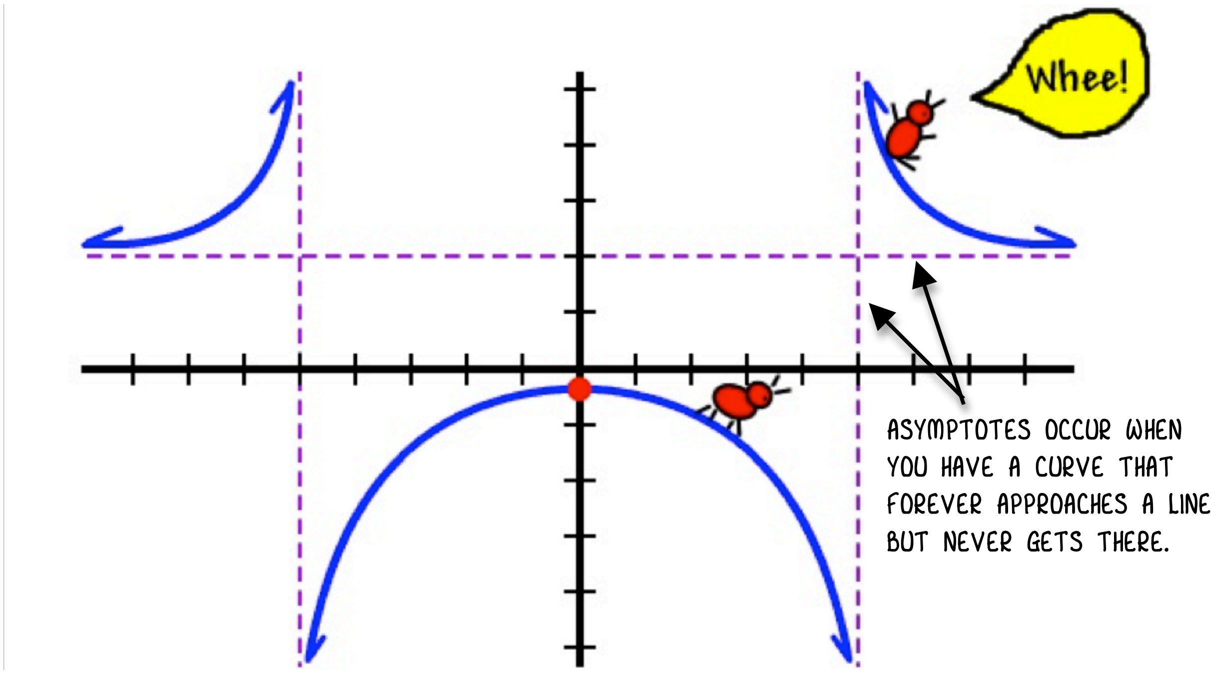
Common Core Math 3

Unit 4 – Rationals & Radicals



"This is the nature of mastery: Mastery is an asymptote. You can approach it. You can home in on it. You can get really, really close to it... The mastery asymptote is a source of frustration. Why reach for something you can never fully attain? But it's also a source of allure. Why not reach for it? The joy is in the pursuit more than the realization. In the end, mastery attracts precisely because mastery eludes."

- Dan Pink from the book DRIVE.

Name: ____

WAKE COUNTY
PUBLIC SCHOOL SYSTEM

APEX HIGH SCHOOL
1501 LAURA DUNCAN ROAD
APEX, NC 27502

Common Core Math 3

Unit 4 Rationals & Radicals

Day	Date	Homework
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Common Core Math 3 – Unit #4 Rationals and Radicals

Topics in this unit:

- **Rationals**
  - Perform operations with rational expressions.
  - Solve rational equations.
  - Graph transformations of 1/x
  - Graph rational functions (honors)
  - Modeling with rational equations
- **Radicals**
  - Add and subtract radical expressions
  - Perform operations with radical expressions.
  - Rationalize the denominator of a radical expression.
  - Graph transformations of $\sqrt{x}$
  - Graph transformation of $\sqrt[3]{x}$ (honors)
  - Simplify expressions with rational exponents.
  - Solve radical equations.

Students will be able to . . .

- Rewrite rational expressions in different forms.
- Understand the analogy between rational numbers and rational expressions - closed under addition, subtraction, multiplication, and division by a non-zero.
- Add, subtract, multiply, and divide rational expressions.
- Describe relationships in applications that can be modeled by rational equations.
- Solve equations involving rational functions using technology.
- Find intersection points of the graphs of two rational functions as solutions to an equation.
- Solve rational equations.
- Graph transformations of 1/x
- Graph rational functions. Find all asymptotes and holes. (honors)
- Add, subtract, multiply, and divide radical expressions.
- Write radical expressions in simplest radical form.
- Write radical expressions in simplest exponential form.
- Write expressions with rational exponents in simplest radical form.
- Write expressions with rational exponents in simplest exponential form.
- Graph transformations of $\sqrt{x}$
- Graph transformation of $\sqrt[3]{x}$ (honors)
- Solve radical equations

VOCABULARY

- A **rational expression** is the quotient of 2 polynomials.

$$\frac{P(x)}{Q(x)} \qquad where \quad Q(x) \neq 0$$

- A **rational function** is a function which is the quotient of 2 polynomials. It has the form:

$$f(x) = \frac{P(x)}{Q(x)} \qquad where \quad Q(x) \neq 0$$

  Graphs of rational functions will either be continuous (no jumps, breaks, or holes) or not continuous.
- An **asymptote** is a line that a curve approaches but never reaches.
- When two variable quantities have a constant ratio, their relationship is called a **direct variation**. The constant ratio *k* is called the constant of variation.
- When the product of the two variables is constant, their relationship is called an **inverse variation**. The constant product *k* is called the constant of variation.
- A **radical expression** is an expression of the type $\sqrt[n]{a}$, where $\sqrt{\ }$ is a **radical sign**, n is the i**ndex**, and *a* is the **radicand**.
- A **radical function** is a function of the type $f(x) = \sqrt[n]{a}$
- A **radical equation** is an equation with a variable in the radicand.
- **Conjugates** are expressions like $\sqrt{m} + \sqrt{n} \quad and \quad \sqrt{m} - \sqrt{n}$ that differ only in the sign of the second term.

Introduction to Rational Functions

You have studied linear, quadratic and polynomial functions and used these to model real world situations. When we have a variable in the denominator of a function, it is referred to as a ***rational function***. Rational functions also commonly occur in mathematical models.

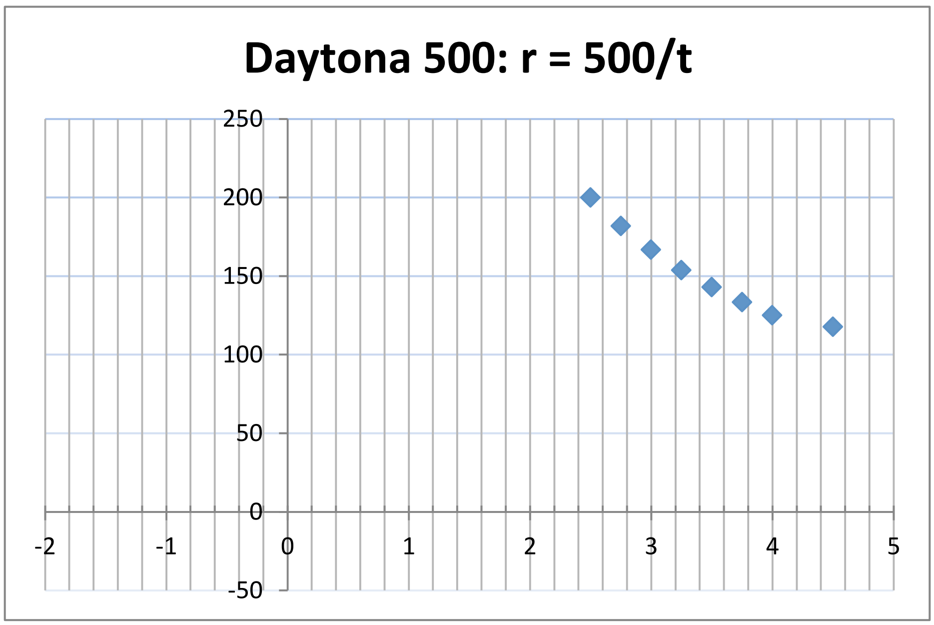
The Daytona 500 is a stock car race that is 500 miles long. You already know that as the speed of a car increases, the time that it takes the car to finish the race will decrease. Another way to say this is that the speed of the car ***varies inversely*** with time.

Since rate * time = distance, we can write the following function for a car at the Daytona 500: $r = 500/t$ where r = rate or speed (mph) and t = time (hrs).

1. Fill in the chart below using the equation above.

Time to finish (hrs)	2.5	2.75	3	3.25	3.5	3.75	4	4.25
Rate of car (mph)								

2. A graph of the data is shown.



3. Consider the equation $r = 500/t$. When we have a variable in the denominator of a function, it almost always means that there is some domain value that just doesn't give us a range value that "makes sense" in context. When this value is placed in the function, the resulting range is said to be ***undefined.*** Are there any domain values (time) that would not make sense in this problem?

4. On the graph, we reflect this domain value with a ***vertical asymptote***. The asymptote is created by drawing a vertical dotted line on the graph at the domain value where the range is undefined. This is a visual "fence" for your graph. It is letting you or anyone else who looks at your work know that no values of the graph will ever go on the line. It can NEVER be touched! It can be "jumped," but you may have to wait for another class to discuss that! Draw the vertical asymptote as a dotted line on the graph above.

5. As the time to finish the race increases think about what happens to the speed of the car. Will the speed of the car ever reach zero (remember the car does finish the race)?

6. As the domain (time) of this function increases, we see the range values (speed) **APPROACHING** zero. In other words as the time to complete the race increases the speed of the car decreases and gets closer and closer to zero, but will never equal zero because then the car could not finish the race! A value that is being approached is called a ***limit***. It can also be called a ***horizontal asymptote*** because we delineate the limit with a horizontal dotted line. What this line on your graph tells anyone looking at the graph is that as your domain values get bigger and bigger (or smaller and smaller) your line will approach this limit value. Draw the horizontal asymptote as a dotted line on the graph above.

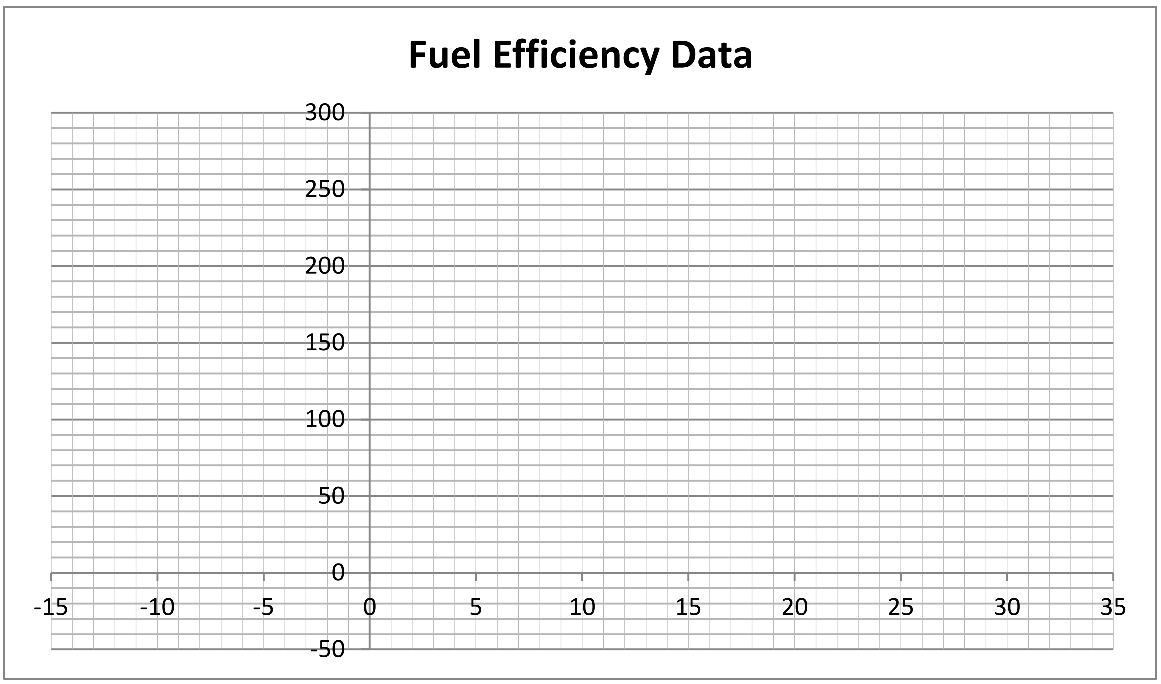
CHECK YOUR UNDERSTANDING

The fuel efficiency of a car on a 300 mile test run can be calculated by the formula $E(g) = \frac{300}{g}$, where E(g) is fuel efficiency and g is the gallons of gas used.

1. Fill in the chart below to reflect fuel efficiency as a function of gallons of gas used.

Gas used (gallons)	5	8	10	13	17	18	20	23
Fuel efficiency (miles per gallon)								

2. Sketch a graph of this data.



3. If there is a vertical asymptote, draw it on your graph as a dotted line.

4. What is the vertical asymptote telling us in context of the problem?

5. Is there a limit to the graph? If so, what is it?

6. What is the limit telling us in context of the problem?

7. Draw the horizontal asymptote on your graph as a dotted line, if there is one.

Graphing Transformations of 1/x

Asymptote: a line that a curve approaches but never reaches

Parent Function $f(x)=\frac{1}{x}$

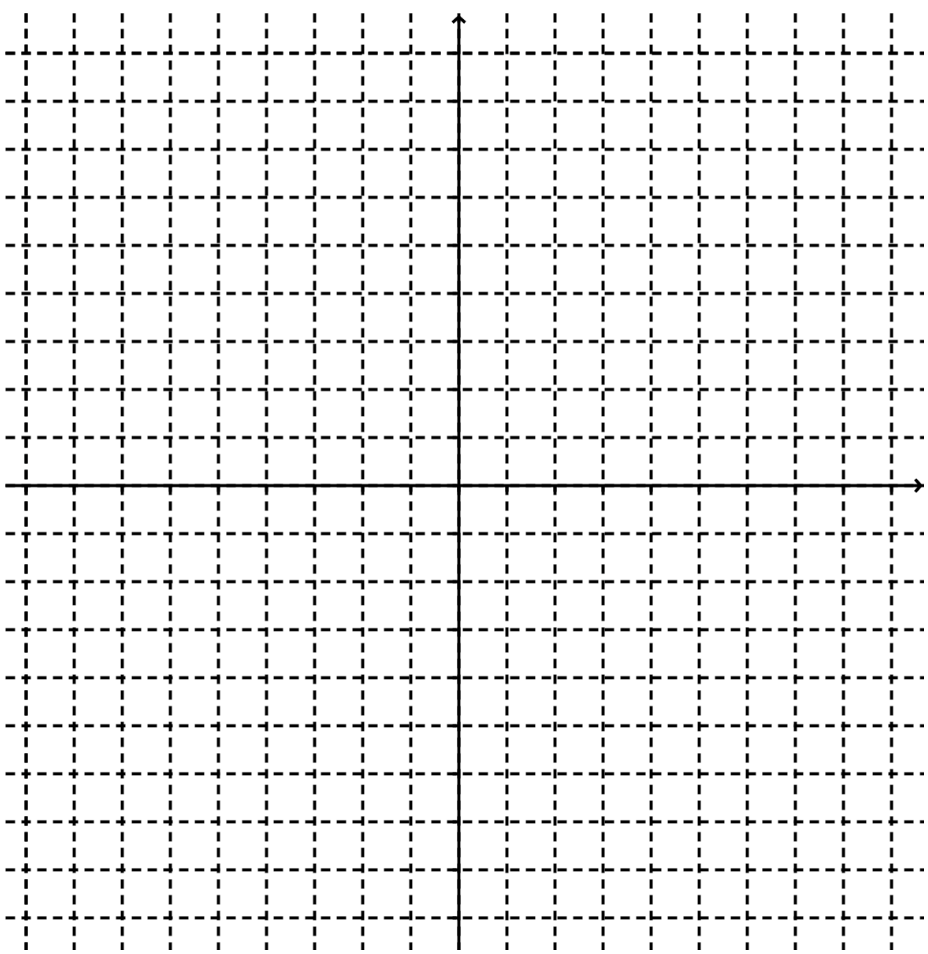
Horizontal Asymptote at y=0

Vertical Asymptote at x=0

General Form: $f(x)=\frac{a}{x-h}+k$

- Horizontal Asymptote at y = k
- Vertical Asymptote at x =h
- (h,k) is the intersection of the asymptote
- "a" vertically stretches of shrinks, if a is negative the graph flips over the x axis

Graph: $f(x)=\frac{1}{x}$



Examples: State the transformations/asymptotes, then graph.

