## Practice

1. A series expansion of $f(x) = e^{x^2}$ is

A. $1 + x + \frac{x^2}{2!} + \ldots + \frac{x^n}{n!} + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

B. $1 + x^2 + \frac{x^4}{2!} + \ldots + \frac{x^{2n}}{n!} + \ldots = \sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$

C. $1 - x + \frac{x^2}{2!} - \ldots + \frac{(-x)^n}{n!} + \ldots = \sum_{n=0}^{\infty} \frac{(-x)^n}{n!}$

D. $1 - x^2 + \frac{x^4}{2!} - \ldots + \frac{(-1)^n x^{2n}}{n!} + \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$

E. $1 + \frac{x^2}{2!} + \frac{x^4}{4!} \ldots + \frac{x^{2n}}{(2n)!} + \ldots = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$

2. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{2^n}{n^3}$ II. $\sum_{n=1}^{\infty} \frac{3}{2n}$ III. $\sum_{n=0}^{\infty} \left( \frac{1}{2^n} - \frac{1}{3^n} \right)$

A. I only B. II only C. III only D. I and II E. I, II, and III

3. What are all the values of $x$ for which the series $\sum_{n=0}^{\infty} \frac{(-1)^n (x-1)^n}{(n+1)^2}$ converges?

A. $(-1,1)$ B. $(0,2)$ C. $[-1,1)$ D. $(0,2]$ E. $[0,2]$

4. The coefficient of the cubic term in the Taylor series for $f(x) = \sin x$ about $x = 0$ is

A. $-\frac{1}{6}$ B. $\frac{1}{6}$ C. $-\frac{1}{3}$ D. $\frac{1}{3}$ E. 1

5. The sum of the infinite geometric series $\sum_{n=0}^{\infty} \frac{5^n}{7^{n+1}}$ is

A. 0 B. 1 C. $\frac{5}{7}$ D. $\frac{1}{2}$ E. $\frac{1}{7}$

**Free Response Prompt**

The function $f(x) = \frac{e^{(x+1)^2} - 1}{(x+1)^2}$ is continuous for $x \neq -1$ and has derivatives of all orders at $x = 0$.

a) Write the first four non-zero terms and the general term of the Maclaurin series for $e^{(x+1)^2}$.

b) Use the Maclaurin series for $f$ to determine whether the graph of $f$ has any points of inflection.

## Answers and Explanations

1. B 2. C 3. E 4. A 5. D

1. The interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n}$ is
(A) $0 \le x < 2$
(B) $0 < x < 2$
(C) $0 < x \le 2$
(D) $0 \le x \le 2$
(E) all real numbers

2. The series $\sum_{n=0}^{\infty} \frac{(2x-1)^n}{n!}$ converges for which of the following values of $x$?
(A) all real numbers
(B) $0 < x < 1$
(C) $0 < x \le 1$
(D) $0 \le x \le 1$
(E) no values of $x$

3. For which of the following values of $x$ does the series $\sum_{n=0}^{\infty} \left(\frac{5x}{3}\right)^n$ converge?
(A) all real numbers
(B) $|x| \le \frac{3}{5}$
(C) $|x| < \frac{3}{5}$
(D) $|x| \le \frac{5}{3}$
(E) $|x| \le 1$

4. The power series for $f(x) = \frac{3}{3-x}$ centered at $x = 0$ is
(A) $\sum_{n=0}^{\infty} \left(\frac{x}{3}\right)^n$
(B) $3\sum_{n=0}^{\infty} \left(\frac{x}{3}\right)^n$
(C) $9\sum_{n=0}^{\infty} \left(\frac{x}{3}\right)^n$
(D) $\frac{1}{3}\sum_{n=0}^{\infty} \left(\frac{x}{3}\right)^n$
(E) $\sum_{n=1}^{\infty} \left(\frac{x}{3}\right)^n$

