The **bisection method** for approximating the real zeros of a continuous function is similar to the method used in Example 8. If you know that a zero exists in the closed interval $[a, b]$, the zero must lie in the interval $[a, (a+b)/2]$ or $[(a+b)/2, b]$. From the sign of $f([a+b]/2)$, you can determine which interval contains the zero. By repeatedly bisecting the interval, you can "close in" on the zero of the function.

TECHNOLOGY You can also use the *zoom* feature of a graphing utility to approximate the real zeros of a continuous function. By repeatedly zooming in on the point where the graph crosses the x-axis, and adjusting the x-axis scale, you can approximate the zero of the function to any desired accuracy. The zero of $x^3 + 2x - 1$ is approximately 0.453, as shown in Figure 1.38.

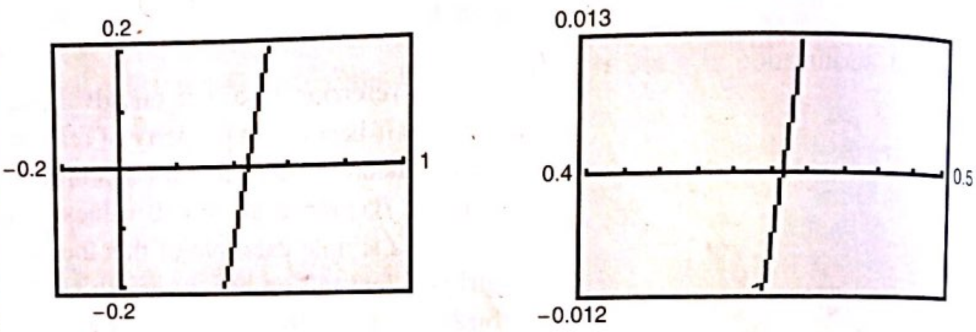


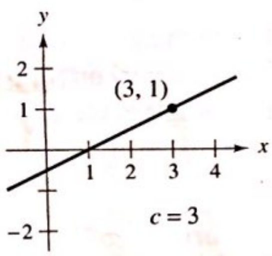
Figure 1.38 Zooming in on the zero of $f(x) = x^3 + 2x - 1$

EXERCISES FOR SECTION 1.4

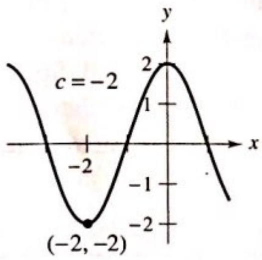
In Exercises 1–6, use the graph to determine the limit, and discuss the continuity of the function.

(a) $\lim_{x \to c^+} f(x)$ (b) $\lim_{x \to c^-} f(x)$ (c) $\lim_{x \to c} f(x)$

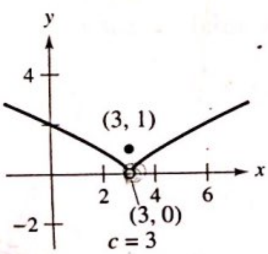
1.



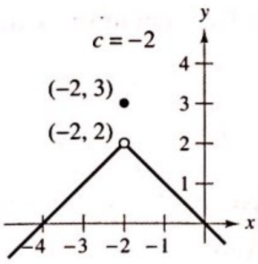
2.



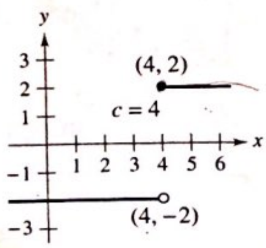
3.



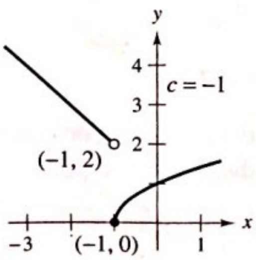
4.



5.



6.



In Exercises 7–24, find the limit (if it exists). If it does not exist, explain why.