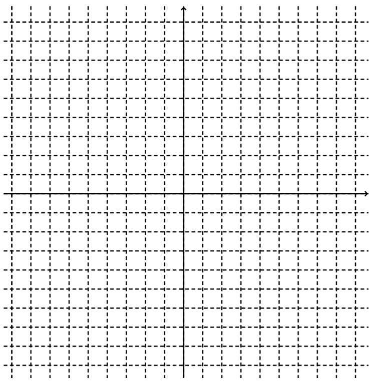
1. $f(x) = \frac{1}{x-3}$

Transformations:

____

V.A. : ________

H.A. : ________



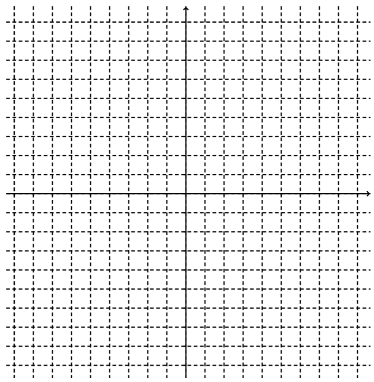
2. $f(x) = \frac{1}{x-4} + 2$

Transformations:

____

V.A. : ________

H.A. : ________



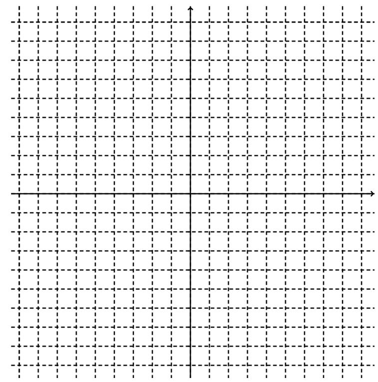
3. $f(x) = \frac{1}{x+3} - 4$

Transformations:

____

V.A. : ________

H.A. : ________



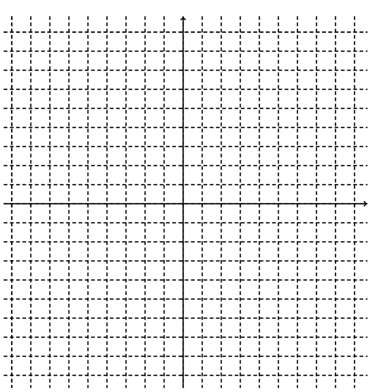
4. $f(x) = \frac{2}{x+3} + 1$

Transformations:

____

V.A. : ________

H.A. : ________



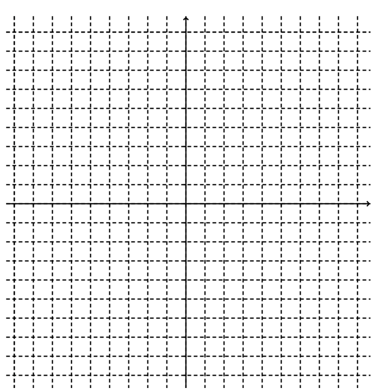
5. $f(x) = \frac{-3}{x-2} - 1$

Transformations:

____

V.A. : ________

H.A. : ________



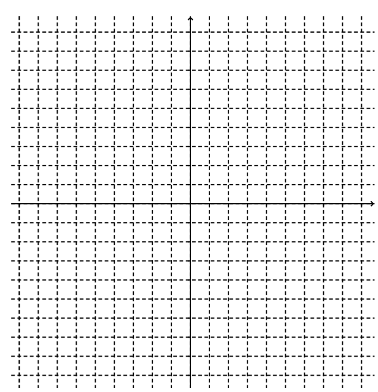
6. $f(x) = \frac{-1}{x+2}$

Transformations:

____

V.A. : ________

H.A. : ________



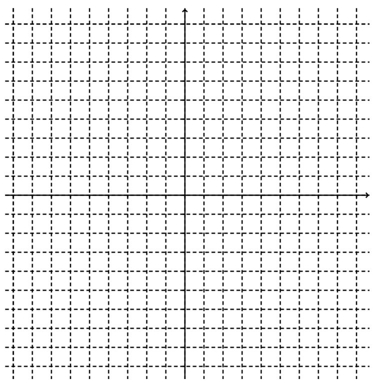
<u>Graphing Worksheet 1/x</u> - State the transformations, and asymptotes, then graph.

1) $f(x) = \frac{4}{x-5}$

Transformations:

____

V.A. : _______H.A. : _______

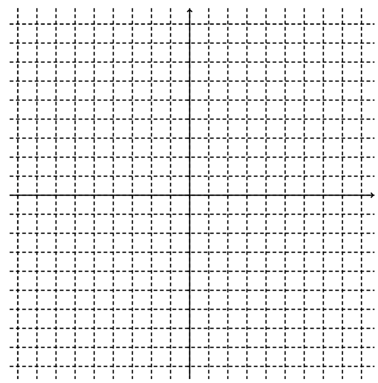


2) $f(x) = \frac{1}{x} - 1$

Transformations:

____

V.A. : _______H.A. : _______

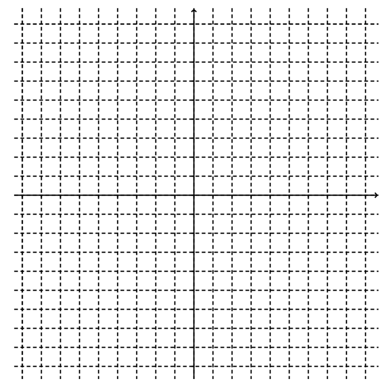


3) $f(x) = \frac{2}{x+1} - 3$

Transformations:

____

V.A. : _______H.A. : _______

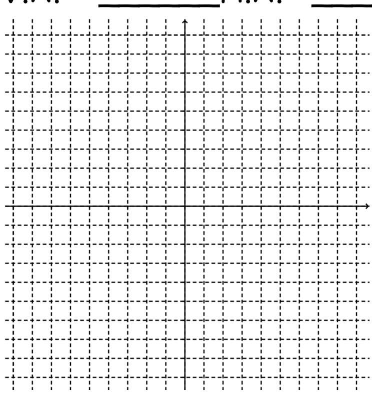


4) $f(x) = \frac{-1}{x-4}$

Transformations:

____

V.A. : _______H.A. : _______

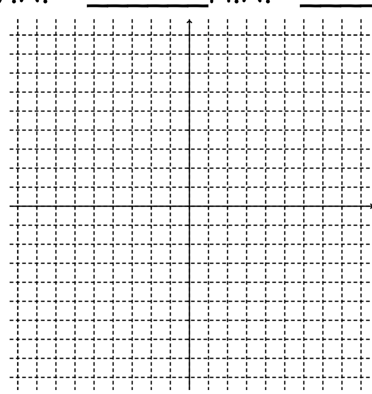


5) $f(x) = \frac{1}{x-2} + 2$

Transformations:

____

V.A. : _______H.A. : _______

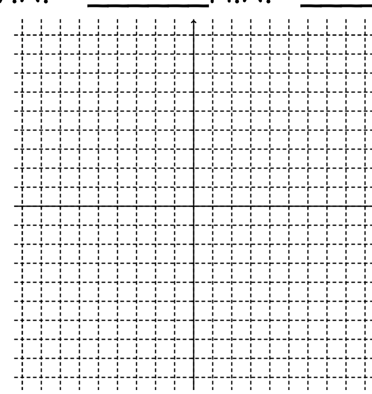


6) $f(x) = \frac{-1}{x+2} - 3$

Transformations:

____

V.A. : _______H.A. : _______



7) $f(x) = \frac{1}{x+1} - 1$

Transformations:

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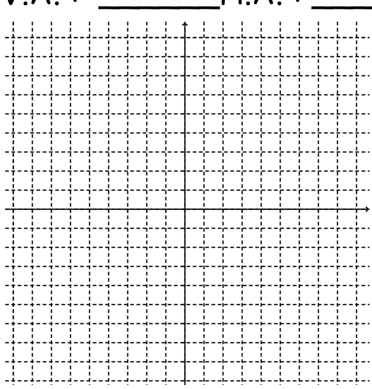
V.A. : _______H.A. : _______

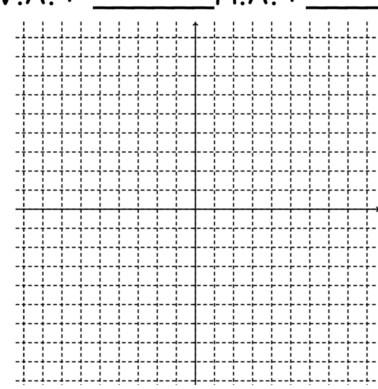


8) $f(x) = \frac{-2}{x-3}$

Transformations:

____

V.A. : _______H.A. : _______



9) $f(x) = \frac{1}{x} + 2$

Transformations:

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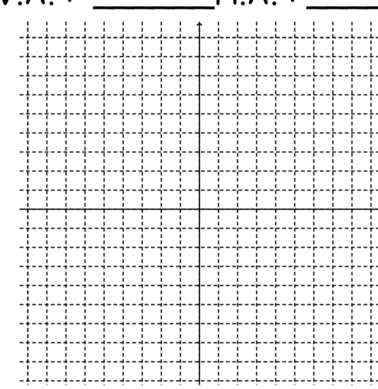
V.A. : _______H.A. : _______



Rational Expressions: A fraction in which the numerator and/or denominator are polynomials.

Multiplying and Dividing Rational Expressions

Rational Expressions are multiplied, and divided just like fractions.

To multiply fractions:

Factor the numerators & denominators, and then cancel factors.

$$\frac{30}{36} * \frac{21}{60} = \frac{5*6*3*7}{6*6*3*4*5} = \frac{7}{24}$$

To divide fractions:

Multiply by the reciprocal. (*Skip, flip, multiply* -or- *keep, change, flip*)

$$\frac{5}{14} \div \frac{15}{22} = \frac{5}{14} * \frac{22}{15} = \frac{5*2*11}{2*7*3*5} = \frac{11}{21}$$

- A fraction is "**simplified**" when the numerator and denominator have no common factors. To simplify a fraction; factor the numerator and denominator, then cancel the common factors.
- **You can only cancel factors!**

To multiply rational expressions:

Factor the numerators & denominators, and then cancel common factors.

$$\frac{(m-3)^2}{m^2-6m+9} \cdot \frac{m^3-9m}{m^2-9} = \frac{(m-3)^2 m(m-3)(m+3)}{(m-3)^2(m-3)(m+3)} = m$$

To divide rational expressions:

Multiply by the reciprocal. (*Skip, flip, multiply* -or- *keep, change, flip*)

$$\frac{x^2+7x+10}{x-6} \div \frac{x+5}{x^2-36} = \frac{x^2+7x+10}{x-6} \bullet \frac{x^2-36}{x+5} = \frac{(x+2)(x+5)(x+6)(x-6)}{(x-6)(x+5)} = (x+2)(x+6)$$

In the rational expression $\frac{x(x-3)}{x+2}$ you CANNOT cancel the x in the numerator (which is a factor) with the x in the denominator because the x in the denominator is NOT A FACTOR (it doesn't multiply everything else in the denominator)! You can only cancel common terms when BOTH are factors!

- **Factors multiply everything else in the numerator/denominator!**

Addition, Subtraction of Rational Expressions

Rational Expressions are simplified, added, and subtracted just like fractions.

To add and subtract fractions:

You **MUST** have a common denominator. Keep the common denominator and add or subtract the numerators.

$$\frac{2}{7}+\frac{4}{7}=\frac{2+4}{7}=\frac{6}{7} \qquad \frac{9}{16}-\frac{3}{16}=\frac{9-3}{16}=\frac{6}{16}=\frac{2*3}{2*8}=\frac{3}{8}$$

When the denominators are not the same:

1) **Find the Least Common Denominator (LCD):**
   - Factor each denominator
   - Take **each unique factor** the **most** number of times that it occurs **in any single denominator**.
2) **Rewrite each fraction with the LCD**
3) **Keep the denominator and add or subtract the numerators**

Example: $\frac{7}{120}+\frac{13}{36}$

1) Find the LCD:

- Factor each denominator

  $120 = 3 \text{ x } 2 \text{ x } 2 \text{ x } 2 \text{ x } 5 = 2^3 \text{ x } 3 \text{ x } 5$

  $36 = 2 \text{ x } 3 \text{ x } 2 \text{ x } 3 = 2^2 \text{ x } 3^2$

- Take each unique factor the most number of times that it occurs in any single denominator.
  - 2 is a factor, the greatest number of 2's is 3
  - 3 is a factor, the greatest number of 3's is 2
  - 5 is a factor, the greatest number of 5's is 1

LCD = 2^3 x 3^2 x 5 = 360

2) Rewrite each fraction with the LCD. $\frac{7}{120}\left(\frac{3}{3}\right)+\frac{13}{36}\left(\frac{10}{10}\right)=\frac{21}{360}+\frac{130}{360}$

3) Keep the common denominator and add the numerators.

$$\frac{21}{360}+\frac{130}{360}=\frac{21+130}{360}=\frac{151}{360}$$

To <u>add and subtract rational expressions:</u>

Example 1: $\frac{1}{x^2+5x+4}+\frac{5x}{3x+3}$

Find the LCD:

$x^2 + 5x + 4 = (x+1)(x+4)$

$3x + 3 = 3(x+1)$

- *(x+1)* is a factor, the greatest number is 1
- *(x+4)* is a factor, the greatest number is 1
- 3 is a factor, the greatest number of 3's is 1

LCD is *3(x+1)(x+4)*

Rewrite each fraction with the LCD, add numerators, keep the denominator, & simplify.

$$\frac{1}{(x+1)(x+4)}\left(\frac{3}{3}\right)+\frac{5x}{3(x+1)}\left(\frac{x+4}{x+4}\right)$$

Write the denominators in factored form!

$$\frac{3}{3(x+1)(x+4)}+\frac{5x(x+4)}{3(x+1)(x+4)}=\frac{3+5x(x+4)}{3(x+1)(x+4)}=\frac{5x^2+20x+3}{3(x+1)(x+4)}$$

Example 2: $\frac{x+1}{x^2+4x+4}-\frac{2}{x^2-4}$

Find the LCD:

$x^2 + 4x + 4 = (x+2)(x+2)$

$x^2 - 4 = (x+2)(x-2)$

- *(x+2)* is a factor, the greatest number is 2
- *(x-2)* is a factor, the greatest number is 1

LCD is *$(x+2)^2(x-2)$*

Rewrite each fraction with the LCD, subtract numerators, keep the denominator, & simplify.

$$\frac{x+1}{(x+2)^2}\left(\frac{x-2}{x-2}\right)-\frac{2}{(x+2)(x-2)}\left(\frac{x+2}{x+2}\right)$$

$$\frac{(x+1)(x-2)}{(x+2)^2(x-2)}-\frac{2(x+2)}{(x+2)^2(x-2)}=\frac{(x+1)(x-2)-2(x+2)}{(x+2)^2(x-2)}$$

$$=\frac{x^2-x-2-2x-4}{(x+2)^2(x-2)}=\frac{x^2-3x-6}{(x+2)^2(x-2)}$$

Rational Expressions – Worksheet 1
Simplify:

1. $\dfrac{1}{x^2+5x+4}+\dfrac{5x}{3x+3}$

2. $\dfrac{5}{6x^2}+\dfrac{x}{4x^2-12x}$

3. $\dfrac{1}{a^2+b^2}+\dfrac{1}{a^2-b^2}+\dfrac{2b^2}{a^4-b^4}$

4. $\dfrac{x+1}{x^2+4x+4}-\dfrac{2}{x^2-4}$

5. $\dfrac{x}{x^2+5x+6}-\dfrac{2}{x^2+3x+2}$

6. $\dfrac{b^2-100}{b^3}\bullet\dfrac{2b^2}{3b^2+31b+10}$

7. $\dfrac{x^2-4}{x+3}\div\dfrac{x^2+4x+4}{x^2+3x}$

8. $\dfrac{\dfrac{2\boldsymbol{x}^2+9\boldsymbol{x}+9}{\boldsymbol{x}+1}}{\dfrac{10\boldsymbol{x}^2+19\boldsymbol{x}+6}{5\boldsymbol{x}^2+7\boldsymbol{x}+2}}$

9. $\dfrac{x^2}{x-y}+\dfrac{y^2}{y-x}$

10. $\dfrac{2y}{1-3y}-\dfrac{y}{3y+1}+\dfrac{10y^2+y}{9y^2-1}$

Rational Expressions – Worksheet 2

Simplify:

