AP Calculus BC: Parametric/Vector Motion

1. A particle is moving such that its position (in cm) at time t seconds is $\langle t^3 - 3t^2 + 4t, \sin t \rangle$.

   a) Find $\vec{v}(t)$ and $\vec{a}(t)$.

   b) Describe the motion of the particle at t = 1 second.

   c) Find the speed at 2 seconds.

   d) Find the total distance traveled on [0, 4].

2. An object moves along a curve in the xy-plane with position $\langle x(t), y(t) \rangle$ at time t seconds for $0 \le t \le 5$ and

   $\dfrac{dx}{dt} = \sin(t^2)$ and $\dfrac{dy}{dt} = t^3 \cos t$. At t = 1, the object is at position $\langle -2, 4 \rangle$.

   a) Find the speed at t = 1 second.

   b) Find the total distance traveled on [0, 5].

   c) Find the displacement on [0, 5].

   d) Find the position at t = 4.

3. The acceleration of an object moving in the xy-plane is given by $\left\langle t - 1, \dfrac{1}{t} \right\rangle$ for the time interval [1, 6]. At t = 1,

   the object is at (2, 1) and at rest.

   a) Find $\vec{v}(t)$ and the position vector $\langle x(t), y(t) \rangle$.

   b) When is the object farthest to the left? What is its position at this time? Justify your answer.

   c) The object starts from rest at t = 1. Is there any other time that the object is stopped on [1, 6]? If so, when? Explain why or why not.

   d) Determine the speed at t = 3 seconds.