[26] $G(t) = 1.1e^{-3.2e^{-1.1t}}$. The y-intercept is $G(0) = 1.1e^{-3.2} \approx 0.045$.

The endpoint value is $G(5) \approx 1.09$. The horizontal asymptote is $\lim_{t\to\infty} G(t) = 1.1$.

The PI occurs at $x = \frac{\ln 3.2}{1.1} \approx 1.057$. $G' > 0 \Rightarrow G$ is $\uparrow$. See *Figure 26.*

7.7 Review Exercises

[1] $y = 10 - 15x \Rightarrow 15x = 10 - y \Rightarrow x = \frac{10 - y}{15} \Rightarrow f^{-1}(x) = \frac{10 - x}{15}$

[2] $y = 9 - 2x^2,\ x \le 0 \Rightarrow 2x^2 = 9 - y \Rightarrow x^2 = \frac{9 - y}{2} \Rightarrow x = \pm\frac{1}{2}\sqrt{18 - 2y}$

{Choose the "$-$" since $x \le 0$} $\Rightarrow f^{-1}(x) = -\frac{1}{2}\sqrt{18 - 2x}$

[3] $f'(x) = 6x^2 - 8 < 0$ for $-1 \le x \le 1 \Rightarrow f$ is $\downarrow \Rightarrow f^{-1}$ exists.

Since $f(0) = 5$, $f^{-1}(5) = 0$. $D_x f^{-1}(5) = \frac{1}{f'(f^{-1}(5))} = \frac{1}{f'(0)} = \frac{1}{-8} = -\frac{1}{8}$.

[4] $f'(x) = 3e^{3x} + 2e^x > 0$ for $x \ge 0 \Rightarrow f$ is $\uparrow \Rightarrow f^{-1}$ exists.

Since $f(0) = -2$, $f^{-1}(-2) = 0$. $D_x f^{-1}(-2) = \frac{1}{f'(f^{-1}(-2))} = \frac{1}{f'(0)} = \frac{1}{5}$.

[5] $f(x) = \ln|4 - 5x^3|^5 = 5\ln|4 - 5x^3| \Rightarrow f'(x) = 5\cdot\frac{1}{4 - 5x^3}\cdot(-15x^2) = \frac{75x^2}{5x^3 - 4}$

[6] $f(x) = \ln|x^2 - 7|^3 = 3\ln|x^2 - 7| \Rightarrow f'(x) = 3\cdot\frac{1}{x^2 - 7}\cdot 2x = \frac{6x}{x^2 - 7}$

[7] $f(x) = (1 - 2x)\ln|1 - 2x| \Rightarrow$

$$f'(x) = \frac{(1 - 2x)(-2)}{1 - 2x} + (-2)\ln|1 - 2x| = -2(1 + \ln|1 - 2x|)$$

[8] $f(x) = \log\left|\frac{2 - 9x}{1 - x^2}\right| = \log|2 - 9x| - \log|1 - x^2| \Rightarrow$

$$f'(x) = D_x\left(\frac{\ln|2 - 9x|}{\ln 10} - \frac{\ln|1 - x^2|}{\ln 10}\right) = \left(\frac{9}{9x - 2} + \frac{2x}{1 - x^2}\right)\frac{1}{\ln 10}$$

[9] $f(x) = \ln\frac{(3x + 2)^4\sqrt{6x - 5}}{8x - 7} = 4\ln(3x + 2) + \frac{1}{2}\ln(6x - 5) - \ln(8x - 7) \Rightarrow$

$$f'(x) = \frac{12}{3x + 2} + \frac{3}{6x - 5} - \frac{8}{8x - 7}$$

[10] $f(x) = \ln\sqrt[4]{\frac{x}{3x + 5}} = \frac{1}{4}\left[\ln x - \ln(3x + 5)\right] \Rightarrow$

$$f'(x) = \frac{1}{4}\left(\frac{1}{x} - \frac{3}{3x + 5}\right) = \frac{5}{4x(3x + 5)}$$