1. $\dfrac{3a-4}{a+b} - \dfrac{3}{a+b}$

2. $\dfrac{6}{3a} - \dfrac{2}{9a}$

3. $\dfrac{10}{3x} + \dfrac{8}{12x^2}$

4. $\dfrac{3}{y+1} - \dfrac{2}{y-1}$

5. $\dfrac{-3a}{a^2-2a} - \dfrac{a}{a^2-4}$

6. $\dfrac{2}{x^2+7x+10} + \dfrac{5}{x^2+3x-10}$

7. $\dfrac{\boldsymbol{c}(\boldsymbol{c}-3)}{\boldsymbol{c}^2-25} \cdot \dfrac{\boldsymbol{c}^2+4\boldsymbol{c}-5}{\boldsymbol{c}^2-4\boldsymbol{c}+3}$

8. $\dfrac{(\boldsymbol{m}-3)^2}{\boldsymbol{m}^2-6\boldsymbol{m}+9} \cdot \dfrac{\boldsymbol{m}^3-9\boldsymbol{m}}{\boldsymbol{m}^2-9}$

9. $\dfrac{x^4y^2z^3}{x^2-4} \div \dfrac{x^8y^4z}{x+2}$

10. $\dfrac{1}{x+3} \div \dfrac{2x}{x^2+5x+6}$

11. $\dfrac{\boldsymbol{c}^3+3\boldsymbol{c}^2}{(\boldsymbol{c}+5)^2} \cdot \dfrac{\boldsymbol{c}^2-25}{\boldsymbol{c}^2}$

12. $\dfrac{x^2+5x+6}{x+7} \div \dfrac{x^2-5x-24}{x^2-x-56} \cdot \dfrac{x+3}{x+2}$

ANSWERS

1) $\dfrac{3a-7}{a+b}$ 2) $\dfrac{16}{9a}$ 3) $\dfrac{10x+2}{3x^2}$ 4) $\dfrac{y-5}{(y+1)(y-1)}$ 5) $\dfrac{-2(2a+3)}{(a-2)(a+2)}$

6) $\dfrac{7x+6}{(x+5)(x+2)(x-2)}$ 7.) $\dfrac{c}{c-5}$ 8.) m 9.) $\dfrac{z^2}{x^4y^2(x-2)}$

10.) $\dfrac{x+2}{2x}$ 11.) $\dfrac{(c+3)(c-5)}{c+5}$ 12.) x+3

Rational Expressions – Worksheet 3
Simplify:

1. $\dfrac{\dfrac{10}{x+1}}{\dfrac{1}{2}+\dfrac{3}{x+1}}$

2. $\dfrac{\dfrac{x^3y^2z}{a^2b^2}}{\dfrac{a^3x^2y}{b^2}}$

3. $\dfrac{\dfrac{2}{x^2-1}+\dfrac{1}{x+1}}{\dfrac{1}{12x^2-3}}$

4. $\dfrac{\dfrac{x^2-4}{x+3}}{\dfrac{x^2+4x+4}{x^2+3x}}$

5. $\dfrac{\dfrac{x}{3}-4}{5+\dfrac{1}{x}}$

6. $\dfrac{\dfrac{1}{x}-\dfrac{1}{y}}{1+\dfrac{1}{x}}$

7. $\dfrac{3z^2+6z-45}{z^2-2z}\cdot\dfrac{10-7z+z^2}{6z^2+33z+15}\div\dfrac{15-8z+z^2}{10z^2+5z}$

8. $\dfrac{a-4}{4a^2-4a}-\dfrac{1}{4a}$

9. $\dfrac{5}{6x}-\dfrac{x-6}{6x^2+6x}$

10. $\dfrac{\dfrac{2x^2+9x+9}{x+1}}{\dfrac{10x^2+19x+6}{5x^2+7x+2}}$

11. $\dfrac{3}{x^2-2x+4}+\dfrac{5}{x+2}$

12. $\dfrac{a^2-b^2}{18-54a}\div\dfrac{a^2+2ab+b^2}{a^2-2ab+b^2}\cdot\dfrac{18a-6}{a^3-b^3}$ (Honors)

13. $\dfrac{4}{x-3}+\dfrac{5}{x^2+3x+9}$

14. $\dfrac{8}{x^2-x-12}-\dfrac{5}{x^2+7x+12}$

ANSWERS: 1) $\dfrac{20}{x+7}$ 2) $\dfrac{xyz}{a^5}$ 3) $\dfrac{12x^2-3}{x-1}$ 4) $\dfrac{x(x-2)}{x+2}$ 5) $\dfrac{x(x-12)}{3(5x+1)}$

6) $\dfrac{y-x}{y(x+1)}$ 7) 5 8) $\dfrac{-3}{4a(a-1)}$ 9) $\dfrac{4x+11}{6x(x+1)}$ 10) x+3 11) $\dfrac{5x^2-7x+26}{(x^2-2x+4)(x+2)}$

12) $\dfrac{-1(a-b)^2}{3(a+b)(a^2+ab+b^2)}$ 13) $\dfrac{4x^2+17x+21}{(x^2+3x+9)(x-3)}$ 14) $\dfrac{3x+52}{(x-4)(x+3)(x+4)}$

COMPLEX FRACTIONS – Worksheet 1

Complex Fraction: A fraction that contains a fraction in its numerator or denominator. To simplify write the numerator as a single fraction, write the denominator as a single fraction, then multiply by the reciprocal of the denominator.
Simplify:

1. $\dfrac{2 + \frac{3}{5}}{5 + \frac{1}{4}}$

2. $\dfrac{\frac{3ab}{x}}{\frac{6a^2b}{x^2}}$

3. $\dfrac{x + \frac{x}{y}}{1 + \frac{1}{y}}$

4. $\dfrac{1 + \frac{1}{2z}}{z - \frac{1}{4z}}$

5. $\dfrac{z + 1 - \frac{20}{z}}{z - 2 - \frac{8}{z}}$

6. $\dfrac{1 + \frac{1}{a} + \frac{2}{a^2}}{2 + \frac{5}{a} + \frac{2}{a^2}}$

7. $\dfrac{1 + \frac{2}{9}}{2 - \frac{1}{3}}$

8. $\dfrac{1 + \frac{1}{a^2}}{1 - \frac{1}{a^2}}$

9. $\dfrac{2 + \frac{1}{x} - \frac{1}{x^2}}{1 + \frac{4}{x} + \frac{3}{x^2}}$

Answers: 1. $\frac{52}{105}$ 2. $\frac{x}{2a}$ 3. x 4. $\frac{2}{2z-1}$ 5. $\frac{z+5}{z+2}$ 6. $\frac{a^2+a+2}{2a^2+5a+2}$

7. $\frac{11}{15}$ 8. $\frac{a^2+1}{a^2-1}$ 9. $\frac{2x-1}{x+3}$

Complex Fractions Worksheet 2 *(HONORS)*

Simplify:

1. $\dfrac{x^{-1}-y^{-1}}{x^{-2}-y^{-2}}$

2. $\dfrac{r^{-1}+s}{r+s^{-1}}$

3. $\dfrac{ab^{-1}-1}{b^{-1}-a^{-1}}$

4. $\dfrac{1+(m-1)^{-1}}{1-(m+1)^{-1}}$

5. $\dfrac{(y-z)^{-3}}{(z-y)^{-3}}$

6. $\dfrac{8s+4t}{4s^{-1}+8t^{-1}}$

7. $\dfrac{\dfrac{1+\frac{b}{a}}{1-\frac{b}{a}}}{\dfrac{\frac{2a}{b}}{\frac{a}{b}-1}}$

8. $\dfrac{\dfrac{1}{c^{-2}}-\dfrac{1}{d^{-2}}}{\dfrac{1}{c^{-2}}+\dfrac{2}{(cd)^{-1}}+\dfrac{1}{d^{-2}}}$

9. $\dfrac{1-\dfrac{1}{x}-\dfrac{12}{x^2}}{3+\dfrac{13}{x}+\dfrac{12}{x^2}}$

10. $\dfrac{2-\dfrac{7}{b}+\dfrac{6}{b^2}}{2+\dfrac{3}{b}-\dfrac{9}{b^2}}$

11. $\dfrac{\dfrac{1}{xy}+\dfrac{2}{yz}+\dfrac{3}{xz}}{\dfrac{2x+3y+z}{xyz}}$

12. $\dfrac{\dfrac{x}{yz}-\dfrac{y}{xz}+\dfrac{z}{xy}}{\dfrac{1}{x^2y^2}-\dfrac{1}{x^2z^2}+\dfrac{1}{y^2z^2}}$

13. $\dfrac{1}{x^{-2}+y^{-2}}$

14. $\dfrac{\dfrac{x+1}{x-1}+\dfrac{x-1}{x+1}}{\dfrac{x+1}{x-1}-\dfrac{x-1}{x+1}}$

Answers: 1. $\dfrac{xy}{y+x}$ 2. $\dfrac{s}{r}$ 3. a 4. $\dfrac{m+1}{m-1}$ 5. -1 6. st 7. $\dfrac{a+b}{2a}$

8. $\dfrac{c-d}{c+d}$ 9. $\dfrac{x-4}{3x+4}$ 10. $\dfrac{b-2}{b+3}$ 11. 1 12. xyz 13. $\dfrac{x^2y^2}{y^2+x^2}$ 14. $\dfrac{x^2+1}{2x}$

Solving Rational Equations

Because division by zero is not allowed, there may need to be restrictions placed on the variable in a rational expression.

Examples: $\frac{z^2-1}{z^2+5}$ *no restrictions on z*

$\frac{m^4+18m+1}{m^2-m-6}$ $m \neq 3,-2$

To Solve Rational Equations:

- Name the restrictions on the variable.
- If the rational equation is written as a proportion (a statement that two ratios are equal):
  - If the denominators are the same, the numerators must be equal.
  - If the denominators are different, cross multiply to solve .
- If the rational equation is not written as a proportion:
  1. Find the LCD of all terms.
  2. Multiply both sides of the equation by the LCD of all terms to eliminate fractions
  3. Solve for the variable.
- Always check solutions to be sure they work.
  - **Solutions cannot be restricted values**.

Ex 1: $\frac{7}{m-3}=\frac{m+2}{m-3}$ $m \neq 3$

$7 = m+2$

$m = 5$

Ex 1: The equation is a proportion and the denominators are the same so the numerators must be equal.

Ex 2: $\frac{5}{x-2}=\frac{x+3}{x-2}$ $x \neq 2$

$5 = x+3$

$x = \not{2}$

No solution.

Ex 2: Since the restriction on x says x cannot be 2, there is no solution.

Ex 3: $\dfrac{7}{x+2} = \dfrac{6}{x-5} \quad x \neq -2,5$

$$7(x-5) = 6(x+2)$$

$$7x - 35 = 6x + 12$$

$$x = 47$$

> **Ex 3:** The equation is a proportion with different denominators, so cross multiply to solve.

Ex 4: $\dfrac{1}{2x} - \dfrac{2}{5x} = \dfrac{1}{2} \quad x \neq 0$

> **Ex 4:** The equation is not a proportion so…
> 1. Find the LCD of all terms
> 2. Multiply both sides of the equation by the LCD to eliminate fractions
> 3. Solve for the variable.

Find the LCD:

2x = 2 * x

5x = 5 * x

2 = 2

- 2 is a factor, the greatest number of 2's is 1
- x is a factor, the greatest number of x's is 1
- 5 is a factor, the greatest number of 5's is 1

LCD is 2*x*5 = 10x

$$10x\left(\frac{1}{2x} - \frac{2}{5x}\right) = 10x\left(\frac{1}{2}\right)$$

$$\frac{10x}{2x} - \frac{20x}{5x} = \frac{10x}{2}$$

$$5 - 4 = 5x$$

$$\frac{1}{5} = x$$

Rational Equations – Worksheet 1
Solve:

1. $\dfrac{3a-1}{4}=2$

2. $y+1-\dfrac{3}{4}y=\dfrac{1}{5}y$

3. $\dfrac{2t}{t-1}=\dfrac{t+6}{1-t}$

4. $\dfrac{10w}{w+2}-\dfrac{2w-3}{w-2}=\dfrac{2w^2-3}{w^2-4}$

5. $\dfrac{4x-36}{x^2-9}+\dfrac{11}{3-x}=\dfrac{11}{x+3}$

6. $\dfrac{b+1}{b^2-b}-\dfrac{b}{b^2-1}=\dfrac{b-1}{b^2+b}$

7. $\dfrac{5}{u+1}+\dfrac{1}{u-1}-\dfrac{7}{3u-5}=0$

8. $\dfrac{5}{x-2}-\dfrac{x}{x+5}=\dfrac{x^2-4}{10-3x-x^2}$

9. $\dfrac{a}{a-1}-\dfrac{a-1}{a}=\dfrac{3}{2}$

10. solve for x: $P=\dfrac{n}{x}-\dfrac{m}{cx}$

11. solve for x: $y=\dfrac{x}{x+b}$

Answers: **1**. 3 **2**. -20 **3**. -2 **4**. $\frac{1}{2}$, 3 **5**. -2 **6**. 0, 4 **7**. $\frac{9}{11}$, 3

8. -3 **9**. 2 , $\frac{1}{3}$ **10**. $x=\dfrac{cn-m}{Pc}$ **11**. $x=\dfrac{by}{1-y}$

Rational Equations - Worksheet 2
Solve.

1. $\frac{12}{x}+\frac{3}{4}=\frac{3}{2}$

2. $\frac{x^2}{8}-4=\frac{x}{2}$

3. $\frac{x+10}{x^2-2}=\frac{4}{x}$

4. $\frac{x}{x+2}+x=\frac{5x+8}{x+2}$

5. $\frac{5}{x-5}=\frac{x}{x-5}-1$

6. $\frac{1}{3x-2}+\frac{5}{x}=0$

7. $\frac{1}{x+3}=\frac{2}{x}-\frac{3}{4x}$

8. $\frac{5}{x+6}=\frac{9x+6}{x^2+6x}+\frac{2}{x}$

9. $\frac{6}{x-1}=\frac{4}{x-2}+\frac{2}{x+1}$

10. $\frac{x+1}{x-3}=4-\frac{12}{x^2-2x-3}$

11. $\frac{1}{x-1}=\frac{2}{x+1}-\frac{1}{x+3}$

12. $\frac{1}{x+2}+\frac{1}{x-2}=\frac{3}{x+1}$

Answers: 1. 16 2. -4, 8 3. $-\frac{2}{3}$, 4 4. 4 5. $x \neq 5$ 6. $\frac{5}{8}$

7. -15 8. -3 9. no soln 10. $-\frac{5}{3}$, 5 11. no soln 12. $1 \pm \sqrt{13}$

Rational Equations - Worksheet 3
Solve.

