

Applications of Piecewise:

Think About It ...

- ① A car company charges \$45 plus \$0.20 per mile over 50 miles.

Give the equation for the cost of driving $0 \leq x \leq 50$ miles.

$$C = 45$$

Give an equation to determine the cost of driving MORE than 50 miles ($x > 50$).

$$\begin{aligned} C &= 45 + .20(x - 50) \\ &= 45 + .2x - 10 = 35 + .2x \end{aligned}$$

Put this together as a piecewise function ...

$$C(x) = \begin{cases} 45 & 0 \leq x \leq 50 \\ 45 + .2(x - 50) & x > 50 \end{cases}$$

How much will it cost you if you drive 20 miles?

\$45

50 miles?

\$45

51 miles?

\$45.20

200 miles?

\$75

- ② A cell phone company charges a monthly fee of \$9.95, and a usage fee as follows:

Less than 150 minutes: \$0.40 per min

150 to 300 minutes: \$0.20 per min

Over 300 minutes: \$0.10 per min

Write the piecewise function, $C(m)$, for the cost of using m minutes.

$$C(m) = \begin{cases} 9.95 + .4m & m < 150 \\ 9.95 + .4(150) + .2(m - 150) & 150 \leq m \leq 300 \\ 99.95 + .10(m - 300) & m > 300 \end{cases}$$

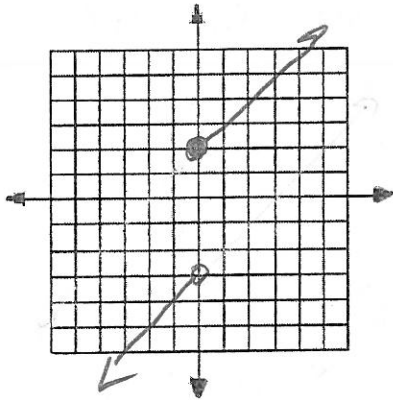
What is the total charge if you use 200 minutes?

350 minutes?

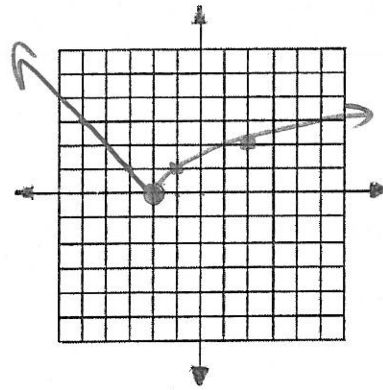
Key

II. Graph the following piecewise functions.

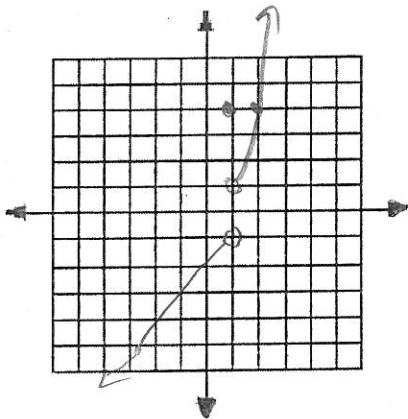
$$f(x) = \begin{cases} x+2, & x \geq 0 \\ x-3, & x < 0 \end{cases}$$



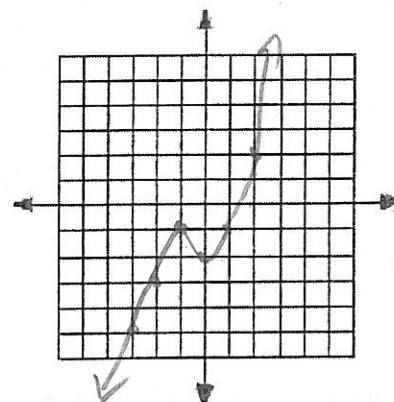
$$f(x) = \begin{cases} \sqrt{x+2}, & x > -2 \\ |x+2|, & x \leq -2 \end{cases}$$



$$f(x) = \begin{cases} x^2, & x > 1 \\ 4, & x = 1 \\ x-2, & x < 1 \end{cases}$$



$$f(x) = \begin{cases} x^2 - 2, & x > -1 \\ 2x+1, & x \leq -1 \end{cases}$$



Day 6 Notes

Applications of Piecewise Functions

1. A long distance telephone charges 99 cents for any call up to 20 minutes in length and 7 cents for each additional minute. Use bracket notation to write a formula for the cost, C , of a call as a function of its length time, t , in minutes. Graph the function. How much does it cost to talk for 10 minutes? 25 minutes?

