Ratio Test $\Rightarrow$ use with factorials + n^{th} powers
does not work with poly / poly $\Rightarrow$ LC

Let $\sum a_n$ be a series with non zero terms

1. $\sum a_n$ converges absolutely if $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$
2. $\sum a_n$ diverges if $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| > 1$ OR ∞
3. The test is inconclusive (you know nothing!) if $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$.

$a_{n+1} \Rightarrow$ substitute $n+1$ for all n's in a_n formula.

Ex. $$\sum_{n=1}^{\infty}\frac{3^n}{n!} = a_n \qquad a_{n+1} = \frac{3^{n+1}}{(n+1)!}$$

$$\lim_{n\to\infty}\left|\frac{\overset{3^n\cdot 3}{\cancel{3^{n+1}}}}{\underset{(n+1)\cancel{n!}}{\cancel{(n+1)!}}}\cdot\frac{\cancel{n!}}{\cancel{3^n}}\right| = \lim_{n\to\infty}\left|\frac{3}{n+1}\right| = 0 < 1$$

$\therefore$ By ratio test $\sum_{n=1}^{\infty} a_n$ converges

Ex. $$\sum_{n=1}^{\infty}\frac{(-n)^n}{n!} = \sum_{n=1}^{\infty}\frac{(-1)^n n^n}{n!} \qquad a_{n+1} = \frac{(-1)^{n+1}(n+1)^{n+1}}{(n+1)!}$$

$$\lim_{n\to\infty}\left|\frac{(n+1)^{n+1}}{(n+1)!}\cdot\frac{n!}{n^n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)^n\cancel{(n+1)}}{\cancel{(n+1)}\cancel{n!}}\cdot\frac{\cancel{n!}}{n^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(n+1)^n}{n^n}\right| = \lim_{n\to\infty}\left|\left(\frac{n+1}{n}\right)^n\right| = \lim_{n\to\infty}\left|\left(1+\frac{1}{n}\right)^n\right| = e > 1$$

By ratio test $\sum a_n$ diverges.

Root Test * only use if only have n^{th} powers

① If $\lim\limits_{n\to\infty} \sqrt[n]{|a_n|} < 1$ then $\sum a_n$ conv. absolutely

② If $\lim\limits_{n\to\infty} \sqrt[n]{|a_n|} > 1$ then $\sum a_n$ diverges

③ If $\lim\limits_{n\to\infty} \sqrt[n]{|a_n|} = 1$ test inconclusive
$\Rightarrow$ Use a diff test!

Ex. $\displaystyle\sum_{n=1}^{\infty} \frac{2^{3n+1}}{n^n} = a_n$

$$\lim_{n\to\infty} \sqrt[n]{\frac{2^{3n+1}}{n^n}} = \lim_{n\to\infty}\left(\frac{2^{3n+1}}{n^n}\right)^{\frac{1}{n}} = \lim_{n\to\infty}\left(\frac{2^{3+\frac{1}{n}}}{n^1}\right) = 0 < 1$$

$\therefore$ By root test $\displaystyle\sum_{n=1}^{\infty} a_n$ converges (abs).

p 603 # 13-27 odd, 35-41 odd, 61-65