[11] $f(x) = \frac{1}{\ln(2x^2 + 3)} \Rightarrow f'(x) = -\frac{\left[1/(2x^2 + 3)\right]\cdot 4x}{\left[\ln(2x^2 + 3)\right]^2} = \frac{-4x}{(2x^2 + 3)\left[\ln(2x^2 + 3)\right]^2}$

[12] $f(x) = \frac{\ln x}{e^{2x} + 1} \Rightarrow f'(x) = \frac{(e^{2x} + 1)(1/x) - (\ln x)(2e^{2x})}{(e^{2x} + 1)^2}$

[13] $f(x) = \frac{x}{\ln x} \Rightarrow f'(x) = \frac{(\ln x)\cdot 1 - x\cdot(1/x)}{(\ln x)^2} = \frac{\ln x - 1}{(\ln x)^2}$

[14] $f(x) = \frac{\ln x}{x} \Rightarrow f'(x) = \frac{x\cdot(1/x) - (\ln x)\cdot 1}{x^2} = \frac{1 - \ln x}{x^2}$

[15] $f(x) = e^{\ln(x^2+1)} = x^2 + 1 \Rightarrow f'(x) = 2x$

16 $f(x) = \ln e^{\sqrt{x}} = \sqrt{x} \Rightarrow f'(x) = \dfrac{1}{2\sqrt{x}}$

17 $f(x) = \ln(e^{4x} + 9) \Rightarrow f'(x) = \dfrac{1}{e^{4x} + 9} \cdot 4e^{4x} = \dfrac{4e^{4x}}{e^{4x} + 9}$

18 $f(x) = 4^{\sqrt{2x+3}} \Rightarrow f'(x) = (4^{\sqrt{2x+3}} \ln 4) \cdot \frac{1}{2}(2x + 3)^{-1/2}(2) = \dfrac{4^{\sqrt{2x+3}} \ln 4}{\sqrt{2x + 3}}$

19 $f(x) = 10^x \log x \Rightarrow f'(x) = \dfrac{10^x}{x \ln 10} + 10^x (\ln 10) \log x$

20 $f(x) = 5^{3x} + (3x)^5 \Rightarrow f'(x) = 5^{3x}(\ln 5)(3) + 5(3x)^4(3) = (3 \ln 5)\, 5^{3x} + 15\,(3x)^4$

21 $f(x) = \sqrt{\ln \sqrt{x}} = (\ln \sqrt{x})^{1/2} \Rightarrow f'(x) = \frac{1}{2}(\ln \sqrt{x})^{-1/2}\left(\dfrac{1}{\sqrt{x}} \cdot \dfrac{1}{2\sqrt{x}}\right) = \dfrac{1}{4x\sqrt{\ln \sqrt{x}}}$

22 $f(x) = (1 + \sqrt{x})^e \Rightarrow f'(x) = e(1 + \sqrt{x})^{e-1} \cdot \dfrac{1}{2\sqrt{x}} = \dfrac{e(1 + \sqrt{x})^{e-1}}{2\sqrt{x}}$

23 $f(x) = x^2 e^{-x^2} \Rightarrow f'(x) = x^2 e^{-x^2}(-2x) + e^{-x^2}(2x) = 2xe^{-x^2}(1 - x^2)$

24 $f(x) = \dfrac{2^{-3x}}{x^3 + 4} \Rightarrow$

$$f'(x) = \frac{(x^3 + 4)(2^{-3x})(\ln 2)(-3) - 2^{-3x}(3x^2)}{(x^3 + 4)^2} = \frac{(-3)\, 2^{-3x}\left[(x^3 + 4) \ln 2 + x^2\right]}{(x^3 + 4)^2}$$

25 $f(x) = \sqrt{e^{3x} + e^{-3x}} \Rightarrow$

$$f'(x) = \tfrac{1}{2}(e^{3x} + e^{-3x})^{-1/2}(3e^{3x} - 3e^{-3x}) = \frac{3(e^{3x} - e^{-3x})}{2\sqrt{e^{3x} + e^{-3x}}}$$