7. $\lim_{x \to 5^+} \dfrac{x - 5}{x^2 - 25}$

8. $\lim_{x \to 2^+} \dfrac{2 - x}{x^2 - 4}$

9. $\lim_{x \to -3^-} \dfrac{x}{\sqrt{x^2 - 9}}$

10. $\lim_{x \to 4^-} \dfrac{\sqrt{x} - 2}{x - 4}$

11. $\lim_{x \to 0^-} \dfrac{|x|}{x}$

12. $\lim_{x \to 2^+} \dfrac{|x - 2|}{x - 2}$

13. $\lim_{\Delta x \to 0^-} \dfrac{\dfrac{1}{x + \Delta x} - \dfrac{1}{x}}{\Delta x}$

14. $\lim_{\Delta x \to 0^+} \dfrac{(x + \Delta x)^2 + x + \Delta x - (x^2 + x)}{\Delta x}$

15. $\lim_{x \to 3^-} f(x)$, where $f(x) = \begin{cases} \dfrac{x + 2}{2}, & x \le 3 \\ \dfrac{12 - 2x}{3}, & x > 3 \end{cases}$

16. $\lim_{x \to 2} f(x)$, where $f(x) = \begin{cases} x^2 - 4x + 6, & x < 2 \\ -x^2 + 4x - 2, & x \ge 2 \end{cases}$

17. $\lim_{x \to 1} f(x)$, where $f(x) = \begin{cases} x^3 + 1, & x < 1 \\ x + 1, & x \ge 1 \end{cases}$

18. $\lim_{x \to 1^+} f(x)$, where $f(x) = \begin{cases} x, & x \le 1 \\ 1 - x, & x > 1 \end{cases}$

19. $\lim_{x \to \pi} \cot x$

20. $\lim_{x \to \pi/2} \sec x$

21. $\lim_{x \to 4^-} (3[\![x]\!] - 5)$

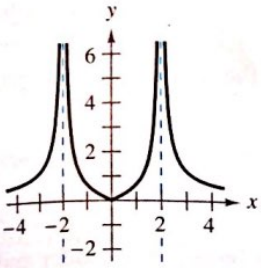
22. $\lim_{x \to 2^+} (2x - [\![x]\!])$

23. $\lim_{x \to 3} (2 - [\![-x]\!])$

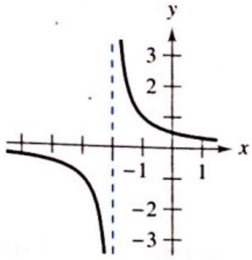
24. $\lim \left(1 - \left[\!\left[-\dfrac{x}{2}\right]\!\right]\right)$

In Exercises 1–4, determine whether $f(x)$ approaches ∞ or $-\infty$ as x approaches -2 from the left and from the right.

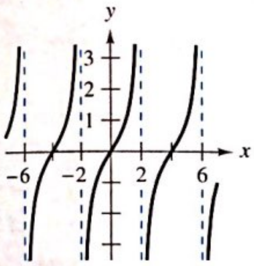
1. $f(x) = 2\left|\frac{x}{x^2 - 4}\right|$



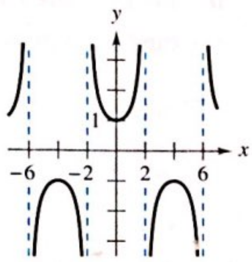
2. $f(x) = \frac{1}{x + 2}$



3. $f(x) = \tan \frac{\pi x}{4}$



4. $f(x) = \sec \frac{\pi x}{4}$



Numerical and Graphical Analysis In Exercises 5–8, determine whether $f(x)$ approaches ∞ or $-\infty$ as x approaches -3 from the left and from the right by completing the table. Use a graphing utility to graph the function and confirm your answer.

x	-3.5	-3.1	-3.01	-3.001
$f(x)$				

x	-2.999	-2.99	-2.9	-2.5
$f(x)$				

5. $f(x) = \frac{1}{x^2 - 9}$

6. $f(x) = \frac{x}{x^2 - 9}$

7. $f(x) = \frac{x^2}{x^2 - 9}$

8. $f(x) = \sec \frac{\pi x}{6}$

In Exercises 9–28, find the vertical asymptotes (if any) of the function.

