Review Key - Parametric & Polar

1. $\cos^2 t = x+2$

$\sin t = y-1$

$\sin^2 t + \cos^2 t = 1$

$(y-1)^2 + (x+2) = 1$

$\boxed{x = -(y-1)^2 - 1}$

init: $(-1,1)$ term: $(-1,1)$

2. $x = \frac{1}{t}+1 \quad y = \frac{2}{t} - t$

$t = \frac{1}{x-1}$

$y = \frac{2}{\frac{1}{x-1}} - \frac{1}{x-1}$

$\boxed{y = 2(x-1) - \frac{1}{x-1}}$

term: $\left(\frac{5}{4}, -\frac{7}{2}\right)$

init not included

3. $\boxed{x^2+y^2 = \frac{y}{x}}$

4. $r^2 = 2r\cos\theta + 3r\sin\theta$

$\boxed{x^2+y^2 = 2x+3y}$

5. $\tan\theta = \tan\frac{\pi}{4}$

$\frac{y}{x} = 1 \quad \boxed{y = x}$

6. $y^2 = 4x$

$r^2\sin^2\theta = 4r\cos\theta$

$r = \frac{4\cos\theta}{\sin^2\theta}$

$\boxed{r = 4\cot\theta\csc\theta}$

7. $x^2+y^2 = 2xy$

$r^2 = 2r\cos\theta \cdot r\sin\theta$

$r^2 = 2r^2\cos\theta\sin\theta$

$\boxed{1 = \sin 2\theta}$

8. $2r\cos\theta - 3r\sin\theta = 8$

$r(2\cos\theta - 3\sin\theta) = 8$

$\boxed{r = \frac{8}{2\cos\theta - 3\sin\theta}}$

9. $(1)^2 + (-\sqrt{3})^2 = r^2$

$r = \pm 2$

$\tan\theta = -\sqrt{3}$ * 4^{th} Q

$\theta = \frac{5\pi}{3} \quad \boxed{\left(2, \frac{5\pi}{3}\right)}$

10. Circle: center $(2,0)$, radius $=2$, $[0,\pi]$, symm to polar

11. Limaçon w/ loop, $[0,2\pi]$, sym to $\theta = \frac{\pi}{2}$

12. Rose curve: 3 petals, length $=2$, $[0,\pi]$, symm to polar

13. Cardioid: $[0,2\pi]$ symm. to $\theta = \frac{\pi}{2}$

14. Line through pole

15. Limaçon w/ dimple, $[0,2\pi]$ sym to polar axis

16. $r = 3\sin\theta \quad [0,\pi]$

a) $r=0 \Rightarrow 3\sin\theta = 0$

$\theta = 0, \pi$

$\frac{dr}{d\theta} = 3\cos\theta \neq 0$ @ $\theta = 0$ or π

$\theta = 0$ & $\theta = \pi$ are same line

$\boxed{\text{Polar} \Rightarrow \theta = 0 \quad \text{Rect: } y = 0}$

b) $x = 3\sin\theta\cos\theta$

$y = 3\sin^2\theta$

horiz: $\frac{dy}{d\theta} = 6\sin\theta\cos\theta = 0$

$\theta = 0, \pi/2 \Rightarrow \boxed{(0,0) \ \left(3, \frac{\pi}{2}\right)}$

vert: $\frac{dx}{d\theta} = 3\cos^2\theta - 3\sin^2\theta = 0$

$\theta = \pi/4$ & $\frac{3\pi}{4} \Rightarrow \boxed{\left(\frac{3\sqrt{2}}{2}, \frac{\pi}{4}\right) \left(\frac{3\sqrt{2}}{2}, \frac{3\pi}{4}\right)}$

c) $\frac{dy}{dx} = \frac{6\cos\theta\sin\theta}{3\cos^2\theta - 3\sin^2\theta}\Big|_{\theta=\frac{\pi}{6}} = \sqrt{3}$