26 $f(x) = (x^2 + 1)^{2x} = e^{2x \ln(x^2+1)} \Rightarrow f'(x) =$

$$e^{2x \ln(x^2+1)}\left[2x \cdot \frac{2x}{x^2 + 1} + 2\ln(x^2 + 1)\right] = (x^2 + 1)^{2x}\left[\frac{4x^2}{x^2 + 1} + 2\ln(x^2 + 1)\right]$$

27 $f(x) = 10^{\ln x} \Rightarrow f'(x) = (10^{\ln x} \ln 10) \cdot \dfrac{1}{x} = \dfrac{10^{\ln x} \ln 10}{x}$

28 $f(x) = 7^{\ln|x|} \Rightarrow f'(x) = (7^{\ln|x|} \ln 7) \cdot \dfrac{1}{x} = \dfrac{7^{\ln|x|} \ln 7}{x}$

29 $y = x^{\ln x} \Rightarrow \ln y = \ln x\,(\ln x) = (\ln x)^2 \Rightarrow \dfrac{y'}{y} = 2(\ln x)^1 \cdot \dfrac{1}{x} \Rightarrow y' = \dfrac{2 \ln x\,(x^{\ln x})}{x}$

30 $y = (\ln x)^{\ln x} \Rightarrow \ln y = (\ln x)\, \ln(\ln x) \Rightarrow$

$$\frac{y'}{y} = \ln x \cdot \frac{1}{x \ln x} + \frac{1}{x} \cdot \ln(\ln x) = \frac{\ln(\ln x) + 1}{x} \Rightarrow y' = (\ln x)^{\ln x}\left[\frac{\ln(\ln x) + 1}{x}\right]$$

31 $f(x) = \ln|\tan x - \sec x| \Rightarrow$

$$f'(x) = \frac{\sec^2 x - \sec x \tan x}{\tan x - \sec x} = \frac{\sec x(\sec x - \tan x)}{\tan x - \sec x} = -\sec x$$

32 $f(x) = \ln \csc \sqrt{x} \Rightarrow f'(x) = \dfrac{1}{\csc \sqrt{x}} \cdot (-\csc \sqrt{x} \cot \sqrt{x}) \cdot \dfrac{1}{2\sqrt{x}} = -\dfrac{\cot \sqrt{x}}{2\sqrt{x}}$

33 $f(x) = \csc e^{-2x} \cot e^{-2x} \Rightarrow$

$$f'(x) = \csc e^{-2x}\left[(-\csc^2 e^{-2x})(-2e^{-2x})\right] + \cot e^{-2x}\left[(-\csc e^{-2x} \cot e^{-2x})(-2e^{-2x})\right]$$
$$= 2e^{-2x} \csc e^{-2x}(\csc^2 e^{-2x} + \cot^2 e^{-2x})$$

34 $f(x) = x^2 e^{\tan 2x} \Rightarrow$

$$f'(x) = x^2 e^{\tan 2x}(\sec^2 2x)(2) + 2x\, e^{\tan 2x} = 2xe^{\tan 2x}(x \sec^2 2x + 1)$$

35 $f(x) = \ln\cos^4 4x = 4\ln\cos 4x \Rightarrow f'(x) = 4 \cdot \frac{1}{\cos 4x} \cdot (-4\sin 4x) = -16\tan 4x$

36 $f(x) = 3^{\sin 3x} \Rightarrow f'(x) = (3^{\sin 3x}\ln 3)(3\cos 3x) = (3\ln 3\ \cos 3x)\, 3^{\sin 3x}$

37 $f(x) = (\sin x)^{\cos x} = e^{\cos x \ln\sin x} \Rightarrow f'(x) =$

$$e^{\cos x\ln\sin x}\left[\cos x \cdot \frac{\cos x}{\sin x} + (-\sin x)\ln\sin x\right] = (\sin x)^{\cos x}(\cos x\cot x - \sin x\ln\sin x)$$