9. $f(x) = \frac{1}{x^2}$

10. $f(x) = \frac{4}{(x - 2)^3}$

11. $h(x) = \frac{x^2 - 2}{x^2 - x - 2}$

12. $g(x) = \frac{2 + x}{x^2(1 - x)}$

13. $f(x) = \frac{x^2}{x^2 - 4}$

14. $f(x) = \frac{-4x}{x^2 + 4}$

15. $g(t) = \frac{t - 1}{t^2 + 1}$

16. $h(s) = \frac{2s - 3}{s^2 - 25}$

17. $f(x) = \tan 2x$

18. $f(x) = \sec \pi x$

19. $T(t) = 1 - \frac{4}{t^2}$

20. $g(x) = \frac{\frac{1}{2}x^3 - x^2 - 4x}{3x^2 - 6x - 24}$

21. $f(x) = \frac{x}{x^2 + x - 2}$

22. $f(x) = \frac{4x^2 + 4x - 24}{x^4 - 2x^3 - 9x^2 + 18x}$

23. $g(x) = \frac{x^3 + 1}{x + 1}$

24. $h(x) = \frac{x^2 - 4}{x^3 + 2x^2 + x}$

25. $f(x) = \frac{x^2 - 2x - 15}{x^3 - 5x^2 + x - 5}$

26. $h(t) = \frac{t^2 - 2t}{t^4 - 16}$

27. $s(t) = \frac{t}{\sin t}$

28. $g(\theta) = \frac{\tan \theta}{\theta}$

In Exercises 29–32, determine whether the function has a ve cal asymptote or a removable discontinuity at $x = -1$. Gr the function using a graphing utility to confirm your answe

29. $f(x) = \frac{x^2 - 1}{x + 1}$

30. $f(x) = \frac{x^2 - 6x - 7}{x + 1}$

31. $f(x) = \frac{x^2 + 1}{x + 1}$

32. $f(x) = \frac{\sin(x + 1)}{x + 1}$

In Exercises 33–48, find the limit.

33. $\lim_{x \to 2^+} \frac{x - 3}{x - 2}$

34. $\lim_{x \to 1^+} \frac{2 + x}{1 - x}$

35. $\lim_{x \to 3^+} \frac{x^2}{x^2 - 9}$

36. $\lim_{x \to 4^-} \frac{x^2}{x^2 + 16}$

37. $\lim_{x \to -3^-} \frac{x^2 + 2x - 3}{x^2 + x - 6}$

38. $\lim_{x \to (-1/2)^+} \frac{6x^2 + x -}{4x^2 - 4x -}$

39. $\lim_{x \to 1} \frac{x^2 - x}{(x^2 + 1)(x - 1)}$

40. $\lim_{x \to 3} \frac{x - 2}{x^2}$

41. $\lim_{x \to 0^-} \left(1 + \frac{1}{x}\right)$

42. $\lim_{x \to 0^-} \left(x^2 - \frac{1}{x}\right)$

43. $\lim_{x \to 0^+} \frac{2}{\sin x}$

44. $\lim_{x \to (\pi/2)^+} \frac{-2}{\cos x}$

45. $\lim_{x \to \pi} \frac{\sqrt{x}}{\csc x}$

46. $\lim_{x \to 0} \frac{x + 2}{\cot x}$

47. $\lim_{x \to 1/2} x \sec \pi x$

48. $\lim_{x \to 1/2} x^2 \tan \pi x$

In Exercises 49–52, use a graphing utility to graph the fun and determine the one-sided limit.

49. $f(x) = \frac{x^2 + x + 1}{x^3 - 1}$

$\lim_{x \to 1^+} f(x)$

50. $f(x) = \frac{x^3 - 1}{x^2 + x + 1}$

$\lim_{x \to 1^-} f(x)$

51. $f(x) = \frac{1}{x^2 - 25}$

$\lim_{x \to 5^-} f(x)$

52. $f(x) = \sec \frac{\pi x}{6}$

$\lim_{x \to 3^+} f(x)$