AP Calculus BC – Review Parametric and Polar

Convert the following parametric and polar equations to rectangular. Give the initial and terminal points if applicable.

1. $x(t) = \cos^2 t - 2$ $y(t) = \sin t + 1$; $[0, 2\pi]$ 2. $x(t) = \frac{1}{t} + 1$ $y(t) = \frac{2}{t} - t$; $(0, 4]$
3. $r^2 = \tan\theta$ 4. $r = 2\cos\theta + 3\sin\theta$ 5. $\theta = \pi/4$

Convert to polar:

6. $y^2 = 4x$ 7. $x^2 + y^2 = 2xy$ 8. $2x - 3y = 8$ 9. $(1, -\sqrt{3})$

Describe the graph of the polar equation: Name it and give appropriate characteristics

10. $r = 4\cos\theta$ 11. $r = 2 - 3\sin\theta$ 12. $r = 2\cos 3\theta$ 13. $r = 1 + \sin\theta$

14. $\theta = \pi/3$ 15. $r = 4 + 2\cos\theta$

16. For $r = 3\sin\theta$, find (a) tangents at the pole, (b) all polar points of horizontal and vertical tangency, and (c) the equation of the tangent line @ $\theta = \pi/6$.

Find the equation of the tangent line and describe the concavity at the given value.

17. $x(t) = 2 + \sec(t)$; $y(t) = 1 + 2\tan(t)$ at $t = \pi/6$ 18. $x(t) = t^2 + 3t$; $y(t) = 2t + 3$ at $(4, 5)$

Determine the rectangular points where the following curves have horizontal and vertical tangents.

19. $r = 2 + 2\cos\theta$ 20. $x(t) = 3t^2 - 6t$; $y(t) = \sqrt{t}$ for $t \geq 0$ 21. $x(t) = 12t - t^3$; $y(t) = t^2 - 5t$ 22. $r = -4\sin\theta$

23. **Find the length of the curve on [0, π]**: $x(t) = \cos(2t)$ $y(t) = \sin^2 t$

24. A football is kicked at a velocity of 80 m/s off of a building 10 m high. If the equations describing the motion of a projectile are given by $x(t) = v_0(\cos\theta)t + x_0$ and $y(t) = \frac{1}{2} gt^2 + v_0(\sin\theta)t + y_0$, answer the following:

a) When will the football strike the ground?

b) What is the maximum height of the football?

c) What is the total distance travelled by the football at the time of impact?

d) What is the speed of the football at 2 seconds?

Find the area of the following regions:

25. bounded by $r = \sin 2\theta$ 26. bounded by $r = 6 - 6\sin\theta$ 27. between the inner and outer loops of $r = 1 - 2\cos\theta$

28. common region bounded by $r = 4\cos 2\theta$ and $r = 2$ 29. outside $r = 6\cos\theta$ and inside $r = 2 + 2\cos\theta$

30. outside $r = 2 + 2\cos\theta$ and inside $r = 3$ 31. inside both $r = 1 + \sin\theta$ and $r = 5\sin\theta$

32. Bounded by $r = 2\sin(3\theta)$

Short Answer AP Exam questions:

1987 – BC 5

The position of a particle moving in the xy-plane at any time t, $0 \leq t \leq 2\pi$, is given by the parametric equations $x = \sin t$ and $y = \cos(2t)$.

a) Find the velocity vector for the particle at any time t, $0 \leq t \leq 2\pi$.
b) For what values of t is the particle at rest?
c) Write an equation for the path of the particle in terms of x and y that does NOT involve trigonometric functions.
d) Sketch the path of the particle.

1994 – BC 3

A particle moves along the graph of $y = \cos x$ so that the x-component of acceleration is always 2. At time t = 0, the particle is at the point $(\pi, -1)$ and the velocity vector of the particle is (0, 0).

a) Find the x- and y- coordinates of the position of the particle in terms of t.
b) Find the speed of the particle when its position is $(4, \cos 4)$.

1995 – BC 1

Two particles move in the xy-plane. For time $t>0$, the position of particle A is given by $x = t - 2$ and $y = (t - 2)^2$, and the position of particle B is given by $x = \frac{3t}{2} - 4$ and $y = \frac{3t}{2} - 2$.

a) Find the velocity vector for each particle at time t = 3.
b) Set up an integral expression that gives the distance traveled by particle A from t = 0 to t = 3. Do not evaluate.
c) Determine the exact time at which the particles collide; that is, when the particles are at the same point at the same time. Justify your answer.

1989 – BC 4

Consider the curve given by the parametric equations $x = 2t^3 - 3t^2$ and $y = t^3 - 12t$.

a) In terms of t, find dy/dx.
b) Write an equation for the line tangent to the curve at the point where t = -1.
c) Rind the x- and y-coordinates for each critical point on the curve and identify each point as having a vertical or horizontal tangent.

1993 – BC 2

The position of a particle at any time $t > 0$ is given by $x(t) = t^2 - 3$ and $y(t) = \frac{2}{3}t^3$.

a) Find the magnitude of the velocity vector at t = 5.
b) Find the total distance traveled by the particle from t = 0 to t = 5.
c) Find dy/dx as a function of x.