$\boxed{y - \frac{3}{4} = \sqrt{3}\left(x - \frac{3\sqrt{3}}{4}\right)}$

17. $\frac{dy}{dx} = \frac{2\sec^2 t}{\sec t\tan t}\Big|_{t=\pi/6} = 4$ $\quad = 2\csc t$ $\quad \boxed{y - 1 - \frac{2\sqrt{3}}{3} = 4\left(x - 2 - \frac{2}{\sqrt{3}}\right)}$

$\frac{d^2y}{dx^2} = \frac{-2\csc t\cot t}{\sec t\tan t}\Big|_{\pi/6} = -6\sqrt{3} < 0$ so conc. down

18. $\frac{dy}{dx} = \frac{2}{2t+3}\Big|_{t=1} = \frac{2}{5}$ $\boxed{y-5 = \frac{2}{5}(x-4)}$

$\frac{d^2y}{dx^2} = \frac{\frac{-4}{(2t+3)^2}}{2t+3} = \frac{-4}{(2t+3)^3}\Big|_{t=1} = \frac{-4}{125} < 0$ conc. down

19. horiz: $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$
$\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)\left(\frac{3}{2}, -\frac{3\sqrt{3}}{2}\right)$
vert $\theta = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$
$(4,0)\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$

20. no horiz
vert @ $t=1 \Rightarrow (-3,1)$

21. horiz @ $t = \frac{5}{2}$ $\left(\frac{115}{8}, \frac{-25}{4}\right)$
vert @ $t = \pm 2$ $(16,-6)(-16,14)$

22. horiz: $\theta = 0, \frac{\pi}{2}$
$(0,0)\ (0,-4)$
vert: $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$
$(-2,-2)\ (2,-2)$

23. $\int_0^{\pi} \sqrt{(-2\sin 2t)^2 + (2\sin t\cos t)^2}\, dt = 4.472$

24. $x(t) = (80\cos 60°)t = 40t$
$y(t) = \frac{1}{2}(-9.8)t^2 + (80\sin 60°)t + 10 = -4.9t^2 + 40\sqrt{3}t + 10$

a) $y(t) = 0 \Rightarrow t = \boxed{14.282 \text{ sec}}$

b) max height $\Rightarrow y'(t) = 0$ @ $t = 7.0696$ sec
$y(7.0696) = \boxed{254.898 m}$

c) $\int_0^{14.282} \sqrt{(40)^2 + (-9.8t + 40\sqrt{3})^2}\, dt = \boxed{792.0926 m}$

d) speed $= \sqrt{(40)^2 + (49.682)^2} = \boxed{63.783 \text{ m/s}}$

25. $4 \cdot \frac{1}{2}\int_0^{\pi/2} (\sin 2\theta)^2 d\theta = \frac{\pi}{2}$

26. $2 \cdot \frac{1}{2}\int_{-\pi/2}^{\pi/2} (6 - 6\sin\theta)^2 d\theta = 54\pi$

27. $\frac{1}{2}\int_0^{2\pi} (1 - 2\cos\theta)^2 d\theta - 2 \cdot \frac{1}{2}\int_{-\pi/3}^{\pi/3} (1 - 2\cos\theta)^2 d\theta$

28. $8\left[\frac{1}{2}\int_0^{\pi/6} (2)^2 d\theta + \frac{1}{2}\int_{\pi/6}^{\pi/4} (4\cos 2\theta)^2 d\theta\right]$

29. $2\left[\frac{1}{2}\int_{\pi/3}^{\pi} (2 + 2\cos\theta)^2 d\theta - \frac{1}{2}\int_{\pi/3}^{\pi/2} (6\cos\theta)^2 d\theta\right]$

30. $2\left[\frac{1}{2}\int_{\pi/3}^{\pi} (3)^2 d\theta - \frac{1}{2}\int_{\pi/3}^{\pi} (2 + 2\cos\theta)^2 d\theta\right] = 14.077$

31. $2\left[\frac{1}{2}\int_0^{\arcsin\frac{1}{4}} (5\sin\theta)^2 d\theta + \frac{1}{2}\int_{\arcsin\frac{1}{4}}^{\pi/2} (1 + \sin\theta)^2 d\theta\right] \approx 4.17$

32. $3\left[\frac{1}{2}\int_0^{\pi/3} (2\sin 3\theta)^2 d\theta\right] = \pi$