Other types – KEY.

1. a) $f'(1) = -2 \quad f''(1) = 12$

   b) $y = 3 - 2(x-1)$

   c) $(1,3)$ decr. b/c $f' < 0$
   c.up b/c $f'' > 0$

2. a) $f'''(-2) = 54$

   b) $(-2, -4)$
   max b/c $f'(-2) = 0$ & $f''(-2) < 0$
   (2^{nd} Deriv test)

   c) $f'(x) \approx -6(x+2) + 27(x+2)^2 - 24(x+2)^3$

3. a) $f(2) = 3 \quad f'(2) = 0 \quad f''(2) = -\frac{2}{3}$

   b) $(2,3)$ max b/c $f'(2) = 0$ & $f''(2) < 0$

   c) $g(x) \approx 3(x-2) - \frac{2}{3!\cdot 3}(x-2)^3 + \frac{6}{5!\cdot 5}(x-2)^5$

4. $f(1) = 6 \quad f'(1) = 2$
   $f''(1) = -16$

   b) $y = 6 + 2(x-1)$

   c) above $\Rightarrow$ since $f'' < 0$
   $f(x)$ is conc. down so
   tangent above curve.

5. a) $f(x) \approx P_3(x) = 4 - 2x + \frac{6x^2}{2!} - \frac{x^3}{3!}$

   $f(0.1) \approx 4 - 2(.1) + \frac{6(.1)^2}{2!} - \frac{(.1)^3}{3!} = \cancel{3.859}\ \ 3.8298\overline{3}$

   b) $g'(x) = f'(x) \approx -2 + \frac{\cancel{12}}{\cancel{2!}}\,6\,x - \frac{\cancel{3}}{\cancel{3!}}\,\frac{1}{2}\,x^2 = -2 + 6x - \frac{1}{2}x^2$

   c) $h(x) = \int_0^{x^2} f(t)\,dt \approx 4x^2 - x^4 + \cancel{2}\,x^6 - \frac{1}{24}x^8$

   d) yes, b/c $h'(x) = 0$ & $h''(x) > 0$
   so by 2^{nd} D. test, local min @ $(0,0)$

6. $f(3) = 2 \quad f'(3) = \frac{-1(2)}{1(2)} = -1 \quad f''(3) = \frac{3!}{2\cdot 2^2} = \frac{3}{4} \quad f'''(3) = \frac{-1\cdot 4!}{3\cdot 8}$

   $f(x) \approx P_3(x) = 2 - (x-3) + \frac{3}{4\cdot 2!}(x-3)^2 - \frac{\cancel{4!}\,4}{3\cdot 8\cdot \cancel{3!}}(x-3)^3$

   b) Need $\sum_{n=1}^{\infty} \frac{(-1)^n (n+1)!}{n\cdot 2^n \cdot n!}(x-3)^n = \sum_{n=1}^{\infty} \frac{(-1)^n (n+1)(x-3)^n}{n\cdot 2^n}$ & do Ratio test

   $\lim_{n\to\infty} \left| \frac{(n+2)\cancel{(x-3)^{n+1}}^{\,x-3}}{(n+1)\cancel{2^{n+1}}_{\,2}} \cdot \frac{n\cancel{2^n}}{(n+1)\cancel{(x-3)^n}} \right| = \frac{1}{2}|x-3| < 1 \Rightarrow |x-3| < 2$

   $\boxed{\text{Radius conv} = 2}$

   c) B/c alt & decr. series, error $= R_5 \le \frac{7}{6\cdot 2^6}(3.1-3)^6 \approx 1.823\times 10^{-8}$