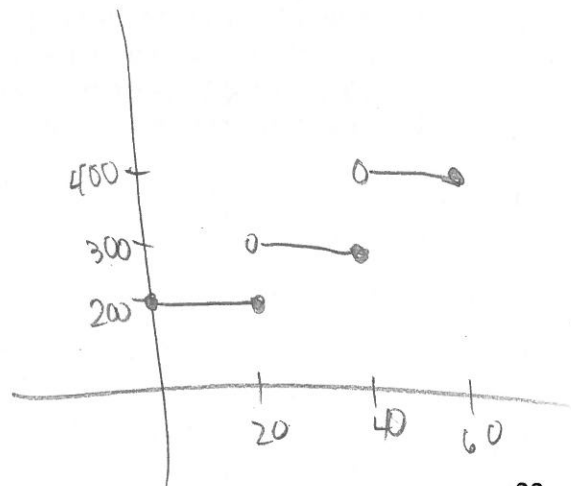
$$C = \begin{cases} .99 & 0 \leq t \leq 20 \\ .99 + .07(t - 20) & t > 20 \end{cases}$$

2. Suppose a carpet store sells carpet for \$10 per square yard for the 100 sq yards purchased, and then lowers the price to \$7 per square yard after the first 100 yards have been purchased. Find a function, $C = f(x)$, that gives the cost of purchasing any number of square yards of carpet between 0 and 200 square yards. How much does it cost for 50 square yards? 150 square yards?

$$C = \begin{cases} 10x & 0 \leq x \leq 100 \\ 1000 + 7(x - 100) & 100 < x \leq 200 \end{cases}$$

3. A company charges \$200 a month to organize a company's payroll for up to 20 employees and an additional \$100 a month for each 20 employees over 20. Find a function, $P = f(x)$, that gives the payroll amount for 100 employees in one month. Graph the function.

$$P = \begin{cases} 200 & 0 \leq x \leq 20 \\ 300 & 20 < x \leq 40 \\ 400 & 40 < x \leq 60 \\ 500 & 60 < x \leq 80 \\ 600 & 80 < x \leq 100 \end{cases}$$



Day 6 Classwork

1. You are a buyer for a grocery store and you are asked to purchase potatoes for the grocery store. The distributor of potatoes tells you that if you buy up to 50 bushels of potatoes, you will pay \$40 per bushel; and for each bushel you purchase above 50 bushels, you will pay \$30 per bushel.

- a. How much will your grocery store pay in total if you decide to purchase 40 bushels? 60 bushels? 100 bushels? $\$3500$ $\$1600$ $\$2300$
- b. Write a function which has as its input values (x-values) the number of bushels of potatoes purchased and outputs the total amount of money that your grocery store will pay for the potatoes.

$$f(x) = \begin{cases} 40x & 0 \leq x \leq 50 \\ 2000 + 30(x-50) & x > 50 \end{cases}$$

2. A certain country taxes the first \$20,000 of an individual's income at a rate of 15%, and all income over \$20,000 is taxed at 20%. $\rightarrow \$2400$ $\rightarrow \$6200$

- a. Al makes \$16,000. Betty makes \$36,000. How much is each taxed?
- b. Write a piecewise function T that specifies the total tax on an income of x dollars.
- c. Make a graph of T. Be sure to plot the points from part a!
- d. Catina is taxed \$5000. What is her income? $\$30000$

$$T = \begin{cases} .15x & 0 \leq x \leq 20000 \\ 3000 + .2(x-20000) & x > 20000 \end{cases}$$

$$3000 + .2(x-20000) = 5000 \quad .2(x-20000) = 2000$$

$$x - 20000 = 10000$$

3. A paperback sells for \$12. The author is paid royalties of 10% on the first 10,000 copies sold, and 15% on any additional copies. $\rightarrow \$1.20$

- a. When the 6,000th book is sold, how much will the author earn on that sale? $\$1.80$
- b. Also, what will the author's total royalties be at that point?
- c. When the 12,000th book is sold, how much will the author earn on that sale?
- d. Also, what will the author's total royalties be at that point?
- e. Let x be the number of copies sold. Write a piecewise function for R (the royalty payment earned on that sale) in terms of x.
- f. How many copies have to be sold in order for the author to have earned \$30,000? 20000

$$R = \begin{cases} 1.2x & 0 \leq x \leq 10000 \\ 12000 + 1.8(x-10000) & x > 10000 \end{cases}$$

Piecewise application extra practice:

Key

1. A parking garage charges \$10 per hour (or part of an hour), with a daily maximum of \$50 per day.

a) How much will a customer pay if he/she parks for 30 minutes? 2 hours? 3.5 hours?