38 $f(x) = \frac{1}{\sin^2 e^{-3x}} \Rightarrow f'(x) = -\frac{2\sin e^{-3x}\ \cos e^{-3x}\,(-3e^{-3x})}{(\sin^2 e^{-3x})^2} = \frac{6e^{-3x}\ \cos e^{-3x}}{\sin^3 e^{-3x}}$

39 $1 + xy = e^{xy} \Rightarrow xy' + y = e^{xy}(xy' + y) \Rightarrow$

$$y'(x - xe^{xy}) = ye^{xy} - y \Rightarrow y' = \frac{y(e^{xy} - 1)}{x(1 - e^{xy})} = -\frac{y}{x}.$$

40 $\ln(x + y) + x^2 - 2y^3 = 1 \Rightarrow \frac{1}{x + y}\cdot(1 + y') + 2x - 6y^2y' = 0 \Rightarrow$

$1 + y' + 2x(x + y) - 6y^2y'(x + y) = 0 \Rightarrow$

$$y'\left[1 - 6y^2(x + y)\right] = -1 - 2x(x + y) \Rightarrow y' = \frac{2x(x + y) + 1}{6y^2(x + y) - 1}$$

41 $y = (x + 2)^{4/3}(x - 3)^{3/2} \Rightarrow \ln y = \frac{4}{3}\ln(x + 2) + \frac{3}{2}\ln(x - 3) \Rightarrow$

$$\frac{y'}{y} = \frac{4}{3(x + 2)} + \frac{3}{2(x - 3)} \Rightarrow y' = \left[\frac{4}{3(x + 2)} + \frac{3}{2(x - 3)}\right](x + 2)^{4/3}(x - 3)^{3/2}$$

42 $y = \sqrt[3]{(3x - 1)\sqrt{2x + 5}} \Rightarrow \ln y = \frac{1}{3}\ln(3x - 1) + \frac{1}{6}\ln(2x + 5) \Rightarrow$

$$\frac{y'}{y} = \frac{3}{3(3x - 1)} + \frac{2}{6(2x + 5)} \Rightarrow y' = \left[\frac{1}{3x - 1} + \frac{1}{3(2x + 5)}\right]\sqrt[3]{(3x - 1)\sqrt{2x + 5}}$$

43 (a) $u = \sqrt{x},\ 2\,du = \frac{1}{\sqrt{x}}\,dx \Rightarrow \int \frac{1}{\sqrt{x}\,e^{\sqrt{x}}}\,dx = 2\int e^{-u}\,du = -2e^{-u} + C$

(b) $\int_1^4 \frac{1}{\sqrt{x}\,e^{\sqrt{x}}}\,dx = -2\left[e^{-\sqrt{x}}\right]_1^4 = -2(e^{-2} - e^{-1}) \approx 0.465$

44 (a) $u = -3x + 2,\ -\frac{1}{3}\,du = dx \Rightarrow \int e^{-3x+2}\,dx = -\frac{1}{3}\int e^u\,du = -\frac{1}{3}e^u + C$

(b) $\int_0^1 e^{-3x+2}\,dx = -\frac{1}{3}\left[e^{-3x+2}\right]_0^1 = -\frac{1}{3}(e^{-1} - e^2) \approx 2.340$

45 (a) $u = -x^2,\ -\frac{1}{2}\,du = x\,dx \Rightarrow \int x\,4^{-x^2}\,dx = -\frac{1}{2}\int 4^u\,du = -\frac{4^u}{2\ln 4} + C$

(b) $\int_0^1 x\,4^{-x^2}\,dx = -\frac{1}{2\ln 4}\left[4^{-x^2}\right]_0^1 = -\frac{1}{2\ln 4}\left(\frac{1}{4} - 1\right) = \frac{3}{8\ln 4} \approx 0.271$

46 (a) $u = x^3 + 3x,\ \frac{1}{3}\,du = (x^2 + 1)\,dx \Rightarrow \int \frac{x^2 + 1}{x^3 + 3x}\,dx = \frac{1}{3}\int \frac{1}{u}\,du = \frac{1}{3}\ln|u| + C$

