \left[\ln\left(\frac{3x-2}{5x+1}\right)\right]$$

$$= \ln\left[\lim_{x \to \infty} \left(\frac{3x-2}{5x+1}\right)\right] = \boxed{\ln\left(\frac{3}{5}\right)}$$

Ex.

$$\lim_{x\to0^+}\left(\frac{1}{e^x-1}-\frac{1}{x}\right)=\lim_{x\to0^+}\frac{x-e^x+1}{x(e^x-1)}=\lim_{x\to0^+}\frac{1-e^x}{e^x+xe^x-1}$$

$\infty-\infty$ $\quad xe^x-x \quad (e^x-1)+xe^x$

$$=\lim_{x\to0^+}\frac{-e^x}{e^x+e^x+xe^x}=\boxed{\frac{-1}{2}}$$

Ex.

$$\lim_{x\to1^+}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)=\lim_{x\to1^+}\frac{x-1-\ln x}{(\ln x)(x-1)}=\lim_{x\to1^+}\frac{1-\frac{1}{x}}{\frac{1}{x}(x-1)+\ln x}$$

$1-\frac{1}{x}+\ln x$

$$=\lim_{x\to1^+}\frac{\frac{1}{x^2}}{\frac{1}{x^2}+\frac{1}{x}}=\boxed{\frac{1}{2}}$$

(3) 0^0 OR ∞^0 OR 1^∞

$$\lim_{x\to a}\left[\underbrace{f(x)^{g(x)}}_{y}\right]=0^0 \text{ OR } \infty^0 \text{ OR } 1^\infty$$

$$\ln y=\lim_{x\to a}\left[\ln\left(f(x)^{g(x)}\right)\right]$$

$$=\lim_{x\to a}\left[g(x)\ln f(x)\right]=L$$

$0\cdot\infty$

$\infty\cdot0$

$$\Rightarrow \lim_{x\to a} f(x)^{g(x)}=\boxed{e^L}$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e^{\boxed{1}}$$

1^∞

$$\lim_{x\to\infty}\ln\left(1+\frac{1}{x}\right)^x = \lim_{x\to\infty} x\ln\left(1+\frac{1}{x}\right) = \lim_{x\to\infty}\frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}}$$

$$= \lim_{x\to\infty}\frac{\cancel{\left(-\frac{1}{x^2}\right)}\frac{1}{1+\frac{1}{x}}}{\cancel{-\frac{1}{x^2}}} = \lim_{x\to\infty}\frac{1}{1+\frac{1}{x}} = 1$$

$$* \boxed{\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e} *$$

0^0

Ex. $$\lim_{x\to\frac{\pi}{2}^-}(\cos x)^{\cos x} = e^{\boxed{0}} = \boxed{1}$$

$$\lim_{x\to\frac{\pi}{2}^-}\cos x\ln(\cos x) = \lim_{x\to\frac{\pi}{2}^-}\frac{\ln(\cos x)}{\sec x} = \lim_{x\to\frac{\pi}{2}^-}\frac{\frac{-\sin x}{\cos x}\ (-\cancel{\tan x})}{\sec x\cancel{\tan x}}$$

$$= \lim_{x\to\frac{\pi}{2}^-}\frac{-1}{\sec x} = \lim_{x\to\frac{\pi}{2}^-}(-\cos x) = 0$$