\$10 \$20 \$40

b) Write a piecewise function where x = # hours and C = the cost of parking for x hours.

$$C = \begin{cases} 10 & 0 < x \leq 1 \\ 20 & 1 < x \leq 2 \\ 30 & 2 < x \leq 3 \\ 40 & 3 < x \leq 4 \\ 50 & 4 < x \leq 24 \end{cases}$$

2. A candy company can produce up to 100 candy bars at a cost of \$1.50 per bar. If they make more than 100 bars, the cost goes down to \$1.10 per bar for each additional bar. Write the piecewise function giving the cost of producing x candy bars.

$$C = \begin{cases} 1.50x & 0 \leq x \leq 100 \\ 150 + 1.1(x - 100) & x > 100 \end{cases}$$

3. Infosystem is a company that charges customers for their service by the minute. The charge is \$5 plus \$0.25 for each minute (or part of a minute). Express the charge as a function of time for up to 5 minutes.

$$C = \begin{cases} 5.25 & 0 < x \leq 1 \\ 5.50 & 1 < x \leq 2 \\ 5.75 & 2 < x \leq 3 \\ 6 & 3 < x \leq 4 \\ 6.25 & 4 < x \leq 5 \end{cases}$$

4. A theme park charges \$40 per person for a group up to 10 people and \$30 per person for each additional group member. Write the piecewise function to represent the cost for a group of people to go to the theme park. How much would it cost a group of 8 people? 15 people?

\$ 320 \$ 550

$$C = \begin{cases} 40x & 0 \leq x \leq 10 \\ 400 + 30(x - 10) & x > 10 \end{cases}$$

Unit 1 Review

Find the DOMAIN for problems 1 - 4. Write in interval notation.

1. $f(x) = \frac{x}{x^2 - 9}$

2. $f(x) = \sqrt{2-x}$

3. $f(x) = 4x + 3$

4. $f(x) = \frac{\sqrt{x+2}}{x^2 + 2x - 3}$

5. The graph of a function f is known. Then the graph of $y = f(x-2)$ may be found by _____.

6. The graph of a function is known. Then the graph of $y=f(-x)$ may be obtained by a reflection about the ___-axis.

7. True or False:

_____ a) The graph of $y = -f(x)$ is the reflection about the x -axis of the graph of $y = f(x)$.

_____ b) To obtain the graph of $y = f(x + 2) - 3$, shift the graph of $y = f(x)$ horizontally to the right 2 units and vertically down 3 units.

8. Find the function that is finally graphed after the following transformations are applied to the graph of $y = \sqrt{x}$.

a) 1. Shift up 2 units.

b) 1. Reflect about the x -axis

c) 1. Reflect about the y -axis.

2. Reflect about the x -axis.

2. Shift up 2 units.

2. Vertically stretch by 3.

3. Shift left 3 units.

3. Shift down 2 units.

4. Shift right 4 units.

$f(x) =$ _____

$f(x) =$ _____

$f(x) =$ _____

9. USE GRAPH PAPER. Draw the parent graph and then show each transformation..

a) $f(x) = x^3 + 4$

b) $f(x) = (x+4)^2$

c) $f(x) = -\frac{1}{2}|x|$

d) $f(x) = -2(x-3)^2 - 1$

e) $f(x) = 2\sqrt{-x} + 1$

10. State the domain in interval notation. Then graph (on graph paper). Then use the graph to state the range.

a) $f(x) = \begin{cases} 3x, & -2 < x \leq 1 \\ x+1, & x > 1 \end{cases}$

b) $f(x) = \begin{cases} x, & -4 \leq x < 0 \\ 1, & x = 0 \\ 3x, & x > 0 \end{cases}$

c) $f(x) = \begin{cases} x^2, & -2 \leq x \leq 2 \\ 2x-1, & x > 2 \end{cases}$

domain: _____

domain: _____

domain: _____

range: _____

range: _____

range: _____

11. Find $\frac{f(a+h)-f(a)}{h}$, where $h \neq 0$, for the following two functions.

a) $f(x) = 2x + 3$

b) $f(x) = x^2 - 2$

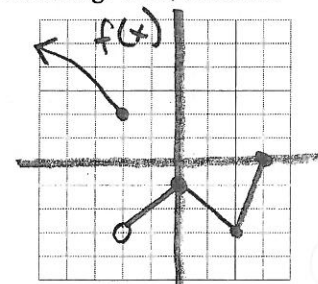
12. Evaluate the piecewise function for $f(-2)$, $f(1)$, and $f(4)$. $f(x) = \begin{cases} x^2 - 2x, & \text{if } x \leq 1 \\ 3x + 1, & \text{if } x > 1 \end{cases}$

13. Use the following graph to evaluate certain values and to find out when it is increasing and decreasing. Also, state the domain and range.

$f(-2) =$ _____ $f(1) =$ _____ $f(x) = 0$ when $x =$ _____

domain _____ Range _____

increasing _____ decreasing _____



14. The domestic postage rate for first class letters weighing 12 oz or less is 33 cents for a letter weighing 1 oz or less and 22 cents for each additional ounce (or part of an ounce). Express the postage P as a function of the weight x of a letter, with $0 < x \leq 12$.