(b) $\int_1^2 \frac{x^2 + 1}{x^3 + 3x}\,dx = \frac{1}{3}\left[\ln\left|x^3 + 3x\right|\right]_1^2 = \frac{1}{3}(\ln 14 - \ln 4) = \frac{1}{3}\ln\frac{7}{2} \approx 0.418$

47 $u = x^2,\ \frac{1}{2}\,du = x\,dx \Rightarrow \int x\tan x^2\,dx = \frac{1}{2}\int \tan u\,du = -\frac{1}{2}\ln|\cos u| + C$

48 $u = x + \frac{\pi}{6},\ du = dx \Rightarrow \int \cot\left(x + \frac{\pi}{6}\right)dx = \int \cot u\,du = \ln|\sin u| + C$

49 $\int x^e\,dx = \frac{x^{e+1}}{e + 1} + C$

[50] $u = 7 - 5x,\ -\frac{1}{5}\,du = dx \Rightarrow \int \frac{1}{7 - 5x}\,dx = -\frac{1}{5}\int \frac{1}{u}\,du = -\frac{1}{5}\ln|u| + C$

[51] $u = 1 - \ln x,\ -du = \frac{dx}{x} \Rightarrow$

$$\int \frac{1}{x - x\ln x}\,dx = \int \frac{1}{x(1 - \ln x)}\,dx = -\int \frac{1}{u}\,du = -\ln|u| + C$$

[52] $u = \ln x,\ du = \frac{dx}{x} \Rightarrow \int \frac{1}{x\ln x}\,dx = \int \frac{1}{u}\,du = \ln|u| + C$

[53] $\int \frac{(1 + e^x)^2}{e^{2x}}\,dx = \int \frac{1 + 2e^x + e^{2x}}{e^{2x}}\,dx = \int (e^{-2x} + 2e^{-x} + 1)\,dx =$

$$-\tfrac{1}{2}e^{-2x} - 2e^{-x} + x + C$$

[54] $\int \frac{(e^{2x} + e^{3x})^2}{e^{5x}}\,dx = \int \frac{e^{4x} + 2e^{5x} + e^{6x}}{e^{5x}}\,dx = \int (e^{-x} + 2 + e^x)\,dx =$

$$-e^{-x} + 2x + e^x + C$$

[55] Using long division,

$$\int \frac{x^2}{x + 2}\,dx = \int \left(x - 2 + \frac{4}{x + 2}\right)dx = \tfrac{1}{2}x^2 - 2x + 4\ln|x + 2| + C$$

[56] Using long division,

$$\int \frac{x^2 + 1}{x + 1}\,dx = \int \left(x - 1 + \frac{2}{x + 1}\right)dx = \tfrac{1}{2}x^2 - x + 2\ln|x + 1| + C$$

[57] $u = 4/x^2,\ -\frac{1}{8}\,du = (1/x^3)\,dx \Rightarrow \int \frac{e^{4/x^2}}{x^3}\,dx = -\frac{1}{8}\int e^u\,du = -\frac{1}{8}e^u + C$

[58] $u = 1/x,\ -du = (1/x^2)\,dx \Rightarrow \int \frac{e^{1/x}}{x^2}\,dx = -\int e^u\,du = -e^u + C$

[59] $u = x^2 + 1,\ \frac{1}{2}\,du = x\,dx \Rightarrow$

$$\int \frac{x}{x^4 + 2x^2 + 1}\,dx = \int \frac{x}{(x^2 + 1)^2}\,dx = \tfrac{1}{2}\int \frac{1}{u^2}\,du = -\frac{1}{2u} + C$$

[60] $u = x^4 + 1,\ \frac{1}{4}\,du = x^3\,dx \Rightarrow \int \frac{5x^3}{x^4 + 1}\,dx = 5\cdot\frac{1}{4}\int \frac{1}{u}\,du = \frac{5}{4}\ln u + C$

