Power Series Practice

I Find the interval of convergence.

1. $\sum_{n=0}^{\infty} (-1)^{n+1} n x^n$
2. $\sum_{n=0}^{\infty} \frac{(3x)^n}{(2n)!}$
3. $\sum_{n=0}^{\infty} \frac{3^{2n}}{n+1} (x-2)^n$
4. $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n(n+1)}$
5. $\sum_{n=1}^{\infty} \frac{n!\, x^n}{(2n)!}$

II Find the power series representation for the following

6. $f(x) = \frac{3}{2x-1}$, $c = 2$
7. $f(x) = \frac{4}{4+x^2}$, $c = 0$
8. $f(x) = xe^{-2x}$, $c = 0$
9. $f(x) = \cos x^2 - 1$, $c = 0$
10. $f(x) = 3x^2 \arctan x$, $c = 0$
11. $f(x) = \cos\sqrt{x}$, $c = 0$

III. Find the given Taylor polynomial to represent the funct

12. $f(x) = \tan x$, $c = \frac{\pi}{4}$, $P_3(x)$
13. $f(x) = \sqrt{x}$, $c = 4$ $P_3(x)$
14. $f(x) = \cos x$, $c = -\frac{\pi}{4}$, $P_4(x)$
15. $f(x) = \sin(x^2)$, $c = 0$, $P_{10}(x)$
16. $f(x) = \frac{1}{x}$, $c = 2$, $P_3(x)$
17. $f(x) = \cos x$, $c = \frac{\pi}{3}$, $P_3(x)$

IV Use a Taylor polynomial to approximate f(x) with error ≤ .
Show all work include function & center used.

18. $\sin 95°$
19. $\ln(1.15)$
20. $\int_0^1 e^{-x^2}\,dx$

V Given Taylor series to represent f(x), describe the graph of f at c

21. $P_3(x) = -4 - 3(x+2) + 4(x+2)^2 - (x+2)^3$
22. $P_2(x) = 3 + \frac{5(x-1)^2}{4}$

23. For #21 above, give values for $f(-2)$, $f'(-2)$, $f''(-2)$ & $f'''(-2)$.

Let f be a function that has derivatives of all orders for all real numbers. Assume $f(0) = 5$, $f'(0) = -3$, $f''(0) = 1$, and $f'''(0) = 4$.

(a) Write the third-degree Taylor polynomial for f about $x = 0$ and use it to approximate $f(0.2)$.

(b) Write the fourth-degree Taylor polynomial for g, where $g(x) = f(x^2)$, about $x = 0$.

(c) Write the third-degree Taylor polynomial for h, where $h(x) = \int_0^x f(t)\,dt$, about $x = 0$.

(d) Let h be defined as in part (c). Given that $f(1) = 3$, either find the exact value of $h(1)$ or explain why it cannot be determined.

The Taylor series about $x = 5$ for a certain function f converges to $f(x)$ for all x in the interval of convergence. The nth derivative of f at $x = 5$ is given by $f^{(n)}(5) = \dfrac{(-1)^n n!}{2^n (n+2)}$, and $f(5) = \dfrac{1}{2}$.

(a) Write the third-degree Taylor polynomial for f about $x = 5$.

(b) Find the radius of convergence of the Taylor series for f about $x = 5$.

(c) Show that the sixth-degree Taylor polynomial for f about $x = 5$ approximates $f(6)$ with error less than $\dfrac{1}{1000}$.

The Maclaurin series for $\ln\left(\dfrac{1}{1-x}\right)$ is $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n}$ with interval of convergence $-1 \le x < 1$.

(a) Find the Maclaurin series for $\ln\left(\dfrac{1}{1+3x}\right)$ and determine the interval of convergence.

(b) Find the value of $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$.

(c) Give a value of p such that $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ converges, but $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ diverges. Give reasons why your value of p is correct.

(d) Give a value of p such that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges, but $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ converges. Give reasons why your value of p is correct.

A function f is defined by

$$f(x) = \frac{1}{3} + \frac{2}{3^2}x + \frac{3}{3^3}x^2 + \cdots + \frac{n+1}{3^{n+1}}x^n + \cdots$$

for all x in the interval of convergence of the given power series.

(a) Find the interval of convergence for this power series. Show the work that leads to your answer.

(b) Find $\displaystyle\lim_{x\to 0} \frac{f(x) - \frac{1}{3}}{x}$.

(c) Write the first three nonzero terms and the general term for an infinite series that represents $\int_0^1 f(x)\,dx$.

(d) Find the sum of the series determined in part (c).

The function f has derivatives of all orders for all real numbers x. Assume $f(2) = -3$, $f'(2) = 5$, $f''(2) = 3$, and $f'''(2) = -8$.

(a) Write the third-degree Taylor polynomial for f about $x = 2$ and use it to approximate $f(1.5)$.

(b) The fourth derivative of f satisfies the inequality $\left|f^{(4)}(x)\right| \le 3$ for all x in the closed interval $[1.5, 2]$. Use the Lagrange error bound on the approximation to $f(1.5)$ found in part (a) to explain why $f(1.5) \ne -5$.

5 (c) Write the fourth-degree Taylor polynomial, $P(x)$, for $g(x) = f(x^2 + 2)$ about $x = 0$. Use P to explain why g must have a relative minimum at $x = 0$.