1. $\frac{2}{x-1}=\frac{x+4}{3}$

2. $\frac{3}{x+1}=\frac{1}{x^2-1}$

3. $\frac{10}{6x+7}=\frac{6}{2x+9}$

4. $\frac{1}{4}-x=\frac{x}{8}$

5. $\frac{1}{x}+\frac{x}{2}=\frac{x+4}{2x}$

6. $\frac{5}{2x}-\frac{2}{3}=\frac{1}{x}+\frac{5}{6}$

7. $\frac{2}{x-3}-\frac{4}{x+3}=\frac{8}{x^2-9}$

8. $\frac{1}{8}+\frac{5x}{x+2}=\frac{5}{2}$

9. $\frac{10}{2y+8}-\frac{7y+8}{y^2-16}=\frac{-8}{2y-8}$

10. $\frac{2}{x+3}-\frac{3}{4-x}=\frac{2x-2}{x^2-x-12}$

Answers: 1. x=2, -5 2. x=4/3 3. x=3 4. x=2/9 5. x = -1, 2
6. x=1 7. x=3, -3 8. x=38/21 9. y=6 10. x=-1

Word Problems

Rate x Time = Work
Rate1 + Rate2 = Combined Rate **(when working at the same time)**
Work1 + Work2 = Total Work

1. A deck of 320 punched cards can be read by one reader in 20 minutes. A second reader can read the same deck in 12 minutes. How many minutes would it take both readers working together to read a deck? {7.5 h}

2. How long would it take to process the deck of cards in example one if the first reader stopped working after 5 minutes? {9 minutes}

3. One pipe can fill a tank in 5 hours. A second pipe can fill it in 3 hours. How long would it take both pipes together to fill the tank $\frac{3}{4}$ of the way full? {$\frac{45}{32} hrs$}

4. Heckel can plow a field in 6 hours. If his brother Jeckel helps him, it will take 4 hours. How long would it take Jeckel to do the job alone? {12 hrs}

5. Carl can paint a house in 8 hours. After he had been working for 1 hour, his brother Gomer started helping him. It then took them 3 more hours to finish. How long would it take Gomer to paint a house by himself? {6h}

Rate of Work Word Problems Worksheet

1. Working alone, a painter can paint a small apartment in 10 hours. Her helper can paint the same apartment in 15 hours. How long would it take the painter and her helper to complete the job if they work together? (6 hr)

2. Pete can do a job in 50 minutes while his brother Jim can do it in ½ the time. How long will it take them to do the job working together? (50/3 min)

3. Pearson College owns a tabulator that can process registration data in 6 hours. Wanting to speed up the processing, the school authorized the purchase of a second tabulator that can do the job in 8 hours. How many hours will it take the two tabulators working together to process all the data? (24/7 hr)

4. A tank can be emptied by one pipe in 40 minutes and by another in 30 minutes. If the tank is 7/8 full, how long will it take to empty it when both pipes are open? (15 min)

5. Shelly can keypunch 300 cards in 1 hour. When she and Linda work together, they can complete the job in 24 minutes. How long would it take Linda to keypunch 300 cards if she worked alone? (40 min)

6. One pipe can fill a tank in 4 hours. A second pipe can also do it in 4 hours, but a third needs 6 hours. How long would it take to fill the tank if all three pipes were open? (1.5 hr)

7. It takes one man 10 hours to do a certain job. After he has been at work for 4 hours, another man is sent to help. The two men then complete the job in 2 more hours. How long would it have taken the second man to do the job alone? (5 hr)

8. Maria can paint the walls of an apartment in 8 hours. After she has worked for 3 hours, Pat joins her and they finish the job in 2 more hours. How long would it take Pat to do the entire job alone? (5 1/3 hr)

9. A tank is fitted with 2 pipes. One can fill it in 8 hours. After it has been open 2 hours, the 2nd pipe is opened and the tank is filled in 2 more hours. How long would it take the second pipe alone to fill the tank? (4 hr)

10. Rosita can wax her car in 2 hr. When she works together with Helga, they can wax the car in 45 min. How long would it take Helga, working by herself, to wax the car? (6/5 hr)

11. Hannah can sand the living room floor in 3 hr. When she works with Henri, the job takes 2 hr. How long would it take Henri, working by himself, to sand the floor? (6 hr)

12. Sara takes 3 hr longer to paint a floor than it takes Kate. When they work together, it takes them 2 hr. How long would each take to do the job alone? (Sara: 6 hr; Kate: 3 hr)

13. A swimming pool can be filled in 12 hr if water enters through a pipe alone or in 30 hr if water enters through a hose alone. If water is entering through both the pipe and the hose, how long will it take to fill the pool? (60/7)

14. Mary takes twice as long as Carole to make the display case at school. Together it takes 4 hours to do the display case. How long does it take Mary to do it herself? (12 hours)

Rational Equations Word Problems – HONORS

1. Sam, an experienced shipping clerk, can fill a certain order in 5 hr. Willy, a new clerk, needs 9 hr to complete the same job. Working together, how long will it take them to fill the order? (45/14)

2. Claudia can paint a neighbor's house 4 times as fast as Jan can. The year they worked together it took them 8 days. How long would it take each to paint the house alone? (Claudia: 10 days; Jan: 40 days)

3. Susanna can deliver papers 3 times as fast as Stan can. If they work together, it takes them 1 hour. How long would it take to deliver the papers alone? (Susanna: 4/3 hr; Stan: 4 hr)

4. A new photocopier works twice as fast as an old one. When the machines work together, a university can produce all its staff manuals in 15 hr. Find the time it would take each machine, working alone, to complete the same job. (New: 22.5 hr; Old: 45 hr)

5. The speed of the stream is 3 mph. A boat travels 4 miles upstream in the same time it takes to travel 10 miles downstream. What is the speed of the boat in still water? (7 mph)

6. The speed of a moving sidewalk at an airport is 7ft/sec. Once on a moving sidewalk, a person can walk 80 ft forward in the same time it takes to walk 15 ft in the opposite direction. At what rate would the person walk on a nonmoving sidewalk? (133/13 ft/sec)

7. Rosanna walks 2 mph slower than Simone. In the time it takes Simone to walk 8 mi, Rosanna walks 5 mi. Find the speed of each person. (Rosanna: 10/3 mph; Simone: 16/3 mph)

8. Train A goes 12 mph slower than the B train. The A train travels 230 mi in the same time that the B train travels 290 mi. Find the speed of each train. (A: 46 mph; B: 58 mph)

9. A steamboat travels 10 kph faster than a freighter. The steamboat travels 75 km in the same time that the freighter travels 50 km. Find the speed of each boat. (Steamboat: 30 kph; Freighter: 20 kph)

10. A barge moves 7 kph in still water. It travels 45 km upriver and 45 km downriver in a total time of 14 hr. What is the speed of the current? (2 kph)

11. A plane travels 100 mph in still air. It travels 240 mi into the wind and 240 mi with the wind in a total time of 5 hr. Find the wind speed. (20 mph)

12. A car traveled 120 mi at a certain speed. If the speed had been 10 mph faster, the trip could have been made in 2 hr less time. Find the speed. (20 mph)

13. A boat travels 45 mi upstream and 45 mi back. The time required for the round trip is 8 hr. The speed of the stream is 3 mph. Find the speed of the boat in still water. (12 mph)

14. A motor boat travels 3 times as fast as the current. A trip up the river and back takes 10 hr, and the total distance of the trip is 100 km. Find the speed of the current. (3.75 mph)

15. A tank can be filled by a hose in 15 hours. It can be emptied by a drainpipe in 25 hours. If the drainpipe is open while the tank is being filled, how long does it take to fill the tank? (37.5 hours)

16. The denominator of a fraction is one less than three times the numerator. If 12 is added to both numerator and denominator, the resulting fraction has a value of $\frac{3}{4}$. Find the original fraction. (3/8)

17. Mike can wash and wax the car in 4.5 hours. Ray can do the job in 3 hours. How long will it take both boys working together to wash and wax the car? (1.8 hours)

18. The numerator of the fraction is 4 more than twice the denominator. If two is subtracted from both the numerator and denominator, the result is 8. Find the original fraction. (10/3)

19. One hose can fill a pool in 8.5 hours. A small hose can fill the same pool 10.75 hours. If both hoses are used, how long will it take to fill the pool? (731/154 hours)

20. It takes Angus three days to cultivate the garden. It takes Helga four days to do the same job. How long does it take them to do the job together? (12/7 days)

21. Mary takes twice as long as Carole to make the display case at school. Together it takes 4 hours to do the display case. How long does it take Mary to do it herself? (12 hours)

22. A machine can cut some wood in 6 minutes, and a man using a hand saw can do it in 18 minutes. After 4 minutes there is a power shortage and the wood must be cut by hand saw. How many minutes must the man work to complete the task? (6 minutes)

23. How many ounces of pure acid must be added to 20 ounces of 20% acid to produce a 50% acid solution? (12 oz)

24. How many ounces of a 75% acid solution must be added to 30 ounces of a 15% acid solution to produce a 50% solution? (42 oz)

25. How many pounds of a 35% silver alloy must be melted with how many pounds of a 65% silver alloy to obtain 20 pounds of a 41% silver alloy? (4 lbs 65%, 16 lbs 35%)

26. How many ounces of a 5% antiseptic solution must be added to 30 ounces of a 10% solution to produce an 8% solution? (20 oz @5%)

27. How many pounds of a 70% copper alloy must be melted with how many pounds of 90% copper alloy to obtain 100 pounds of an 81% copper alloy? (45 lbs@70%, 55 lbs @90%)

28. A tank contains 20 gallons of a mixture of alcohol and water which is 40% alcohol. How much of the mixture should be removed and replaced by an equal volume of water so that the resulting solution will be 25% alcohol? (7.5 gal)

29. How many quarts of an antifreeze solution containing 40% glycerine should be drawn from a radiator containing 32 quarts and replaced with water in order that the radiator be filled with a 20% glycerine solution? (16 qts)

Variation

Direct Variation:

When two variable quantities have a constant ratio, their relationship is called a direct variation.

$\frac{y}{x} = k$ or $y = kx$ (where $k \neq 0$)

The equation is read:

"*y* varies directly as *x*"

"*y* is directly proportional to *x*"

"*y* is directly related to *x*"

The graph of a direct variation:

- is a line
- passes through the origin (0,0)
- with slope = k ,the constant of variation

The constant ratio *k* is called the constant of variation.

Example: If y varies directly as x and y = 4 when x = -2, find x when y = 6

For a direct variation: $\frac{y}{x} = k$

$$\frac{4}{-2} = \frac{6}{x}$$

$$4x = -12$$

$$x = 3$$

Inverse Variation

When the product of the two variables is constant, one of them is said to vary inversely as the other.

$xy = k$ or $y = \frac{k}{x}$ (where $k \neq 0$)

The equation is read:

"*y* varies inversely as *x*"

"*y* is inversely proportional to *x*"

"*y* is inversely related to *x*"

The graph of an inverse variation:

- never crosses the x-axis $(y \neq 0)$
- never crosses the y-axis $(x \neq 0)$
- does not pass through the origin

The constant product *k* is called the constant of variation.

Examples: If y varies inversely as x and y = 3 when x = 4, find y when x = 18 .

For an inverse variation $xy = k$

$$3 \cdot 4 = y \cdot 18$$

$$12 = 18y$$

$$\frac{2}{3} = y$$

If y varies inversely as x and y = 100 when x = 2.5, find x when y=125

$$100 \cdot 2.5 = 125 \cdot x$$

$$250 = 125x$$

$$2 = x$$

The constant in a <u>direct variation</u> is the <u>quotient</u> of the variables.	$\frac{y}{x} = k$
The constant in an <u>inverse variation</u> is the <u>product</u> of the variables.	$xy = k$

Combined or Joint Variation

A combined variation combines direct and inverse variation in more complicated relationships.

Direct Variation	Equation form
y varies directly as x	$y = kx$
Inverse Variation	Equation form
y varies inversely as x	$y = \frac{k}{x}$
Combined Variation Examples	Equation form
y varies directly as the square of x	$y = kx^2$
y varies inversely as the cube of x	$y = \frac{k}{x^3}$
z varies jointly as x and y	$z = kxy$
z varies jointly as x and y, and inversely as w	$z = \frac{kxy}{w}$
z varies directly as x, and inversely as the product of w and y	$z = \frac{kx}{wy}$

VARIATION Worksheet #1

1. If y varies inversely as x and $y = 4$ when $x = 2$, find y when $x = 6$.

2. **If x varies inversely as the square of y and $x = 4$ when $y = 2$, find x when $y = 3$.**

3. **If y varies directly as x and $y = 6$ when $x = 4$, find y when $x = 8$.**

4. **If y varies directly as x and $y = 12$ when $x = -6$, find x when $y = 22$.**

5. **If z varies jointly as x and y, and $z = 12$ when $x = 3$ and $y = 4$, find z when $x = 5$ and $y = 6$.**

6. **If z varies jointly as x and y, and $z = 15$ when $x = 5$ and $y = 2$, find z when $x = 7$ and $y = 4$.**

7. **If y varies directly as x and inversely as z, and $y = 49$ when $x = 14$ and $z = 4$, find y when $x = 16$ and $y = 7$.**

8. **If a varies directly as b and inversely as the square of c, and $a = 46$ when $b = 12$ and $c = 6$, find b when $a = 23$ and $c = 6$.**

9. **If y varies directly as x and inversely as the square of z, and $x = 48$ when $y = 8$ and $z = 3$, find x when $y = 12$ and $z = 2$.**

10. **If d varies jointly as r and t, and $d = 110$ when $r = 55$ and $t = 2$, find r when $d = 40$ and $t = 3$.**

VARIATION WORKSHEET #2

1. If r varies directly as s, and $r=2$ when $s=10$, find r when $s=30$.

2. If y varies directly as x, and $y=24$ when $x=8$, find y when $x=50$.

3. If p is directly proportional to t and $p=2$ when $t=10$, find p when $t=-1$.

4. If y varies inversely as x and $y=27$ when $x=3$, find y when $x=9$.

5. If y varies inversely as x and $y=15$ when $x=-2$, find y when $x=-5$.

6. If y varies inversely as x and $y=9$ when $x=4$, find y when $x=6$.

7. If y varies jointly as x and z and $y=12$ when $x=4$ and $z=3$, find y when $x=9$ and $z=8$.

8. If y varies jointly as x and z and $y=72$ when $x=3$ and $z=8$, find y when $x=-2$ and $z=-3$.

9. If y varies jointly as x and z and $y=24$ when $x=2$ and $z=3$, find y when $x=4$ and $z=7$.

10. A fish with a mass of $3kg$ causes a fishing pole to bend $9cm$. If the amount of bending varies directly as the mass, how much will the pole bend for a $2kg$ fish?

11. The mass of a copper bar varies directly as its length. If a bar long $40cm$ long has a mass of approximately $420g$, find the mass of a bar $136cm$ long.

12. The interest earned on an investment varies directly with the interest rate. If a 9% rate yields $\$279$, what interest rate yields $\$341$?

13. The illumination i from a light varies inversely as the square of its distance d from an object. $i=8ft$ candles when $d=3ft$, find i when $d=4ft$.

14. The pressure P of a gas at a constant temperature varies inversely as the volume V. If $V=450in^3$ when $P=30lb/in^2$, find P when $V=750in^3$.

15. The frequency of a radio wave is inversely proportional to its wave length. If a radio wave, $30m$ long has a frequency of $1200 kilocycles$ per second, what is the length of a wave with a frequency of $900 kilocycles$?

16. The area A of a triangle varies jointly as the length of its base b and the length of its corresponding altitude h. If $A=15cm$ when $b=10cm$ and $h=3cm$, find A when $b=25cm$ and $h=6cm$.

17. The distance D traveled at a uniform rate varies jointly as the rate r and the time t. If $D=120$ when $r=60$ and $t=2$, find D when $r=80$ and $t=3$.

18. The area A of a parallelogram varies jointly as the length of a base b and the length of a corresponding altitude h. If $A=16$ when $b=2$ and $h=8$, find A when $b=8$ and $h=16$.

Closure Tables

In this activity, you will explore:

- *the properties of some sets under the operations of addition, subtraction, multiplication, and division*
- *the concept of closure and closure tables*

.

Definition:
A set is ***closed*** under an operation if for any two numbers in the set, the result of the operation is also in the set. This definition will take on more meaning as you explore examples

Problem 1 – The set {0, 1}

Let's begin with the set {0, 1}. This set has only two elements. Is it closed under multiplication?

Make all the multiplication problems you can using two numbers from this set. You can use a number twice. There are 4 possible multiplication problems. Look at their answers. Do the answers belong to the set {0, 1}?

If all of the answers belong to the set {0, 1}, then the set is closed under that operation. If any of the answers do not belong to the set, then the set is not closed under that operation.

0 * 0 = 0

0 * 1 = 0

1 * 0 = 0

1 * 1 = 1

Is the set {0, 1} closed under addition?

List all the addition problems you can using two numbers from this set. You can use a number twice. There are 4 possible addition problems. Do the answers belong to the set {0, 1}? Is this set closed under addition?

0 + 0 = 0

0 + 1 = 0

1 + 0 = 1

1 + 1 = 2

Closure tables are a way to organize your work when checking to see if sets are closed under operations. The closure table for {0, 1} under multiplication is shown.
Closure tables are similar to multiplication tables, but they can show any operation. To read them, find a number in the first column and a number in the first row. The intersection of these gives the result of the two numbers under that operation.
To use the closure table to determine if this set is closed under multiplication, look at each of the products in the table. All of the products belong to the set {0, 1} so this set is closed under multiplication.

x	**0**	**1**
0	0	0
1	0	1

- Is the set {0, 1} closed under subtraction? Use the closure table to the right to explain your answer.

–	**0**	**1**
0	0	–1
1	1	0

Problem 2 – The whole numbers {0,1,2,3,…},

In this problem, you will examine a larger set of numbers, the whole numbers, and determine if they are closed under addition, subtraction, multiplication, and division.

A closure table with all of the whole numbers would be infinitely large! For our purposes, it is enough to use just a few examples of the whole numbers. Complete the table.

- Do you see any patterns in the table? Do any of the sums repeat?
- What property of addition causes this pattern?
- Are all of the sums whole numbers?
- Is the set of whole numbers closed under addition?

+	0	1	2	3	4
0					
1					
2					
3					
4					

Addition is commutative, so it doesn't matter what order you add two numbers. Subtraction is **not** commutative. It does matter in what order you subtract. Complete the table.

- Do you see any patterns in the table?
- Are all of the differences whole numbers?
- Is the set of whole numbers closed under subtraction?

–	0	1	2	3	4
0					
1	1				
2		1			
3			1		
4				1	

Complete the closure tables for multiplication and division of whole numbers.

×	0	1	2	3	4
0					
1					
2					
3					
4					

÷	0	1	2	3	4
0					
1					
2					
3					
4					

- Do you see a pattern in the table?
- What property causes this pattern?
- Is the set of whole numbers closed under multiplication?

- Is the set of whole numbers closed under division?