[61] $u = 1 + e^x,\ du = e^x\,dx \Rightarrow \int \frac{e^x}{1 + e^x}\,dx = \int \frac{1}{u}\,du = \ln u + C$

[62] $\int (1 + e^{-3x})^2\,dx = \int (1 + 2e^{-3x} + e^{-6x})\,dx = x - \frac{2}{3}e^{-3x} - \frac{1}{6}e^{-6x} + C$

[63] $\int 5^x e^x\,dx = \int (5e)^x\,dx$ { treat $5e$ as a constant } $= \frac{(5e)^x}{\ln(5e)} + C$

[64] $u = x^2,\ \frac{1}{2}\,du = x\,dx \Rightarrow \int x\,10^{(x^2)}\,dx = \frac{1}{2}\int 10^u\,du = \frac{10^u}{2\ln 10} + C$

[65] $u = \log x,\ du = \frac{1}{x\ln 10}\,dx \Rightarrow \int \frac{1}{x\sqrt{\log x}}\,dx = \ln 10\int u^{-1/2}\,du = 2\ln 10\sqrt{u} + C$

[66] $u = 1 + 7^x,\ du = (7^x\ln 7)\,dx \Rightarrow \int 7^x\sqrt{1 + 7^x}\,dx = \frac{1}{\ln 7}\int u^{1/2}du = \frac{2u^{3/2}}{3\ln 7} + C$

[67] $u = e^{-x},\ -du = e^{-x}\,dx \Rightarrow \int e^{-x}\sin e^{-x}\,dx = -\int \sin u\,du = \cos u + C$

[68] $u = \sec x,\ du = \sec x\tan x\,dx \Rightarrow \int \tan x\,e^{\sec x}\sec x\,dx = \int e^u\,du = e^u + C$

[69] $u = 1 + \cot x,\ -du = \csc^2 x\,dx \Rightarrow \int \frac{\csc^2 x}{1 + \cot x}\,dx = -\int \frac{1}{u}\,du = -\ln|u| + C$

[70] $u = \sin x - \cos x,\ du = (\cos x + \sin x)\,dx \Rightarrow$

$$\int \frac{\cos x + \sin x}{\sin x - \cos x}\,dx = \int \frac{1}{u}\,du = \ln|u| + C$$

[71] $u = 1 - 2\sin 2x,\ -\frac{1}{4}\,du = \cos 2x\,dx \Rightarrow$

$$\int \frac{\cos 2x}{1 - 2\sin 2x}\,dx = -\tfrac{1}{4}\int \frac{1}{u}\,du = -\tfrac{1}{4}\ln|u| + C$$

[72] $u = 3^x,\ du = (3^x \ln 3)\,dx \Rightarrow \int 3^x(3 + \sin 3^x)\,dx = \frac{1}{\ln 3}\int (3 + \sin u)\,du =$

$$\frac{1}{\ln 3}(3u - \cos u) + C = \frac{1}{\ln 3}(3^{x+1} - \cos 3^x) + C$$

[73] $u = e^x,\ du = e^x\,dx \Rightarrow \int e^x \tan e^x\,dx = \int \tan u\,du = -\ln|\cos u| + C$

[74] $u = 1/x,\ -du = (1/x^2)\,dx \Rightarrow \int \frac{\sec(1/x)}{x^2}\,dx = -\int \sec u\,du = -\ln|\sec u + \tan u| + C$

[75] $u = 3x,\ \frac{1}{3}\,du = dx \Rightarrow \int (\csc 3x + 1)^2\,dx = \frac{1}{3}\int (\csc^2 u + 2\csc u + 1)\,du =$

$$\tfrac{1}{3}(-\cot u + 2\ln|\csc u - \cot u| + u) + C$$

[76] $\int \cos 2x \csc 2x\,dx = \int \frac{\cos 2x}{\sin 2x}\,dx = \int \cot 2x\,dx = \frac{1}{2}\ln|\sin 2x| + C$

