Practice 3.4

Locate local extrema. Find intervals where $f(x)$ is concave up or down and find x-coordinates of points of inflection

1. $f(x) = 3x^4 - 4x^3 + 6$ 2. $f(x) = 3x^5 - 5x^3$ 3. $f(x) = \sqrt[5]{x} - 1$

4. $f(x) = x^{2/3}(1-x)$ 5. $f(x) = 8x^{1/3} + x^{4/3}$ 6. $f(x) = x\sqrt{4-x}$

7. $f(x) = \sin 2x \quad [0, 2\pi]$ 8. $f(x) = \sin x - \cos x \quad [0, 2\pi]$

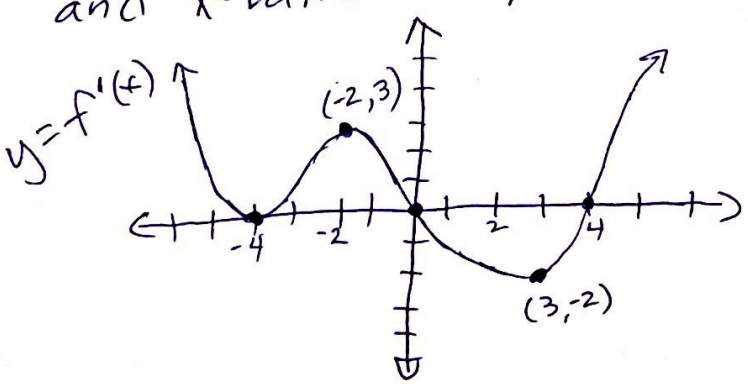
Sketch a graph of a continuous function that satisfies the following:

9. $f(0)=1$; $f(2)=3$; $f'(0) = f'(2) = 0$; $f'(x) < 0$ on $(-\infty, 0) \cup (2, \infty)$, $f'(x) > 0$ on $(0,2)$; $f''(x) > 0$ $(-\infty, 1)$; $f''(x) < 0$ on $(1, \infty)$.

10. $f(0)=4$; $f(2)=2$; $f(5)=6$; $f'(0) = f'(2) = 0$; $f'(x) > 0$ $(-\infty, 0) \cup (2, \infty)$; $f'(x) < 0$ $(0,2)$; $f''(x) < 0$ on $(-\infty, 1) \cup (3, 5)$; $f''(x) > 0$ on $(1,3) \cup (5, \infty)$.

11. $f(1)=4$; $f'(x) > 0$ if $x < 1$; $f'(x) < 0$ if $x > 1$; $f''(x) > 0$ for every $x \neq 1$.

Given graph of $f'(x)$, determine where $f(x)$ is increasing, decreasing, x values of local extrema, concave up, c. down, and x-values of poi.



f ↑
↓
x max
x min
c up
c down
x poi

1. CN: 0, 1 Lmin @ (1, 5) No max
cu $(-\infty, 0) \cup (\frac{2}{3}, \infty)$ cd $(0, \frac{2}{3})$ poi @ $x = 0, \frac{2}{3}$

2. CN: $0, \pm 1$ Lmin @ $(1, -2)$ Lmax @ $(-1, 2)$
CD $(-\infty, -\sqrt{1/2}) \cup (0, \sqrt{1/2})$ cup $(-\sqrt{1/2}, 0) \cup (\sqrt{1/2}, \infty)$ poi: $x = 0; \pm\sqrt{\frac{1}{2}}$

3. CN: 0 No extrema cup $(-\infty, 0)$ cdown $(0, \infty)$ poi $x = 0$

4. CN: $\frac{2}{5}, 0$ Lmax @ $x = \frac{2}{5}$ Lmin $(0, 0)$
cup $(-\infty, -\frac{1}{5})$ cd $(-\frac{1}{5}, 0) \cup (0, \infty)$ poi $x = -\frac{1}{5}$