Problem 3 – The integers {…-3, -2, -1, 0, 1, 2, 3, …}

In this problem, you will determine if the set of all integers is closed under addition, subtraction, multiplication, and division.

Choose some integers to use in the closure tables. Be sure to choose some negative and some positive integers.

Complete the closure tables for addition, subtraction, multiplication, and division of integers.

+			**0**		
0					

–			**0**		
0					

×			**0**		
0					

÷			**0**		
0					

- Do you see any patterns in the tables?

- Under which operations are the integers closed?

Problem 4 – The counting numbers or natural numbers {1,2,3,…},

In this problem, you will examine the set of counting numbers, and determine if they are closed under addition, subtraction, multiplication, and division.
Choose some counting numbers to use in the closure tables.

Complete the closure tables for addition, subtraction, multiplication, and division of counting numbers.

+					

-					

x					

÷					

- Do you see any patterns in the tables?
- Under which operations are the counting numbers closed?

Problem 5 – The rational numbers. A rational number is a number that can be expressed as a fraction, the numerator and the denominator are both integers. Rational numbers are terminating or repeating decimals.

Under which operations are the rational numbers closed?

Closure Worksheet

For each set given below, tell whether the set is closed under (a) addition and (b) multiplication. Complete closure tables if needed. When the set is not closed, give an example to show this.

1. {0}

2. {0,1,2}

3. the odd counting numbers

4. the odd counting numbers

5. $\left\{1,\frac{1}{2},\frac{1}{3},\frac{1}{4},...\right\}$

6. {4,8,12,16,...}

7. the even integers

8. the odd integers

9. the natural numbers divisible by 11

10. the positive real numbers that are less than 1

The following tables define the operations of ❖ and ★ on a set of four elements: {0,1,2, 3}. For example, 1❖ 3 = 2, because 2 is in the box to the right of 1 and below 3 in the ❖ table.

❖	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

★	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	1	3
3	0	3	3	0

11. Is the set closed with respect to ❖? With respect to ★ ?

12. Is ❖ commutative? Is ★ commutative? (Show work to support your answer)

13. What is the multiplicative identity for ❖ ? What is the multiplicative identity for ★?

14. Show that ❖ is associative in the case: (1 ❖2) ❖ 3 = 1 ❖ (2 ❖ 3)

15. Show that ★ is associative in the case: (1 ★ 2) ★ 3 = 1 ★ (2 ★ 3)

Rational Functions: A rational function is the quotient of 2 polynomials. It has the form:

$$f(x)=\frac{P(x)}{Q(x)} \qquad where \;\; Q(x)\neq 0$$

Graphs of rational functions will either be continuous (no jumps, breaks, or holes) or not continuous.

Asymptote: A line that a curve approaches but never reaches.

Vertical Asymptotes and Holes

Points of discontinuity exist for each real zero of Q(x). Each real zero will either be a vertical asymptote or a hole in the graph.

- Common real zeros of P(x) and Q(x) are holes.
- All other real zeros of Q(x) are vertical asymptotes.

Examples:

$y=\frac{x}{x+2}$ V.A. x=-2

$y=\frac{(x+5)(x-2)}{(x-2)}$ hole at x=2

$y=\frac{(x-4)(x-2)}{x(x-3)^2(x-4)}$ hole at x=4, V.A. x=0, x=3

- Remember, vertical lines start with “x=”
- Vertical Asymptotes and Holes are the values of x that make the denominator 0 (what x can’t be).

Horizontal Asymptotes

The graph of a rational function has at most one horizontal asymptote.

- y=0 if degree in denominator > degree in numerator
- y=a/b if degree in denominator = degree in numerator
  a = coefficient of term in numerator with highest degree
  b = coefficient of term in denominator with highest degree
- none if degree in denominator < degree in numerator

Examples:

$y=\frac{x}{x^2+2}$ H.A. is y=0 (degree in den. > degree in num.)

$y=\frac{2x^2+1}{3x^2+2}$ H.A. is y=2/3 (degree in den.=degree in num.)

$y=\frac{2x^3+1}{x+2}$ no H.A. (degree in den. < degree in num.)

- Remember, horizontal lines start with “y=”
- Horizontal asymptotes are the values that y approaches as x –> ∞ or x –> -∞
- Is there any easier way to find them?

Slant Asymptotes

Slant asymptotes occur when the degree of the numerator is exactly one greater than the degree of the denominator.

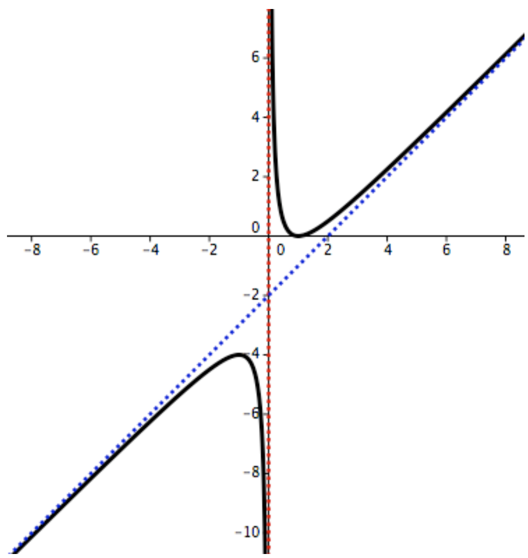
Example: $y = \frac{x^2 - 2x + 1}{x}$

1) Use division to rewrite as a quotient $y = x - 2 + \frac{1}{x}$

2) As x –> ∞ or x –> -∞ , $\frac{1}{x}$ –> 0 and $x - 2 + \frac{1}{x}$ –> $x - 2$.

The line $y = x - 2$ is the slant asymptote.

V.A. x=0
H.A. none



Examples:

Find all asymptotes and holes, state the domain, and find the x and y intercepts.

1. $y = \frac{x}{x^2 - 4}$

2. $y = \frac{x^2 - 9}{x + 3}$

3. $y = \frac{2x^2 - 10}{x^2 + 4}$

4. $y = \frac{x^2 + 3x - 10}{x^2 + 2x - 8}$

5. $y = \frac{x^3 + x}{x^2 - 4}$

6. $y = \frac{3x + 1}{2x - 5}$

MORE…..

7. $f(x) = \frac{5x}{x-4}$

8. $f(x) = \frac{-2x+1}{3x+5}$

9. $f(x) = \frac{3}{x^2+4x}$

10. $h(x) = \frac{x^2-1}{x^2-6x-7}$

11. $g(x) = \frac{x^2-6x+9}{x^2-x-6}$

12. $m(x) = \frac{x^2+6x+8}{x+4}$

13. $f(x) = \frac{12x}{3x^2+1}$

14. $f(x) = \frac{2x^3}{x^2+1}$

15. $f(x) = \frac{x^2-9}{x+2}$

To graph rational functions:

1) find asymptotes and holes
2) plot points (pick x values on either side of V.A.s)

Examples:

1. $y = \dfrac{x}{x^2 - 4}$

2. $y = \dfrac{x^2 - 9}{x + 3}$

3. $y = \dfrac{x^2 + 3x - 10}{x^2 + 2x - 8}$

4. $y = \dfrac{x^3 + x}{x^2 - 4}$

5. $y = \dfrac{3}{(x+1)(x-1)}$

Rational Functions Worksheet 1 - HONORS

List all the asymptotes (i.e. horizontal, vertical, slant), holes, state the domain, find the x-intercept(s), and find the y-intercept.

1. $f(x) = \frac{2x}{x+4}$

2. $h(x) = \frac{x-1}{(2x-1)(x-5)}$

3. $g(x) = \frac{x-2}{x^2+4x+3}$

4. $h(x) = \frac{x^2}{x^2+1}$

5. $f(x) = \frac{(x+1)^2}{x^2-1}$

6. $g(x) = \frac{x^2+3x-3}{x+4}$

7. $f(x) = \frac{x^2+3x-4}{x}$

8. $y = \frac{2x}{2x-8}$

9. $h(x) = \frac{x^2-9}{x-3}$

10. $y = \frac{2}{x-4}$

11. $g(x) = \frac{x^2-6x+9}{x^2-x-6}$

12. $f(x) = \frac{(x-2)^2(x+1)^2}{(x-2)(x-1)}$

Graphing Rational Functions Worksheet 1 - HONORS

Create a function of the form $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x})$ that satisfies each set of conditions.

1. Vertical asymptotes at $x = 4$, hole at $x = 0$

2. Vertical asymptotes at $x = -5$ and $x = 1$, hole at $x = -1$

3. Holes at $x = 3$ and $x = -7$, resembles $y = x$

Graph each function.(Use your own graph paper) First find the asymptotes, x-intercepts, y-intercepts, and holes.

4. $y = \frac{3}{x+2}$

5. $y = \frac{x-5}{x+1}$

26. $y = \frac{(x+2)(x-2)}{x-2}$

7. $y = \frac{x}{x-5}$

8. $y = \frac{-2}{(x-3)^2}$

9. $y = \frac{x^2-x}{x}$

10. $y = \frac{-5}{(x-3)(x+1)}$

11. $y = \frac{x^2+3x-4}{x}$

12. $y = \frac{x}{1-x^2}$

Graphing Rational Functions Worksheet 2 - Honors

Graph each rational function <u>on separate graph paper.</u>

1) $f(x) = \frac{4}{x-5}$

2) $f(x) = \frac{x-1}{x^2+3x-4}$

3) $f(x) = \frac{2}{x^2+3x-10}$

4) $f(x) = \frac{x^2-4x+3}{x^2-x-6}$

5) $f(x) = \frac{x^2+2x-15}{x-3}$

6) $f(x) = \frac{x^2-x-6}{x^2+3x+2}$

7) $y = \frac{1}{x+1}$

8) $y = \frac{1}{x} - 5$

9) $y = \frac{1}{x-2} - 3$

10) $y = \frac{1}{(x+2)^2}$

11) $y = \frac{5}{2x-4} + 1$

12) $y = \frac{3x^2}{x^2-9}$

13) $y = \frac{6x+7}{2x-3}$

Rationals – REVIEW 1
Simplify.

1. $\dfrac{x-5}{2x-6}+\dfrac{x-7}{4x-12}$

2. $\dfrac{x^2-5x+6}{x^2-4}\bullet\dfrac{x^2+3x+2}{x^2-2x-3}$

3. $\dfrac{x}{x-1}-\dfrac{x-1}{x}-\dfrac{1}{x^2-x}$

4. $\dfrac{b^2-25}{(b+5)^2}\div\dfrac{2b+10}{4b+20}$

5. $\dfrac{x^2+3x-10}{x^2+8x+15}\bullet\dfrac{x^2+5x+6}{x^2+4x+4}$

6. $\dfrac{\dfrac{2}{x^2}+4}{\dfrac{2}{x^2}-\dfrac{2}{3x}}$

7. $\dfrac{x}{3x+9}-\dfrac{x+8}{x^2+3x}$

8. $\dfrac{\dfrac{1}{n^2-6n+9}}{\dfrac{n+3}{2n^2-18}}$

Solve.

9 . $\dfrac{2x}{x-3}-\dfrac{1}{2}=\dfrac{2}{2x-6}$

10. $\dfrac{1}{x+3}=\dfrac{2}{x}-\dfrac{3}{4x}$

11. $\dfrac{9}{t+5}-\dfrac{1}{t-5}=\dfrac{3t}{t^2-25}$

12. $\dfrac{5-x}{x-3}=\dfrac{1}{x-3}$

13. $\dfrac{10}{5x-10}=\dfrac{7}{3x-5}$

14. $\dfrac{1}{r-2}+\dfrac{1}{r^2-7r+10}=\dfrac{6}{r-2}$

15. $\dfrac{x+5}{x^2-2x}-1=\dfrac{1}{x^2-2x}$

16. If 2 fractions have denominators x^2+4x+4 and x^2+5x+6, what is the least common denominator?

17. Cark can wash and wax his car in 3 hours while it takes his daughter 6 hours to do the same job. How long would it take them if they worked together?

18. Mary can wash dishes in 15 minutes, but if she is helped by her sister, they can do the job in 10 minutes. How long would it take her sister alone?

19. Pipe A can fill a tank in 4 hours and Pipe B can fill the tank in 3 hours. With the tank empty, Pipe A is turned on and one hour later, Pipe B is turned on. How long will Pipe B run before the tank is full?

20. A painter works on a job for 10 days and is then joined by an associate. Together they finish the job in 6 more days. The associate could have done the job alone in 30 days. How long would it have taken the painter to do the job alone?

ANSWERS

1. $\dfrac{x-17}{4(x-3)}$ **2.** 1 **3.** $\dfrac{2}{x}$ **4.** $\dfrac{2(b-5)}{b+5}$ **5.** $\dfrac{x-2}{x+2}$ **6.** $\dfrac{3(1+2x^2)}{3-x}$ **7.** $\dfrac{x^2-3x-24}{3x(x+3)}$ **8.** $\dfrac{2}{n-3}$

9. x = $-\dfrac{1}{3}$ **10.** x = -15 **11.** t = 10 **12.** x=4 **13.** x = 4 **14.** r = 26/5 **15.** x = 4, -1

16. $(x+2)^2(x+3)$ **17.** 2 hr. **18.** 30 min. **19.** 9/7 hrs **20.** 20 days

Rationals – REVIEW 2

1. Cark can wash and wax his car in 3 hours while it takes his daughter 6 hours to do the same job. How long would it take them if they worked together?

2. Mary can wash dishes in 15 minutes, but if she is helped by her sister, they can do the job in 10 minutes. How long would it take her sister alone?

3. Pipe A can fill a tank in 4 hours and Pipe B can fill the tank in 3 hours. With the tank empty, Pipe A is turned on and one hour later, Pipe B is turned on. How long will Pipe B run before the tank is full?

4. A painter works on a job for 10 days and is then joined by an associate. Together they finish the job in 6 more days. The associate could have done the job alone in 30 days. How long would it have taken the painter to do the job alone?

#5-12 HONORS

5. How many ounces of water must be mixed with 2 ounces of a 30% antifreeze solution to produce a 20% solution?

6. How many ounces of a 75% acid solution must be added to 30 ounces of a 15% acid solution to produce a 50% acid solution?

7. The denominator of a fraction is 1 less than twice the numerator. If 7 is added to both the numerator and denominator, the resulting fraction has a value of $\frac{7}{10}$. Find the original fraction.

8. The ratio of 4 less than a number to 26 more than that number is 1 to 3. What is the number?

9. Five times the multiplicative inverse of a number is added to the number and the result is 10.5. What is the number?

10. How many pounds of a 70% copper alloy must be melted with how many pounds of a 90% copper alloy to obtain 100 pounds of an 81% copper alloy?

11. Amy-Marie can ride 17 km on her bicycle in the same time that it takes her to walk 9 km. If her riding rate is 4 km/h faster than her walking rate, how fast does she walk?

12. Stephanie rows 4 km upstream in twice the time it takes her to row 12 km downstream. If the current in the river flows at a rate of 6 km/h, how fast does Stephanie row in still water?