5. The number $e$ can be represented by which of the following power series?
(A) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$
(B) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$
(C) $\sum_{n=0}^{\infty} \frac{1}{n!}$
(D) $\sum_{n=1}^{\infty} \frac{1}{n!}$
(E) $\sum_{n=0}^{\infty} \frac{e^n}{n!}$

6. The power series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(3x-1)^n}{n}$ is equivalent to $f(x) =$
(A) $e^{3x-1}$
(B) $\ln(3x-1)$
(C) $\ln(3x)$
(D) $\ln(3x) - 1$
(E) $e^{3x} - 1$

7. For values of $x$ in the interval $-5 < x < 5$, the sum of the series $\sum_{n=1}^{\infty} \left(\frac{x}{5}\right)^n$ is equal to
(A) $\frac{x}{5-x}$
(B) $\frac{5}{5-x}$
(C) $\frac{1}{5-x}$
(D) $\frac{5}{x-5}$
(E) $\frac{x}{x-5}$

8. On the interval $(-3, 3)$, the sum of the series $\sum_{n=1}^{\infty} n\left(\frac{x}{3}\right)^{n-1}$ can be represented by
(A) $f(x) = \frac{9}{(3-x)^2}$
(B) $f(x) = \frac{3}{(3-x)^2}$
(C) $f(x) = -\frac{3}{(3-x)^2}$
(D) $f(x) = \frac{x}{3-x}$
(E) $f(x) = \frac{3}{3-x}$

## Free-Response Questions

*A graphing calculator is required for some questions.*

1. (a) Write a power series for $f(x) = e^{2x} - 1$ and state its interval of convergence.
(b) Find the power series for $f'(x)$ by differentiating the power series found in part (a).
(c) Use the power series for $f(x)$ to find the power series for $f'(x)$ by a method other than taking the derivative.
[illegible] an expression for $f'(0.5)$ in sigma

$\frac{3}{5} - \frac{4}{7} + \cdots$

## Multiple-Choice Questions

*No calculator is allowed for these questions.*

**1.** $\sum_{n=1}^{\infty} \left(-\frac{1}{5}\right)^n$

(A) converges to $-\frac{1}{6}$
(B) converges to $-\frac{1}{5}$
(C) converges to $-\frac{1}{4}$
(D) converges to 0
(E) diverges

**2.** For what values of $x$ does $\sum_{n=0}^{\infty} (2x)^{2n}$ converge?

(A) for no values of $x$
(B) $|x| < \frac{1}{2}$
(C) $|x| \leq \frac{1}{2}$
(D) $|x| < 2$
(E) for all real numbers

**3.** The series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$

(A) converges to $\frac{2}{3}$
(B) converges to $\ln 2$
(C) converges to 2
(D) converges to $e$
(E) diverges

**4.** Find the sum $\sum_{n=1}^{\infty} n\left(\frac{1}{4}\right)^{n-1}$.

(A) $\frac{9}{16}$
(B) $\frac{16}{25}$
(C) $\frac{3}{4}$
(D) $\frac{4}{3}$
(E) $\frac{16}{9}$