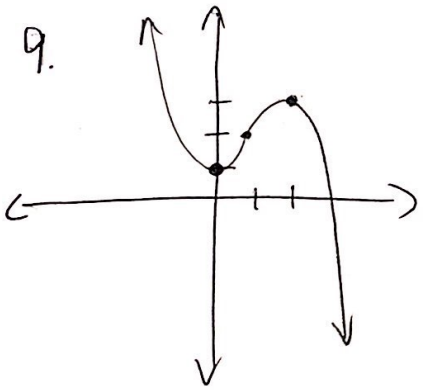
5. CN: $x = -2, 0$ Lmin $(-2, -6\sqrt[3]{2})$ cup $(-\infty, 0) \cup (4, \infty)$ CD $(0, 4)$ poi $x = 0, 4$

6. CN: $x = \pm\sqrt{2}, \pm 2$ Lmin $(-\sqrt{2}, -2)$ Lmax $(\sqrt{2}, 2)$
cup $(-2, 0)$ cd $(0, 2)$ poi @ $x = 0$

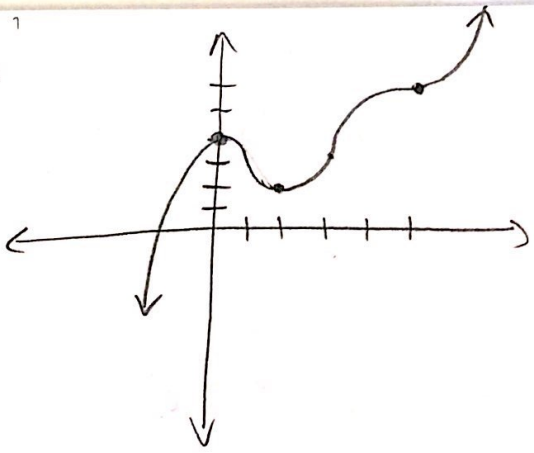
7. CN: $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ Lmax $(\frac{\pi}{4}, 1)$ $(\frac{5\pi}{4}, 1)$ Lmin: $(\frac{3\pi}{4}, -1)$ $(\frac{7\pi}{4}, -1)$
CDown: $(0, \pi/2) \cup (\pi, \frac{3\pi}{2})$ cup $(\frac{\pi}{2}, \pi) \cup (\frac{3\pi}{2}, 2\pi)$ poi $x = \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

8. CN: $\frac{3\pi}{4}, \frac{7\pi}{4}$ Lmax $(\frac{3\pi}{4}, \sqrt{2})$ Lmin $(\frac{7\pi}{4}, -\sqrt{2})$
cup $(0, \frac{\pi}{4}) \cup (\frac{5\pi}{4}, 2\pi)$ cdown $(\frac{\pi}{4}, \frac{5\pi}{4})$ poi $x = \frac{\pi}{4}, \frac{5\pi}{4}$

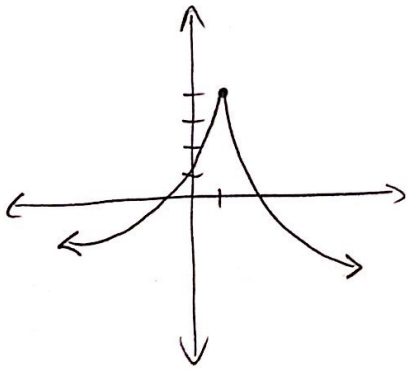
9.



10.



11.



12. ↑ $(-\infty, -4) \cup (-4, 0) \cup (4, \infty)$ b/c $f' > 0$
↓ $(0, 4)$ b/c $f' < 0$
Xmax: $x = 0$ b/c f' ch + to −
min: $x = 4$ b/c f' ch − to +
cup: $(-4, -2) \cup (3, \infty)$ b/c f' ↑
cd: $(-\infty, -4) \cup (-2, 3)$ b/c f' ↓
X poi: $x = -4, -2, 3$ b/c f' has extrema