13. $\frac{18}{x} = \frac{3+3x}{x}$

14. $\frac{x}{x+3} = \frac{2}{5}$

15. $\frac{2x-3}{x-3} - 2 = \frac{12}{x+3}$

16. $\frac{y+3}{y-1} + \frac{y+1}{y-3} = 2$

17. $\frac{1}{x-4} + \frac{2}{x^2-16} = \frac{3}{x+4}$

18. $\frac{3x-4}{x-1} = 2 + \frac{x+4}{x+1}$

19. $\frac{5}{y-1} + \frac{60}{1-y^2} = \frac{10}{y+1}$

20. $\frac{8}{x-1} + \frac{30}{1-x^2} = \frac{6}{x+1}$

Answers: 1. 2 hrs 2. 30 min. 3. $\frac{9}{7}$ *hrs* 4. 20 days 5. 1 oz 6. 42 oz 7. $\frac{7}{13}$ 8. 19 9. 10 or $\frac{1}{2}$

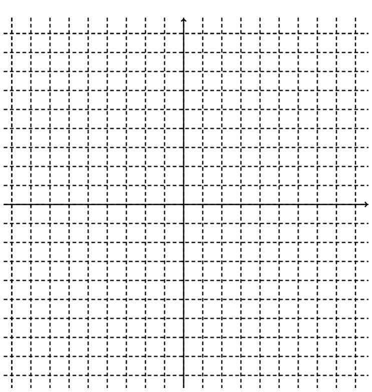
10. 45; 55 11. 4.5 km/h 12. 8.4 km/h 13. 5 14. 2 15. 5 16. 2 17. 9 18. $\frac{1}{2}$ 19. -9 20. 8

21. $f(x) = \frac{-2}{x+3}$

Transformations:

____

V.A. : _____ H.A. : _____

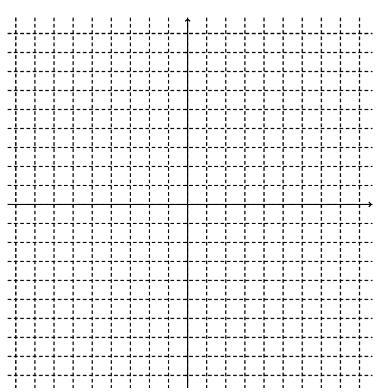


22. $f(x) = \frac{1}{x} + 4$

Transformations:

____

V.A. : _____ H.A. : _____

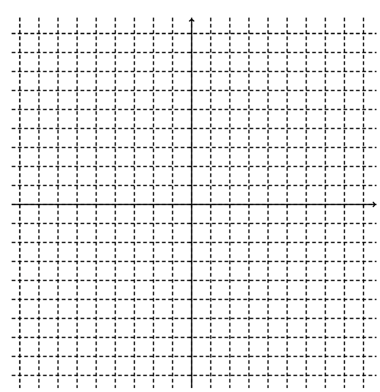


23. $f(x) = \frac{1}{x-1} + 2$

Transformations:

____

V.A. : _____ H.A. : _____



Rationals Review 3

Simplify

1. $\dfrac{x-5}{2x-6}-\dfrac{x-7}{4x-12}$

2. $\dfrac{x+\dfrac{x}{3}}{x-\dfrac{x}{6}}$

3. $\dfrac{x^3-2x^2}{x^4-x^2}\cdot\dfrac{x^2-2x+1}{4x-8}$

4. $\dfrac{\dfrac{2x}{y}+1-\dfrac{y}{x}}{\dfrac{2x}{y}+\dfrac{y}{x}-3}$

5. Solve for a: $\dfrac{1}{a}-\dfrac{1}{b}=c$

6. $\dfrac{x+2}{x-5}+6$

7. $\dfrac{a^3+b^3}{a^2-b^2}\div\dfrac{a^2+2ab+b^2}{a+b}$ **(Honors)**

Solve

8. $\dfrac{2}{r^2-2r}-\dfrac{1}{r}=\dfrac{1}{3}$

9. $\dfrac{5}{3-x}=\dfrac{10}{x+3}-\dfrac{7x+1}{x^2-9}$

#10 – 12 HONORS

10. $\dfrac{a-1}{3a+2}+\dfrac{3a+4}{1-2a}=\dfrac{3a^2-5}{6a^2+a-2}$

Solve for x: 11. $t=\dfrac{x-\pi r^2}{\pi x}$

12. $R=\dfrac{x}{a+\dfrac{x}{t}}$

Answers: 1 $\dfrac{1}{4}$ 2. $\dfrac{8}{5}$ 3. $\dfrac{x-1}{4(x+1)}$ 4. $\dfrac{x+y}{x-y}$ 5. $a=\dfrac{b}{bc+1}$

6. $\dfrac{7(x-4)}{x-5}$ 7. $\dfrac{a^2-ab+b^2}{(a-b)(a+b)}$ 8. $\{-4,3\}$ 9. $\{2\}$ 10. $\{-\dfrac{1}{10},-2\}$

11. $x=\dfrac{-\pi r^2}{\pi t-1}$ or $\dfrac{\pi r^2}{1-\pi t}$ 12. $\dfrac{Rat}{t-r}=x$

Radical Expressions

Squaring a # and finding the square root and are inverse operations.

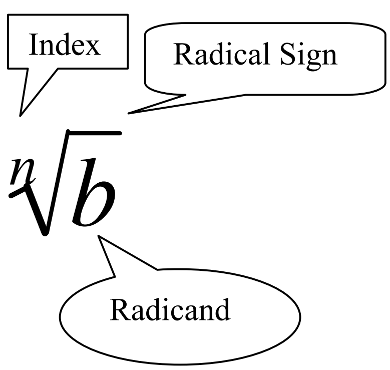
Since $5^2 = 25$ a square root of 25 is 5.
Since $(-5)^2 = 25$, -5 is also a square root.
Meaning if $x^2 = 25$ then $x = 5$ and $x = -5$

Cubing a # and finding the cube root are inverse operations.

Since $2^3 = 8$ a cube root of 8 is 2.
Since $(-2)^3 = -8$ a cube root of -8 is -2.
Meaning if $x^3 = 8$ then $x = 2$
and if $x^3 = -8$ then $x = -2$

If $a^n = b$ then a is an n^{th} root of b. (For n a positive integer.)

Another way to write "**a is an nth root of b**" is $a = \sqrt[n]{b}$



If there is no index it is assumed to be 2 (the square root).

When a number has 2 roots, the radical sign indicates the principal root. The principal root is positive when the index is even. When the index is odd there is only one root.

Examples:

1. $\sqrt{9} = 3$
2. $-\sqrt{9} = -3$
3. $\sqrt{-9} = 3i$
4. $\sqrt[3]{8} = 2$
5. $-\sqrt[3]{8} = -2$
6. $\sqrt[3]{-8} = -2$

The value of an unknown variable might be positive or negative, it's unknown! When the radicand contains variables, absolute value signs may be needed to ensure that the principal root is positive. If variables are assumed to be positive the absolute value signs are not needed

- If n is even then $\sqrt[n]{b^n} = |b|$ (because the principal root is positive)
- If n is odd $\sqrt[n]{b^n} = b$ (because there is only one root)

Examples:

7. $\sqrt{x^3} = |x|\sqrt{x}$ 8. $\sqrt{49m^2t^8} = 7|m|t^4$ 9. $\sqrt{49m^2t^8} =$

10. $\sqrt{49m^2t^8} =$ 11. $\sqrt[3]{80n^5} =$ 12. $\sqrt[3]{80n^5} =$

13. $\sqrt[3]{-27x^6} =$ 14. $\sqrt{y^3 + 8y^2 + 16y} =$ 15. $\sqrt[3]{-27x^6} =$

19. $\sqrt[3]{-27x^6} =$ 20. $\sqrt[3]{-27x^6} =$ 21. $\sqrt[3]{-27x^6} =$

Operations with Radicals

Adding & Subtracting

In order to add/subtract radicals, they must be "like" radicals. "Like" radicals have the same radicand and the same index.

- **The radicand and the index must be the same**.

Note: $\sqrt{9+16} \neq \sqrt{9} + \sqrt{16}$

Examples:

1. $2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$ 2. $7\sqrt[3]{5} - 3\sqrt[3]{5} = 4\sqrt[3]{5}$

3. $7\sqrt{3} - \sqrt{12} = 7\sqrt{3} - 2\sqrt{3} = 5\sqrt{3}$

4. $5\sqrt[3]{x} - 3\sqrt[3]{x} =$ 5. $7\sqrt[4]{5} - 2\sqrt[3]{5} =$ 6. $4\sqrt[3]{81} + 2\sqrt[3]{72} + 3\sqrt[3]{24} =$

Multiplying

If $\sqrt[n]{a}$ *and* $\sqrt[n]{b}$ *are real numbers, then* $\sqrt[n]{a}\cdot\sqrt[n]{b}=\sqrt[n]{ab}$.

- **The index must be the same**.

Examples: $\sqrt{2}\cdot\sqrt{8}=\sqrt{16}=4$

$$\sqrt[3]{-5}\cdot\sqrt[3]{25}=\sqrt[3]{-125}=\sqrt[3]{(-5)^3}=-5$$

Dividing

If $\sqrt[n]{a}$ *and* $\sqrt[n]{b}$ *are real numbers, and* $b\neq 0$ *,then* $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}=\sqrt[n]{\dfrac{a}{b}}$

- **The index must be the same**.

Examples:

1. $\dfrac{\sqrt{36}}{\sqrt{25}}=\sqrt{\dfrac{36}{25}}=\dfrac{6}{5}$

2. $\dfrac{\sqrt[3]{32}}{\sqrt[3]{-4}}=\sqrt[3]{\dfrac{32}{-4}}=\sqrt[3]{-8}=-2$

3. $3\sqrt{7x^3}\bullet 2\sqrt{21x^3y^2}=$

4. $\sqrt[3]{54x^2y^3}\bullet\sqrt[3]{5x^3y^4}=$

5. $\dfrac{\sqrt{12a^2b^5}}{\sqrt{6a^2b}}=$

6. $\dfrac{\sqrt[4]{1024x^{15}}}{\sqrt[4]{4x}}=$

To **rationalize the denominator** means to rewrite the expression so there are no radicals in any denominator, and no denominators in any radical.

Examples:

1. $\dfrac{\sqrt{2}}{\sqrt{3}} = \dfrac{\sqrt{2}\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \dfrac{\sqrt{2\cdot 3}}{\sqrt{3\cdot 3}} = \dfrac{\sqrt{6}}{3}$

2. $\sqrt[3]{\dfrac{2}{5}} = \sqrt[3]{\dfrac{2\cdot 5\cdot 5}{5\cdot 5\cdot 5}} = \sqrt[3]{\dfrac{50}{5^3}} = \dfrac{\sqrt[3]{50}}{\sqrt[3]{5^3}} = \dfrac{\sqrt[3]{50}}{5}$

3. $\dfrac{\sqrt[3]{x}}{\sqrt[3]{2}} =$

4. $\dfrac{\sqrt[3]{2x^3}}{\sqrt[3]{9}}$

5. $\sqrt[4]{\dfrac{3}{4x}}$

Multiplying Binomial Radical Expressions

Multiply radical expressions that are binomials the same way that you multiply binomials.

Examples:

1. $(3+2\sqrt{5})(2+4\sqrt{5}) = 6+12\sqrt{5}+4\sqrt{5}+40$

$= 46+16\sqrt{5}$

2. $\sqrt{3}(\sqrt{6}+7) =$

3. $\left(\sqrt{5}-2\sqrt{15}\right)\left(\sqrt{5}+\sqrt{15}\right) =$

Conjugates

Conjugates are expressions like $\sqrt{m}+\sqrt{n}$ *and* $\sqrt{m}-\sqrt{n}$ that differ only in the sign of the second term.

Remember that $(a+b)(a-b)=a^2-b^2$ (the difference of squares)

Then $\left(\sqrt{m}+\sqrt{n}\right)\left(\sqrt{m}-\sqrt{n}\right)=\left(\sqrt{m}\right)^2-\left(\sqrt{n}\right)^2=m-n$

When m and n are rational numbers, the product is a rational number.

Examples:

1. $\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)=(2)^2-\left(\sqrt{3}\right)^2=4-3=1$
2. $\left(\sqrt{5}+4\right)\left(\sqrt{5}-4\right)=\left(\sqrt{5}\right)^2-(4)^2=5-16=-11$
3. $\left(3\sqrt{2}+\sqrt{7}\right)\left(3\sqrt{2}-\sqrt{7}\right)=\left(3\sqrt{2}\right)^2-\left(\sqrt{7}\right)^2=3^2\left(\sqrt{2}\right)^2-\left(\sqrt{7}\right)^2=18-7=11$

Conjugates can be used to **rationalize binomial radical denominators**.

$$\frac{3+\sqrt{5}}{1-\sqrt{5}}=\frac{3+\sqrt{5}}{1-\sqrt{5}}\infty\frac{1+\sqrt{5}}{1+\sqrt{5}}=\frac{\left(3+\sqrt{5}\right)\left(1+\sqrt{5}\right)}{\left(1-\sqrt{5}\right)\left(1+\sqrt{5}\right)} \quad 4.$$

$$=\frac{3+3\sqrt{5}+\sqrt{5}+(\sqrt{5})^2}{1^2-\left(\sqrt{5}\right)^2}$$

$$=\frac{8+4\sqrt{5}}{-4}=\frac{8}{-4}+\frac{4\sqrt{5}}{-4}=-2-\sqrt{5}$$

5. $\dfrac{2-\sqrt{3}}{4+\sqrt{3}}=$

Properties of Exponents

- Anything to the zero power is equal to 1 $a^0 = 1$
- When you multiply with the same base, keep the base and add the exponents $a^m \bullet a^n = a^{m+n}$
- Power to a power, keep the base and multiply the exponents $(a^m)^n = a^{mn}$

> An exponent is distributed to everything inside ( )
>
> $(a^m b^n)^p = a^{mp} b^{np}$ $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
>
> ***BUT** Remember – you can't distribute an exponent across addition or subtraction (you have to FOIL)*
>
> $(a + b)^2 \neq a^2 + b^2$
>
> $(a + b)^2 = (a + b)(a + b)$

- When you divide with the same base, keep the base and subtract the exponents $\frac{a^m}{a^n} = a^{m-n}$
- To change the sign of an exponent change the position (numerator <-> denominator)

$$a^{-n} = \frac{1}{a^n} \quad \text{-or -} \quad \frac{a^{-m}}{a^{-n}} = \frac{a^n}{a^m}$$

- Switch between exponential and radical forms.

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

Rational Exponents

Radical Form		Exponential Form
$\sqrt[b]{a^c}$	$\Longleftrightarrow$	$a^{\frac{c}{b}}$

$\sqrt{x} \quad \Longleftrightarrow$

$\Longleftrightarrow \quad x^{\frac{5}{3}}$

$2\sqrt[4]{2z^3n} \quad \Longleftrightarrow$

Fractional exponents must have the same denominator to be able to write as a single radical!

$$3^{\frac{2}{3}}x^{\frac{1}{2}}y^{\frac{5}{6}} = 3^{\frac{4}{6}}x^{\frac{3}{6}}y^{\frac{5}{6}} = \sqrt[6]{3^4x^3y^5}$$

$$4^{\frac{2}{5}}x^{\frac{1}{2}}y^{\frac{7}{10}} =$$

Simplifying Radicals:
(You are finished when these conditions are met.)

1. The radicand cannot contain a factor that is a perfect root of the index.
2. The radicand cannot be a fraction
3. There cannot be a radical in the denominator.
4. The index is as small as possible.

To make the index as small as possible:

1. Write all radicals in exponential form. ($\sqrt[b]{a^c} = a^{\frac{c}{b}}$)
2. Change all bases (big number) to the smallest base to a power.

   Ex. $9 = 3^2$ $\quad 16 = 4^2 = (2^2)^2 = 2^4$ $\quad 729 = 27^2 = (3^3)^2 = 3^6$
3. Multiply powers.
4. Simplify fraction.
5. Write in new radical form.

Examples:

$\sqrt[6]{36} = 36^{(1/6)} = (6^2)^{(1/6)} = 6^{(2/6)} = 6^{(1/3)} = \sqrt[3]{6}$

$\sqrt[10]{81} = 81^{(1/10)} = (3^4)^{(1/10)} = 3^{(4/10)} = 3^{(2/5)} = \sqrt[5]{3^2} = \sqrt[5]{9}$

Simplest Exponential Form:
(You are finished when these conditions are met.)

1. There are no negative exponents.
2. There are no rational exponents in the denominator.
3. There are no radicals.

Simplest Radical & Exponential Forms:

Write in simplest exponential form:

1. $\sqrt[3]{x}$

2. $\sqrt[6]{5}$

3. $\sqrt[4]{x^3}$

4. $\sqrt[3]{y^2z^6}$

5. $\sqrt[4]{7^3x^2y^7z}$

6. $3\sqrt[5]{3a^2c^9}$

7. $4\sqrt[3]{2x^2y^7}$

Write as a single radical in simplest form.

8. $a^{\frac{1}{4}}$

9. $3^{\frac{1}{2}}$

10. $x^{\frac{4}{5}}$

11. $x^{\frac{3}{2}}y^{\frac{5}{2}}$

12. $3^{\frac{2}{3}}x^{\frac{1}{2}}p^{\frac{1}{6}}$

13. $a^{\frac{3}{2}}b^{\frac{1}{4}}$

14. $2^{\frac{1}{3}}x^{\frac{3}{4}}y^{\frac{7}{6}}$

Write in simplest radical form:
(The index should be as small as possible.)

15. $\sqrt[4]{x^2}$

16. $\sqrt[8]{16}$

17. $\sqrt[6]{8}$

18. $\sqrt[8]{64}$

19. $\sqrt[6]{81}$

29. $y^{\frac{8}{3}}$

Simplify. If not perfect, leave in simplest radical form.

21. $81^{\frac{3}{4}}$

22. $\left(36^{\frac{5}{3}}\right)^{\frac{3}{10}}$

23. $36^{\frac{3}{2}}$

24. $64^{\frac{5}{6}}$

Solving Equations Containing Radicals

Sometimes an equation contains a radical and there are no variables under the radical. Treat the radical like a number (because it is just a number) and solve the equation.