[77] $u = 9x,\ \frac{1}{9}\,du = dx \Rightarrow \int (\cot 9x + \csc 9x)\,dx = \frac{1}{9}\int (\cot u + \csc u)\,du =$

$$\tfrac{1}{9}\ln|\sin u| + \tfrac{1}{9}\ln|\csc u - \cot u| + C$$

[78] $\int \frac{\sin x + 1}{\cos x}\,dx = \int (\tan x + \sec x)\,dx = -\ln|\cos x| + \ln|\sec x + \tan x| + C$

[79] $y'' = -e^{-3x} \Rightarrow y' = \frac{1}{3}e^{-3x} + C.\ \ y' = 2$ if $x = 0 \Rightarrow 2 = \frac{1}{3} + C \Rightarrow C = \frac{5}{3}.$

$y' = \frac{1}{3}e^{-3x} + \frac{5}{3} \Rightarrow y = -\frac{1}{9}e^{-3x} + \frac{5}{3}x + C.$

$$y = -1 \text{ if } x = 0 \Rightarrow -\tfrac{1}{9} + C = -1 \Rightarrow C = -\tfrac{8}{9}.$$

[80] (a) $q'(t) = q(t)\,k\sin 2\pi t$. Since $q(t) > 0\ \forall t$ and $k > 0$, the sign of $q'(t)$ is determined by the sign of $\sin 2\pi t$. $\sin 2\pi t > 0$ if $0 < 2\pi t < \pi$, $2\pi < 2\pi t < 3\pi$, etc. Thus, $q'(t) > 0$ on $(n, n + \frac{1}{2})$ and $q'(t) < 0$ on $(n + \frac{1}{2}, n + 1)$, where n is a nonnegative integer. Hence, $q(t)$ increases during spring and summer ($t = 0$ to $t = \frac{1}{2}$) and decreases during fall and winter ($t = \frac{1}{2}$ to $t = 1$).

(b) $\frac{q'(t)}{q(t)} = k\sin 2\pi t \Rightarrow \ln q(t) = -\frac{k}{2\pi}\cos 2\pi t + c \Rightarrow q(t) = e^c e^{-\frac{k}{2\pi}\cos 2\pi t}.$

$q(0) = e^c e^{-\frac{k}{2\pi}} = q_0 \Rightarrow e^c = q_0 e^{\frac{k}{2\pi}} \Rightarrow$

$$q(t) = q_0 e^{\frac{k}{2\pi}} e^{-\frac{k}{2\pi}\cos 2\pi t} = q_0 e^{\frac{k}{2\pi}(1 - \cos 2\pi t)}.$$

[81] $a(t) = e^{t/2} \Rightarrow v(t) = 2e^{t/2} + C.\ \ v(0) = 6 \Rightarrow C = 4$ and $v(t) = 2e^{t/2} + 4.$

Since $v(t) > 0$, the distance traveled from $t = 0$ to $t = 4$ is

$$s(4) - s(0) = \int_0^4 v(t)\,dt = \left[4e^{t/2} + 4t\right]_0^4 = 4e^2 + 12 \approx 41.56 \text{ cm.}$$

82 $f(x) = x^2 \ln x \Rightarrow f'(x) = x(1 + 2\ln x)$.

$f'(x) = 0 \Rightarrow x = e^{-1/2} \approx 0.607$.

$f''(x) = 3 + 2\ln x$. $f''(e^{-1/2}) = 2 > 0 \Rightarrow$

$f(e^{-1/2}) = -1/(2e) \approx -0.184$ is a *LMIN*.

$f''(x) = 0 \Rightarrow x = e^{-3/2} \approx 0.223$.

$f''(x) < 0$ on $(0, e^{-3/2})$ and f is *CD*.

$f''(x) > 0$ on $(e^{-3/2}, \infty)$ and f is *CU*. *PI* at $x = e^{-3/2}$.