**5.** The series $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{(2n+1)!}$ has a sum of

(A) $-1$
(B) 0
(C) 1
(D) $\pi$
(E) no value

6. $\sum_{n=1}^{\infty} \frac{(-1)^n \left(\frac{\pi}{3}\right)^{2n}}{(2n)!} =$

(A) $-\frac{1}{2}$

(B) $\frac{1}{2}$

(C) $\frac{3}{3-\pi}$

(D) $\frac{3}{3+\pi}$

(E) no value

7. The series $\sum_{n=0}^{\infty} \left(\frac{2^n}{3^n} - 1\right)$

(A) converges to $\frac{2}{3}$

(B) converges to 2

(C) converges to 3

(D) converges to 0

(E) diverges

8. Determine the sum of the series $\frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \cdots$

(A) 1

(B) 2

(C) $\frac{1}{e}$

(D) $e$

(E) $\frac{3}{2}$

9. The error in estimating $e^{-2}$ using five terms of the Taylor series for $e^{-x}$ is not greater than

(A) $\frac{2^4}{4!}$

(B) $\frac{2^5}{5!}$

(C) $\frac{2^6}{6!}$

(D) $\frac{2^5}{5}$

(E) $\frac{2^6}{6}$

10. The interval of convergence of the series $\sum_{n=0}^{\infty} \frac{3^n(x-2)^n}{n!}$ is

(A) $(-\infty,\infty)$

(B) $[2,\infty)$

(C) $(-\infty, 2]$

(D) $\left(\frac{5}{3}, \frac{7}{3}\right)$

(E) $x = 2$ only

11. Find the values of $x$ for which the seri $\sum_{n=0}^{\infty} \frac{5^n}{2^n}(x+1)^n$ converges.

(A) no values of $x$

(B) $-\frac{7}{5} < x < -\frac{3}{5}$

(C) $-\frac{5}{2} < x < \frac{5}{2}$

(D) $x = 0$ only

(E) $x = -1$ only

12. For what value(s) of $p$ does $\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^p}$ co verge?

(A) $p \neq 0$

(B) $p < 0$

(C) $p > 0$

(D) $p \geq 1$

(E) $p > 1$

13. The series represented by $\sum_{n=0}^{\infty} \left(\frac{1}{5}\right)^n$ is

(A) an alternating series

(B) a $p$-series

(C) a geometric series

(D) a telescoping series

(E) a power series

14. $\sum_{n=1}^{\infty} \frac{2^n}{n}$ can be described as a series that is

(A) geometric and convergent

(B) not geometric but convergent

(C) alternating

(D) a power series

(E) divergent

15. The series $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n$

(A) converges to 1

(B) diverges

(C) converges to $e$

(D) is geometric with $r > 1$

(E) is alternating

16. The fourth-degree Taylor polynomial fo $\frac{1}{1-x}$ centered at $x = -1$ is

(A) $\sum_{n=0}^{4} x^n$

(B) $\sum_{n=0}^{4} (x+1)^n$

(C) $\sum_{n=0}^{4} \frac{(x+1)^n}{2^{n+1}}$

(D) $\sum_{n=0}^{4} \frac{(x+1)^n}{2^n}$

(E) $2 \sum^{4} (x+1)^n$

17. What is the sum of the series $\sum_{n=0}^{\infty} \frac{\left(\frac{\pi}{6}\right)^{2n}}{n!}$, if it exists?

(A) $\frac{6}{6-\pi}$

(B) $\frac{1}{2}$

(C) $\frac{\sqrt{3}}{2}$

(D) $e^{\pi^2/36}$

(E) does not exist

18. $\frac{d}{dx}\left(\sum_{n=1}^{\infty} \frac{x^{2n}}{n!}\right) =$

(A) $-\sin(x^2)$

(B) $-2x\sin(x^2)$

(C) $\cos(x^2)$

(D) $e^{x^2}$

(E) $2xe^{x^2}$

19. The series $\sum_{n=0}^{\infty}\left(\frac{2^n - n^2}{3^n}\right)$

(A) converges to 0

(B) converges to $\frac{32}{81}$

(C) converges to $\frac{7}{243}$

(D) converges but the sum can only be approximated

(E) diverges

20. What does the series $\frac{\pi}{2}\sum_{n=0}^{\infty} \cos n\pi \frac{\left(\frac{\pi}{2}\right)^{2n}}{(2n+1)!}$ converge to?

(A) $-1$

(B) 0

(C) 1

(D) $\frac{2}{2-\pi}$

(E) It diverges.

**Free-Res**

(a) Write the e
$f(x)$ at $x = 2$

(b) Express $P_4($

(c) Find the v

(d) Is $\int_2^3 f(x)$
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