Example 1:

$$2x\sqrt{5}-3=0$$

$$2x\sqrt{5}=3$$

$$x=\frac{3}{2\sqrt{5}}$$

$$x=\frac{3}{2\sqrt{5}}\bullet\frac{\sqrt{5}}{\sqrt{5}}=\frac{3\sqrt{5}}{10}$$

Example 2:

$$2x-3x\sqrt{5}=4$$

$$x\left(2-3\sqrt{5}\right)=4$$

$$x=\frac{4}{2-3\sqrt{5}}$$

$$x=\frac{4}{2-3\sqrt{5}}\bullet\frac{2+3\sqrt{5}}{2+3\sqrt{5}}=\frac{8+12\sqrt{5}}{-41}$$

Practice:

$$5x\sqrt{3}-4=2$$

$$3x-x\sqrt{2}=5$$

Sometimes an equation contains a radical with variables under the radical. This is called a ***radical equation.*** To solve a radical equation, isolate the radical and take it out! (by raising to a power). If there is more than one radical with a variable, isolate and take out each radical one at a time. When solving radical equations you **MUST check you solutions** to make sure they work.

Example 3: $3\sqrt{2x-1}=15$

1. $\sqrt{2x-1}=5$

2. $\left(\sqrt{2x-1}\right)^2=(5)^2$

3. $2x-1=25$

4. $x=13$

5. $3\sqrt{2(13)-1}=15$

$3\sqrt{25}=15$

$3(5)=15$

$15=15$

It Checks! **The solution is {13}**

Steps to solve:
1. Isolate the radical
2. Raise both sides of the equation to a power equal to the index
3. Simplify
4. Solve for the variable
5. CHECK answer!!!

Example 4: $\sqrt{x+6}-\sqrt{x}=\sqrt{2}$

1. $\sqrt{x+6}=\sqrt{2}+\sqrt{x}$

2. $\left(\sqrt{x+6}\right)^2=\left(\sqrt{2}+\sqrt{x}\right)^2$

3. $x+6=2+2\sqrt{2x}+x$

4. $2=\sqrt{2x}$

5. $2^2=\left(\sqrt{2x}\right)^2$

6. $4=2x$

7. $x=2$

8. $\sqrt{x+6}-\sqrt{x}=\sqrt{2}$

$\sqrt{2+6}-\sqrt{2}=\sqrt{2}$

$\sqrt{8}-\sqrt{2}=\sqrt{2}$

$2\sqrt{2}-\sqrt{2}=\sqrt{2}$

$\sqrt{2}=\sqrt{2}$

It Checks! **The solution is {2}**

Steps to solve:
1. Isolate 1 of the radicals
2. Raise both sides of the equation to a power equal to the index
3. Simplify
4. Isolate the second radical
5. Raise both sides of the equation to a power equal to the index.
6. Simplify
7. Solve for the variable
8. CHECK answer!!!

Example 5: $5+\sqrt{x-4}=2$

1. $\sqrt{x-4}=-3$
2. $\left(\sqrt{x-4}\right)^2=(-3)^2$
3. $x-4=9$
4. $x=13$
5. $5+\sqrt{13-4}=2$

$5+\sqrt{9}=2$

$8\neq 2$

It didn't work! **Solution: {No Real Solution} or** $\varnothing$

Another way to solve a radical equation is to graph on your calculator. For Example 5 above you would enter in your calculator:

$Y_1=5+\sqrt{x-4}$

$Y_2=2$

Adjust the window as needed and you should see the graphs never intersect, which confirms there is no real solution.

Sometimes an equation has rational exponents. This is just a radical equation in disguise! Change to radical form and solve like a radical equation.

Example 6: $2(7n-1)^{1/3}-4=0$

1. Change to radical form:

$2\sqrt[3]{7n-1}-4=0$

2. $\sqrt[3]{7n-1}=2$

3. $\left(\sqrt[3]{7n-1}\right)^3=(2)^3$

4. $7n-1=8$

5. $n=\dfrac{9}{7}$

Solution: $\left\{\dfrac{9}{7}\right\}$

6. Check:

$2(7n-1)^{1/3}-4=0$

$2\left(7\left(\frac{9}{7}\right)-1\right)^{\frac{1}{3}}-4=0$

$2(9-1)^{\frac{1}{3}}-4=0$

$2(8)^{\frac{1}{3}}-4=0$

$2(2)-4=0$

$4-4=0$

$0=0$

It checks!

Using nth roots

Use ± when you introduce a radical with an even index or raise to a power with an even denominator.

Example 7:

$$5y^4 = 80$$

$$y^4 = 16$$

$$\sqrt[4]{y^4} = \pm\sqrt[4]{16}$$

$$y = \pm 2$$

Check:

$5(+2)^4 = 5 \bullet 16 = 80$

$5(-2)^4 = 5 \bullet 16 = 80$

Example 8:

$$(x-4)^4 = 81$$

$$\left((x-4)^4\right)^{\frac{1}{4}} = \pm(81)^{\frac{1}{4}}$$

$$x-4 = \pm(3^4)^{\frac{1}{4}}$$

$$x-4 = \pm 3$$

$$x = 4 \pm 3$$

$$x = 7,1$$

Check:

$(7-4)^4 = 3^4 = 81$

$(1-4)^4 = (-3)^4 = 81$

Practice:

1. $\sqrt{2x+8} - 4 = 6$
2. $\sqrt{2x+3} = x$
3. $\sqrt[3]{3x} + 6 = 10$
4. $x - 3 = \sqrt{4x}$
5. $\sqrt[5]{2x+1} + 5 = 9$
6. $6\sqrt{x} - \sqrt{x-1} = 0$
7. $3x^{\frac{1}{4}} = 4$
8. $2x^{\frac{3}{2}} = 250$
9. $x^{\frac{4}{3}} + 9 = 25$
10. $(x+9)^{\frac{5}{2}} - 1 = 31$
11. $5(x-8)^{\frac{3}{4}} = 40$
12. $3(x-1)^{\frac{2}{3}} + 4 = 52$
13. $2(x-5)^4 = 32$

Square Root Worksheet
Simplest radical form

1. $\sqrt{\frac{2}{7}}$

2. $\frac{\sqrt{7}}{\sqrt{3}}$

3. $\frac{3\sqrt{10}}{6\sqrt{2}}$

4. $\sqrt{84}$

5. $-5\sqrt{28x^3y^8z^{10}}$

6. $\sqrt{150}-3\sqrt{24}+4\sqrt{54}$

7. $\left(2\sqrt{14ab^3}\right)\left(-5\sqrt{7a^3b}\right)$

8. $(3+\sqrt{5})(3-\sqrt{5})$

9. $\left(2+\sqrt{7}\right)^2$

10. $\sqrt{72}-\sqrt{\frac{1}{2}}+\sqrt{\frac{9}{2}}$

11. $\frac{2}{3-\sqrt{2}}$

12. $\left(2\sqrt{3}\right)^2$

13. $(5-\sqrt{10})(6+2\sqrt{2})$

14. $\sqrt{\frac{5}{8}}$

15. $\sqrt{3}(6+\sqrt{12})$

16. $(4\sqrt{5}+3)^2$

17. $(3-\sqrt{x+5})^2$

18. $\sqrt{200}-\frac{2}{3}\sqrt{162}$

19. $(7+3\sqrt{5})(7-3\sqrt{5})$

20. $\frac{2-\sqrt{7}}{3+\sqrt{6}}$

Answers

1. $\frac{\sqrt{14}}{7}$
2. $\frac{\sqrt{21}}{3}$
3. $\frac{\sqrt{5}}{2}$
4. $2\sqrt{21}$
5. $-10xy^4|z^5|\sqrt{7x}$
6. $11\sqrt{6}$
7. $-70a^2b^2\ \sqrt{2}$
8. 4
9. $11+4\sqrt{7}$
10. $7\sqrt{2}$
11. $\frac{6+2\sqrt{2}}{7}$
12. 12
13. $30-6\sqrt{10}+10\sqrt{2}-4\sqrt{5}$
14. $\frac{\sqrt{10}}{4}$
15. $6\sqrt{3}+6$
16. $89+24\sqrt{5}$
17. $14-6\sqrt{x+5}+x$
18. $4\sqrt{2}$
19. 4
20. $\frac{6-2\sqrt{6}-3\sqrt{7}+\sqrt{42}}{3}$

Simplifying Radicals Worksheet 0

Simplify the following over the complex numbers.

1. $\pm\sqrt{81}$

2. $\sqrt{196}$

3. $\sqrt{256}$

4. $\sqrt[4]{81}$

5. $\sqrt[3]{-216}$

6. $\sqrt[3]{-1000}$

7. $-\sqrt{0.49}$

8. $\sqrt{121n^2}$

9. $\sqrt{25y^6}$

10. $\sqrt{576}$

11. $-\sqrt{144a^2b^4}$

12. $\sqrt[3]{-8b^3c^3}$

13. $\pm\sqrt[3]{27r^3s^3}$

14. $\sqrt[3]{64a^6b^3}$

15. $\sqrt[4]{625n^8m^4}$

16. $\sqrt{(3s)^4}$

17. $\sqrt{(a+b)^2}$

18. $\sqrt{(3x+y)^2}$

19. $\sqrt[3]{2x-y)^3}$

20. $\sqrt{x^2+10x+25}$

21. $\sqrt{x^2+6x+9}$

22. $\sqrt{9a^2+6a+1}$

23. $\sqrt{4y^2+12y+9}$

24. $\sqrt{-9}$

25. $\sqrt{-48}$

26. $\sqrt{-150}$

27. $\sqrt{-400}$

28. $\sqrt[3]{-64}$

29. $5\sqrt{-12}$

30. $-3\sqrt{-72}$

Radical Expressions Practice 1 - Simplify

1. $\sqrt{81}$

2. $\sqrt[3]{-343}$

3. $\sqrt{144p^6}$

4. $\pm\sqrt{4a^{10}}$

5. $\sqrt[5]{243p^{10}}$

6. $-\sqrt[3]{m^6n^9}$

7. $\sqrt[3]{-b^{12}}$

8. $\sqrt{16a^{10}b^8}$

9. $\sqrt{121x^6}$

10. $\sqrt{(4k)^4}$

11. $\pm\sqrt{169r^4}$

12. $-\sqrt[3]{-27p^6}$

13. $-\sqrt{625y^2z^4}$

14. $\sqrt{36q^{34}}$

15. $\sqrt{100x^2y^4z^6}$

16. $\sqrt[3]{-0.027}$

17. $-\sqrt{-0.36}$

18. $\sqrt{0.64p^{10}}$

19. $\sqrt[4]{(2x)^8}$

20. $\sqrt{(11y^2)^4}$

21. $\sqrt[3]{(5a^2b)^6}$

22. $\sqrt{(3x - 1)^2}$

23. $\sqrt[3]{(m - 5)^6}$

24. $\sqrt{36x^2 - 12x + 1}$

Radical Expressions Practice 2 – Simplify

1. $3\sqrt{2} + \sqrt{50} - 4\sqrt{8}$

2. $\sqrt{20} + \sqrt{125} - \sqrt{45}$

3. $\sqrt{300} - \sqrt{27} - \sqrt{75}$

4. $\sqrt[3]{81} \cdot \sqrt[3]{24}$

5. $\sqrt[3]{2}(\sqrt[3]{4} + \sqrt[3]{12})$

6. $2\sqrt{3}(\sqrt{15} + \sqrt{60})$

7. $(2 + 3\sqrt{7})(4 + \sqrt{7})$

8. $(6\sqrt{3} - 4\sqrt{2})(3\sqrt{3} + \sqrt{2})$

9. $(4\sqrt{2} - 3\sqrt{5})(2\sqrt{20} + 5)$

10. $\dfrac{5\sqrt{48} + \sqrt{75}}{5\sqrt{3}}$

11. $\dfrac{4 + \sqrt{2}}{2 - \sqrt{2}}$

12. $\dfrac{5 - 3\sqrt{3}}{1 + 2\sqrt{3}}$

Radical Expressions Practice 3 - Simplify

1. $5\sqrt{54}$

2. $\sqrt[4]{32a^9b^{20}}$

3. $\sqrt{75x^4y^7}$

4. $\sqrt{\frac{36}{125}}$

5. $\sqrt{\frac{a^6b^3}{98}}$

6. $\sqrt[3]{\frac{p^5q^3}{40}}$

Radical Expressions Practice 4 – Simplify

1. $\sqrt{540}$

2. $\sqrt[3]{-432}$

3. $\sqrt[3]{128}$

4. $-\sqrt[4]{405}$

5. $\sqrt[3]{-5000}$

6. $\sqrt[5]{-1215}$

7. $\sqrt[3]{125t^6w^2}$

8. $\sqrt[4]{48v^8z^{13}}$

9. $\sqrt[3]{8g^3k^8}$

10. $\sqrt{45x^3y^8}$

11. $\sqrt{\frac{11}{9}}$

12. $\sqrt[3]{\frac{216}{24}}$

13. $\sqrt{\frac{1}{128}c^4d^7}$

14. $\sqrt{\frac{9a^5}{64b^4}}$

15. $\sqrt[4]{\frac{8}{9a^3}}$

16. $(3\sqrt{15})(-4\sqrt{45})$

17. $(2\sqrt{24})(7\sqrt{18})$

18. $\sqrt{810} + \sqrt{240} - \sqrt{250}$

19. $6\sqrt{20} + 8\sqrt{5} - 5\sqrt{45}$

20. $8\sqrt{48} - 6\sqrt{75} + 7\sqrt{80}$

21. $(3\sqrt{2} + 2\sqrt{3})^2$

22. $(3 - \sqrt{7})^2$

23. $(\sqrt{5} - \sqrt{6})(\sqrt{5} + \sqrt{2})$

24. $(\sqrt{2} + \sqrt{10})(\sqrt{2} - \sqrt{10})$

25. $(1 + \sqrt{6})(5 - \sqrt{7})$

26. $(\sqrt{3} + 4\sqrt{7})^2$

27. $(\sqrt{108} - 6\sqrt{3})^2$

28. $\frac{\sqrt{3}}{\sqrt{5} - 2}$

29. $\frac{6}{\sqrt{2} - 1}$

30. $\frac{5 + \sqrt{3}}{4 + \sqrt{3}}$

31. $\frac{3 + \sqrt{2}}{2 - \sqrt{2}}$

32. $\frac{3 + \sqrt{6}}{5 - \sqrt{24}}$

33. $\frac{3 + \sqrt{x}}{2 - \sqrt{x}}$

Radical Equations Practice 5 – Solve

1. $3 + 2x\sqrt{3} = 5$ **2.** $2\sqrt{3x + 4} + 1 = 15$ **3.** $8 + \sqrt{x + 1} = 2$

4. $\sqrt{5 - x} - 4 = 6$ **5.** $12 + \sqrt{2x - 1} = 4$ **6.** $\sqrt{12 - x} = 0$

7. $\sqrt{21} - \sqrt{5x - 4} = 0$ **8.** $10 - \sqrt{2x} = 5$ **9.** $\sqrt{x^2 + 7x} = \sqrt{7x - 9}$

10. $4\sqrt[3]{2x + 11} - 2 = 10$ **11.** $2\sqrt{x + 11} = \sqrt{x + 2} + \sqrt{3x - 6}$ **12.** $\sqrt{9x - 11} = x + 1$

Rational Exponents Practice 6

Write each expression in radical form.

1. $11^{\frac{1}{7}}$ **2.** $15^{\frac{1}{3}}$ **3.** $300^{\frac{3}{2}}$

Write each radical using rational exponents.

4. $\sqrt{47}$ **5.** $\sqrt[3]{3a^5b^2}$ **6.** $\sqrt[4]{162p^5}$

Evaluate each expression.

7. $-27^{\frac{2}{3}}$ **8.** $\dfrac{5^{-\frac{1}{2}}}{2\sqrt{5}}$ **9.** $(0.0004)^{\frac{1}{2}}$

10. $8^{\frac{2}{3}} \cdot 4^{\frac{3}{2}}$ **11.** $\dfrac{144^{-\frac{1}{2}}}{27^{-\frac{1}{3}}}$ **12.** $\dfrac{16^{-\frac{1}{2}}}{(0.25)^{\frac{1}{2}}}$

Rational Exponents Practice 7 – Simplify

1. $x^{\frac{4}{5}} \cdot x^{\frac{6}{5}}$

2. $\left(y^{\frac{2}{3}}\right)^{\frac{3}{4}}$

3. $p^{\frac{4}{5}} \cdot p^{\frac{7}{10}}$

4. $\left(m^{-\frac{6}{5}}\right)^{\frac{2}{5}}$

5. $x^{-\frac{3}{8}} \cdot x^{\frac{4}{3}}$

6. $\left(s^{-\frac{1}{6}}\right)^{-\frac{4}{3}}$

7. $\dfrac{p}{p^{\frac{1}{3}}}$

8. $\left(a^{\frac{2}{3}}\right)^{\frac{6}{5}} \cdot \left(a^{\frac{2}{5}}\right)^{3}$

9. $\dfrac{x^{-\frac{1}{2}}}{x^{-\frac{1}{3}}}$

10. $\sqrt[6]{128}$

11. $\sqrt[4]{49}$

12. $\sqrt[5]{288}$

13. $\sqrt{32} \cdot 3\sqrt{16}$

14. $\sqrt[3]{25} \cdot \sqrt{125}$

15. $\sqrt[6]{16}$

16. $\dfrac{x - \sqrt[3]{3}}{\sqrt{12}}$

17. $\sqrt{\sqrt[3]{48}}$

18. $\dfrac{a\sqrt[3]{b^4}}{\sqrt{ab^3}}$

6 – 1 Simplifying Radicals Worksheet 1

Simplify.