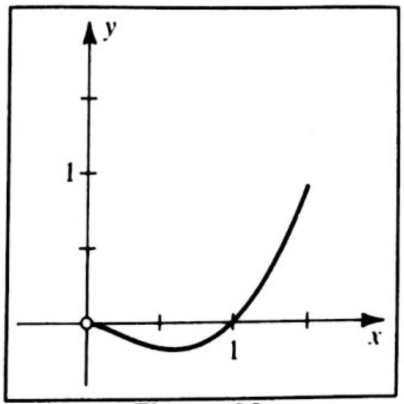

Figure 82

83 $y = xe^{1/x^3} + \ln|2 - x^2| \Rightarrow y' = xe^{1/x^3}(-3/x^4) + e^{1/x^3} - \dfrac{2x}{2 - x^2}$.

At $x = 1$, $y' = -3e + e - 2 = -2(1 + e)$. Tangent line at $P(1, e)$:

$y - e = -2(1 + e)(x - 1)$, or approximately, $y = -7.44x + 10.15$.

84 $e^{2x} > \dfrac{x}{x^2 + 1}$ on $[0, 1] \Rightarrow A = \displaystyle\int_0^1 \left(e^{2x} - \frac{x}{x^2 + 1}\right) dx = \left[\tfrac{1}{2}e^{2x} - \tfrac{1}{2}\ln(x^2 + 1)\right]_0^1 =$

$\tfrac{1}{2}(e^2 - \ln 2 - 1) \approx 2.848$.

85 Using disks, $V = \pi \displaystyle\int_{-3}^{-2} (e^{4x})^2\, dx = \pi\left[\tfrac{1}{8}e^{8x}\right]_{-3}^{-2} = \tfrac{\pi}{8}(e^{-16} - e^{-24}) \approx 4.42 \times 10^{-8}$.

86 From (7.33), $N(t) = 651e^{0.02t}$.

$t = 20$ corresponds to the year 2000. $N(20) = 651e^{0.4} \approx 971.2$ million.

$N'(t) = 651(0.02)e^{0.02t}$. $N'(20) = 13.02e^{0.4} \approx 19.4$ million/yr.

87 $q(t) = Ae^{ct}$. A half-life of 5 days $\Rightarrow e^{c(5)} = \tfrac{1}{2} \Rightarrow e^c = (\tfrac{1}{2})^{1/5}$.

Thus, $q(t) = A(\tfrac{1}{2})^{t/5}$. $q(t) = 1\%\, A \Rightarrow 0.01A = A(\tfrac{1}{2})^{t/5} \Rightarrow (\tfrac{1}{2})^{t/5} = \tfrac{1}{100} \Rightarrow$

$t = \dfrac{5\ln(1/100)}{\ln(1/2)} \approx 33.2$ days.

88 (a) $T = -8310 \ln x$. $T(0.04) = -8310 \ln(0.04) \approx 26{,}749$ yr, or about 27,000 yr.

(b) $dT = -8310 \cdot \frac{1}{x}\, dx = -8310 \cdot \frac{1}{0.04}(\pm 0.005) = \pm 1038.75 \approx \pm 1040$ yr.

89 The rate at which sugar *does not dissolve* is also directly proportional to the amount that does not dissolve. Thus, $q(t) = q_0 e^{ct}$, where q represents the amount of sugar that remains *undissolved*. $q(0) = 10 \Rightarrow q_0 = 10$.

$q(3) = \tfrac{1}{2}q_0 \Rightarrow e^{c(3)} = \tfrac{1}{2} \Rightarrow e^c = (\tfrac{1}{2})^{1/3} \Rightarrow q(t) = 10(\tfrac{1}{2})^{t/3}$.

(a) If 2 more pounds dissolve, 3 pounds will remain.

$q(t) = 3 \Rightarrow (\tfrac{1}{2})^{t/3} = \tfrac{3}{10} \Rightarrow t = \dfrac{3\ln(3/10)}{\ln(1/2)} \approx 5.2$ or 2.2 additional hr.

(b) $t = 7$ for 8:00 P.M. $10 - q(7) = 10 - 10(\tfrac{1}{2})^{7/3} = 10\left[1 - (\tfrac{1}{2})^{7/3}\right] \approx 8.016$ lb.