Power Series Practice Key

1. $\lim_{n\to\infty}\left|\dfrac{(n+1)\cancel{x^{n+1}}\,x}{\underset{4}{\cancel{4^{n+1}}}((n+1)^2+1)}\cdot\dfrac{\cancel{4^n}(n^2+1)}{n\cancel{x^n}}\right| = \lim_{n\to\infty}\left|\dfrac{(n+1)(n^2+1)}{4n((n+1)^2+1)}\right||x|$

$= \frac{1}{4}|x|<1 \Rightarrow |x|<4$

By ratio test, Conv. $(-4,4)$

$\boxed{\text{Center} = 0 \quad \text{Radius} = 4}$

$x=-4: \displaystyle\sum_{n=0}^{\infty}\frac{n(-1)^n\cancel{4^n}}{\cancel{4^n}(n^2+1)} = \sum_{n=0}^{\infty}\frac{(-1)^n n}{n^2+1}$

AST

$\lim_{n\to\infty}\frac{n}{n^2+1}=0$ ✓

$f' = \dfrac{n^2+1-n(2n)}{(n^2+1)^2} = \dfrac{1-n^2}{(n^2+1)^2} < 0$ when $n>1$

decr

so conv. @ $x=-4$

$x=4: \displaystyle\sum_{n=0}^{\infty}\frac{n}{n^2+1} = a_n \quad b_n = \frac{1}{n}$ div. harmonic

$\lim_{n\to\infty}\frac{n}{n^2+1}\cdot\frac{n}{1} = 1 > 0$ so $\sum a_n$ div by LCT.

$\boxed{\text{IOC}: [-4,4)}$

2. $\lim_{n\to\infty}\left|\dfrac{\overset{10}{\cancel{10^{n+1}}}\,\overset{(x+2)}{\cancel{(x+2)^{n+1}}}}{\underset{(n+1)}{\cancel{(n+1)!}}}\cdot\dfrac{\cancel{n!}}{\cancel{10^n}\cancel{(x+2)^n}}\right| = \lim_{n\to\infty}\left|\dfrac{10}{n+1}\right||x+2| = 0\cdot|x+2|<1$

so converges by ratio test

$\boxed{\text{IOC}: (-\infty,\infty)}$ $\boxed{\text{Center} = -2 \quad \text{Rad} = \infty}$

3. $\lim_{n\to\infty}\left|\dfrac{(n+1)!\,(x-1)^{n+1}}{n!\,(x-1)^n}\right| = \lim_{n\to\infty}|n+1|\cdot|x-1| = \infty|x-1|<1$

By ratio conv, only converges when $x=1$.

$\boxed{\text{Center} = 1 \quad \text{Rad} = 0 \quad \text{IOC} \Rightarrow x=1 \text{ only}}$

4. $f'(x) = \displaystyle\sum_{n=1}^{\infty}\frac{10^n n(x+2)^{n-1}}{n!} = \sum_{n=1}^{\infty}\frac{10^n(x+2)^{n-1}}{(n-1)!}$

$\displaystyle\int_{-2}^{x} f(t)\,dt = \sum_{n=0}^{\infty}\frac{10^n(x+2)^{n+1}}{(n+1)!}$

This is $= n!(n+1)$

5. $\frac{2}{2-3x} = \frac{2}{2-3(x+2)+6} = \frac{2}{8-3(x+2)} = \frac{1/4}{1-3/8(x+2)}$ → 1st term

r

$= \frac{1}{4} + \frac{3}{32}(x+2) + \frac{9}{256}(x+2)^2 + \cdots = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)\left(\frac{3}{8}\right)^n (x+2)^n$

Geo, so IOC $\left|\frac{3}{8}(x+2)\right| < 1 \Rightarrow |x+2| < \frac{8}{3} \Rightarrow \boxed{\left(-\frac{14}{3}, \frac{2}{3}\right)}$

6. $f(x) = \frac{1}{x} = \frac{1}{1+(x-1)} = 1-(x-1)+(x-1)^2 - \cdots = \sum_{n=0}^{\infty}(-1)^n (x-1)^n$

Geo so IOC $|x-1|<1 \Rightarrow \boxed{(0,2)}$

7. $\ln x = \int_1^x \frac{1}{t}\,dt = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n (x-1)^{n+1}}{n+1}$

Use IOC from #6 & check endpts

$x=0 \quad \sum \frac{(-1)^n(-1)^{n+1}}{n+1} = \sum_{n=0}^{\infty} \frac{(-1)}{n+1} = -\sum_{n=1}^{\infty}\frac{1}{n}$ div. b/c harmonic

$x=2 \quad \sum_{n=0}^{\infty} \frac{(-1)^n(1)^{n+1}}{n+1} = \sum \frac{(-1)^n}{n+1}$ AST $\Rightarrow \lim_{n\to\infty} \frac{1}{n+1} = 0 \quad \frac{1}{n+2} < \frac{1}{n+1}$ dec.

∴ Conv by AST

$\boxed{\text{IOC}: (0,2]}$

8. $\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 \cdots = \sum_{n=0}^{\infty}(-1)^n x^n \rightarrow$ IOC $(-1,1)$ b/c Geo

$\ln(1+x) = \int_0^x \frac{1}{1+t}\,dt = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 \cdots = \boxed{\sum_{n=0}^{\infty}\frac{(-1)^n x^{n+1}}{n+1}}$

chk endpts of $(-1,1)$

$x=-1 \quad \sum_{n=0}^{\infty} \frac{-1}{n+1}$

$x=1 \quad \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$

} see work for #7

$\boxed{\text{IOC } (-1,1]}$

9. $\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \dots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}$

$4\ln(1+x) = 4x - 2x^2 + \frac{4}{3}x^3 - \dots = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot 4x^{n+1}}{n+1}$

$\frac{4\ln(1+x)}{x} = 4 - 2x + \frac{4}{3}x^2 - \dots = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot 4 \cdot x^n}{n+1}$

$\frac{4\ln(1+x)}{x} - 4 = -2x + \frac{4}{3}x^2 - \dots = \boxed{\sum_{n=1}^{\infty} \frac{(-1)^n \cdot 4 \cdot x^n}{n+1}}$

10. $\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \dots$

$\int_0^{.2} \ln(1+x)\,dx = \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{12}x^4 - \dots \Big|_0^{.2}$

$= \frac{1}{2}(.2)^2 - \frac{1}{6}(.2)^3 + \frac{1}{12}(.2)^4 - \dots$

$\approx \frac{1}{2}(.2)^2 - \frac{1}{6}(.2)^3 \approx \boxed{.018\overline{6}}$

B/c alt & conv, error $\leq \left|\frac{1}{12}(.2)^4\right| \approx .0001\overline{3} \leq .001$