1. $\sqrt[5]{32}$

2. $-\sqrt[4]{256}$

3. $\sqrt{x^2 + 10x + 25}$

4. $\sqrt[6]{(m+4)^6}$

5. $\sqrt[3]{-64r^6w^{15}}$

6. $\sqrt{49m^2t^8}$

7. $\sqrt[4]{81}$

8. $\sqrt[3]{-64}$

9. $\sqrt{(2x)^8}$

10. $-\sqrt[4]{625}$

11. $\sqrt[3]{216}$

12. $\sqrt{676x^4y^6}$

13. $\sqrt[3]{(2x+1)^3}$

14. $\sqrt[5]{-32x^5y^{10}}$

15. $-\sqrt{144m^8n^6}$

16. $\sqrt[3]{-27x^9y^{12}}$

17. $\sqrt[5]{243x^{10}}$

18. $-\sqrt{49a^{10}b^{16}}$

19. $\sqrt[4]{(x-5)^8}$

20. $\sqrt[3]{343d^6}$

21. $\sqrt{.81}$

22. $-\sqrt{0.0016}$

23. $\sqrt[3]{0.512}$

24. $-\sqrt[4]{0.6561}$

6 – 2 Simplifying Radicals Worksheet 2

Simplify.

1. $\sqrt[3]{-432}$

2. $\sqrt{540}$

3. $\sqrt{5}\,(\sqrt{10}-\sqrt{45})$

4. $\sqrt[3]{6}\,(4\sqrt[3]{12}+5\sqrt[3]{9})$

5. $\left(2\sqrt[3]{24}\right)\left(7\sqrt[3]{18}\right)$

6. $\sqrt[4]{32\boldsymbol{x}^4\boldsymbol{y}^5\boldsymbol{n}^{10}}$

7. $\sqrt{1792}$

8. $\sqrt[3]{-6750}$

9. $\sqrt{3\boldsymbol{x}^2\boldsymbol{y}^3}\cdot\sqrt{75\boldsymbol{xy}^5}$

10. $\sqrt[3]{9\boldsymbol{t}^5\boldsymbol{v}^8}\cdot\sqrt[3]{6\,\boldsymbol{t}\,\boldsymbol{v}^4}$

11. $\sqrt{60}\cdot\sqrt{105}$

12. $\sqrt[3]{3600}\cdot\sqrt[3]{165}$

13. $\dfrac{\sqrt{35}}{\sqrt{7}}$

14. $\dfrac{\sqrt[4]{42}}{\sqrt[4]{7}}$

15. $\sqrt{\dfrac{3}{5}}$

16. $\sqrt{\dfrac{6}{\boldsymbol{w}}}$

17. $\sqrt[4]{\dfrac{5}{27}}$

18. $\sqrt[4]{\dfrac{8}{9\boldsymbol{a}^3}}$

19. $\dfrac{\sqrt{20}}{\sqrt{5}}$

20. $\sqrt{\dfrac{11}{9}}$

21. $\sqrt[3]{\dfrac{2}{9}}$

22. $\sqrt[3]{\dfrac{9}{25}}$

23. $\dfrac{\sqrt[3]{16}}{\sqrt[3]{4}}$

24. $\dfrac{\sqrt[3]{9}}{\sqrt[3]{4}}$

6 – 3 Computing with Radicals Worksheet

1. $4\sqrt{24}+\sqrt{18}-5\sqrt{54}-4\sqrt{450}$

2. $\sqrt{45}-\left(\sqrt{5}\right)^2+\sqrt{180}$

3. $\sqrt[3]{56}+\sqrt[3]{24}-\sqrt{28}$

4. $9\sqrt[4]{5}-\sqrt[4]{5}+11\sqrt[4]{5}$

5. $\sqrt[4]{x^8}+2\sqrt[3]{x^6}-\sqrt{x^2}+\sqrt[3]{x^3}$

6. $\sqrt{75v^5t^3}-\sqrt{48v^3t^7}$

7. $\left(6-\sqrt{3}\right)^2$

8. $\left(4\sqrt{7}+5\sqrt{2}\right)\left(2\sqrt{7}-3\sqrt{2}\right)$

9. $\left(x^2+\sqrt[3]{a^2}\right)\left(x^4-\sqrt[3]{a^2}x^2+\sqrt[3]{a^4}\right)$

10. $\left(6-\sqrt[3]{4}\right)\left(\sqrt[3]{32}+\sqrt[3]{16}\right)$

11. $2\sqrt{48}-\sqrt{12}-3\sqrt{63}+\sqrt{112}$

12. $\sqrt[3]{108}+2\sqrt[3]{32}+2\sqrt[3]{500}-2\sqrt[3]{4}$

13. $\sqrt{810}+\sqrt{240}+\sqrt{135}-\sqrt{250}$

14. $\sqrt[3]{216}-\sqrt[3]{48}+\sqrt[3]{432}$

15. $\left(\sqrt{12}-2\sqrt{3}\right)^2$

16. $\left(\sqrt{18}+2\sqrt{3}\right)^2$

17. $\left(\sqrt{5}-\sqrt{6}\right)\left(\sqrt{5}+\sqrt{2}\right)$

18. $\left(\sqrt{50}+\sqrt{27}\right)\left(\sqrt{2}-\sqrt{6}\right)$

19. $\dfrac{3}{2-\sqrt{5}}$

20. $\dfrac{6}{\sqrt{2}-1}$

21. $\dfrac{5+\sqrt{3}}{4+\sqrt{3}}$

22. $\dfrac{6}{2-\sqrt{7}}$

23. $\sqrt[3]{144}+\sqrt[3]{\dfrac{2}{3}}-5\sqrt[3]{18}$

24. $\sqrt{\dfrac{3}{8}}+\sqrt{54}-\sqrt{6}$

6 – 4 Rational Exponent Worksheet 1

Express using rational exponents.

1. $\sqrt[3]{26}$
2. $\sqrt[5]{8}$
3. $\sqrt{36x^5y^6}$
4. $\sqrt[7]{y^3}$
5. $\sqrt[10]{x^6}$
6. $\sqrt[3]{28x^2y^3t^{11}}$
7. $3\sqrt[4]{27n^{10}w}$
8. $2\sqrt[4]{2z^{\frac{1}{2}}n^{\frac{6}{5}}}$
9. $4\sqrt{2a^{10}b^3}$
10. $\sqrt[3]{27m^6n^4}$

Express in simplest radical form. (one radical per answer!)

11. $x^{\frac{3}{5}}$
12. $27^{\frac{1}{6}}$
13. $2^{\frac{5}{7}}a^{\frac{3}{7}}y^{\frac{9}{7}}$
14. $(3w)^{\frac{2}{3}}m^{\frac{7}{3}}$
15. $a^{\frac{2}{3}}g^{\frac{1}{4}}e^{\frac{1}{2}}$
16. $w^{\frac{3}{7}}n^{\frac{5}{3}}$
17. $\sqrt[6]{36}$
18. $\sqrt[10]{81}$
19. $m^{\frac{1}{3}}v^{\frac{3}{4}}z^{\frac{5}{6}}$
20. $27^{\frac{1}{2}}b^{\frac{2}{3}}c^{\frac{7}{6}}$

Evaluate each expression WITHOUT a calculator. (then use calc. to check your answer)

21. $32^{\frac{2}{5}}$
22. $(25^{\frac{2}{3}})^{\frac{3}{4}}$
23. $(0.216)^{\frac{1}{3}}$
24. $\left(\frac{27}{125}\right)^{\frac{2}{3}}$
25. $\sqrt[4]{2401}$
26. $36^{2.5}$

6 – 6 Rational Exponent Worksheet 2

Express in Simplest Exponential Form.

(No negative exponents, no rational exponents in the denominator, no radicals)

1. $y^{-\frac{1}{2}}$

2. $b^{-\frac{3}{5}}$

3. $v^{-\frac{4}{9}}$

4. $t^{-\frac{3}{2}}$

5. $\frac{1}{w^{\frac{4}{5}}}$

6. $\frac{1}{b^{\frac{4}{7}}}$

7. $\frac{2}{4^{\frac{2}{3}}}$

8. $\frac{3}{6^{\frac{1}{2}}}$

9. $\frac{14}{7^{\frac{2}{3}}}$

10. $\frac{12}{3^{\frac{5}{2}}}$

11. $x^{-\frac{3}{5}}$

12. $\frac{r^2 t^3}{\sqrt[4]{a^3}}$

13. $\frac{m^{\frac{7}{4}} + 3m^{-\frac{1}{4}}}{m^{\frac{3}{4}}}$

14. $\frac{7y^{\frac{4}{5}} + y^{\frac{6}{5}}}{y^{-\frac{1}{5}}}$

15. $(w^{-\frac{3}{8}})^{-\frac{4}{9}}$

16. $\left(\sqrt[8]{11}x^{\frac{3}{4}}y^{-\frac{1}{2}}\right)^4$

17. $\frac{x^{\frac{3}{2}}}{x^{\frac{1}{2}} + y^{\frac{1}{2}}}$

18. $\frac{2 - n^{\frac{1}{2}}}{6 - n^{\frac{1}{2}}}$

19. $\frac{r^{-\frac{3}{4}}y^{-\frac{3}{2}}}{\sqrt{yr^{-\frac{1}{2}}}}$

20. $\left(\frac{n^{-\frac{4}{5}}}{x^{-10}n^{\frac{2}{5}}}\right)^{-5}$

6-7 Solving Equations Containing Radicals Worksheet

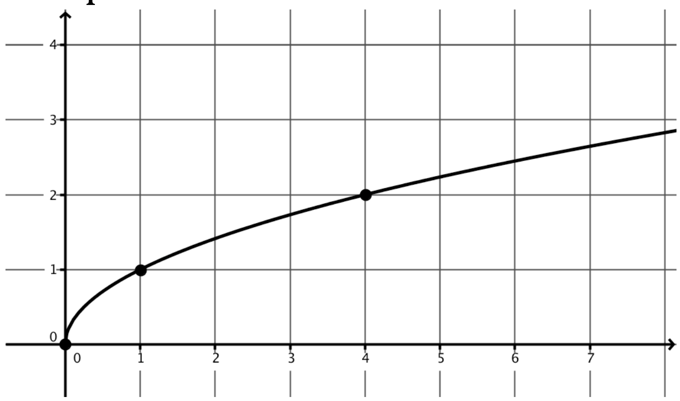
Solve each equation. Check for extraneous solutions!)

1. $7x\sqrt{3}-5=0$
2. $4x-x\sqrt{3}=6$
3. $18-3x=x\sqrt{2}$
4. $\sqrt{x+8}-5=0$
5. $\sqrt[3]{y-7}=4$
6. $\sqrt[4]{3x}-2=0$
7. $\sqrt{8n-5}-1=2$
8. $\sqrt{1-4t}-8=-6$
9. $\sqrt[4]{7v-2}+12=7$
10. $\sqrt[3]{6u-5}+2=-3$
11. $\sqrt{6x-4}=\sqrt{2x+10}$
12. $5(x-8)^{\frac{3}{4}}=40$
13. $\sqrt{k+9}-\sqrt{k}=\sqrt{3}$
14. $\sqrt{x+10}+\sqrt{x-6}=8$
15. $\sqrt{x+2}-7=\sqrt{x+9}$
16. $\sqrt{4x^2-3x+2}-2x-5=0$

Graphing Radical Functions

Square Roots

The parent function is $y = \sqrt{x}$



Notice the pattern:
- over 1 up 1
- over 4 up 2

$$y = a\sqrt{x - h} + k$$

- h shifts the graph horizontally
- k shifts the graph vertically
- a vertically stretches or shrinks the graph

Graph the following: State the domain and range of each.

1. $y = \sqrt{x}$

2. $y = \sqrt{x+1} - 3$

3. $y = 2\sqrt{x}$

4. $y = -\sqrt{x}$

5. $y = \sqrt{x-3} + 2$

6. $y = 2\sqrt{x+4} - 1$

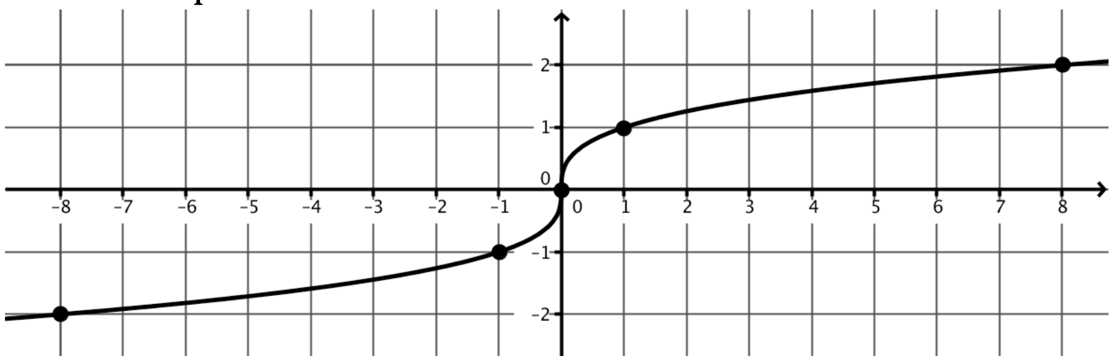
7. $y = -3\sqrt{x-2} + 1$

8. $y = \sqrt{x} - 6$

Graphing Radical Functions **HONORS**

Cube Roots

The parent function is $y = \sqrt[3]{x}$



Notice the pattern:
- right 1 up 1
- right 8 up 2
- left 1 down 1
- left 8 down 2

$y = a\sqrt[3]{x-h} + k$

- h shifts the graph horizontally
- k shifts the graph vertically
- a vertically stretches or shrinks the graph

Graph the following: Tell the domain and range of each.

1. $y = \sqrt[3]{x}$

2. $y = \sqrt[3]{x+2} - 1$

3. $y = 3\sqrt[3]{x-2} + 1$

4. $y = -2\sqrt[3]{x-1} + 1$

5. $y = -\frac{1}{2}\sqrt[3]{x+3} + 1$

Review Operations with Radicals

Perform the indicated operation and write radicals in simplest form.

1) $\sqrt{12m^7n^8}$

2) $-4\sqrt[3]{189}$

3) $\sqrt[3]{-64x^6y^9}$

4) $\sqrt{9x^2+6x+1}$

5) $5\sqrt{12} - 3\sqrt{18} + 2\sqrt{108} - 3\sqrt{8}$

6) $\dfrac{4-\sqrt{6}}{2+\sqrt{6}}$

7) $(5-\sqrt{5})(4+2\sqrt{5})$

8) $\dfrac{\sqrt{5}}{\sqrt{3w}}$

9) $(-2\sqrt{15})(4\sqrt{20})$

10) $\sqrt[3]{3}\,(\sqrt[3]{16} + \sqrt[3]{9})$

Radical Review 1

Write in simplest radical form.

1. $\sqrt{625x^6y^3}$
2. $\sqrt{-27w^9y^6}$
3. $8\sqrt[3]{189}$
4. $4\sqrt{12}-\sqrt{18}+\sqrt{108}+7\sqrt{72}$
5. $(7-\sqrt{5})(3+2\sqrt{5})$
6. $\sqrt{\frac{5}{2w}}$
7. $\frac{2-\sqrt{6}}{4+\sqrt{6}}$
8. $y^{\frac{-1}{2}}$
9. $\frac{14}{7^{\frac{2}{3}}}$
10. $\frac{r^2t^3}{\sqrt[4]{a}}$

Write in simplest exponential form.

11. $\left(\frac{n^{\frac{-4}{5}}}{x^{-10}n^{\frac{2}{5}}}\right)^{-5}$
12. $\sqrt[3]{27m^6n^4}$
13. $\sqrt[5]{8}$

Write in simplest radical form.

14. $x^{\frac{5}{8}}w^{\frac{1}{2}}y^{\frac{3}{4}}$
15. $27^{\frac{1}{6}}$
16. $w^{\frac{3}{7}}n^{\frac{5}{3}}$

Solve.

17. $\sqrt{5y-3}=\sqrt{7y+9}$
18. $\sqrt[3]{2x-7}=-2$
19. $\sqrt{4x^2-3x+2}-2x-5=0$
20. $4x + x\sqrt{3} = 6$
21. $\sqrt{x+10}+\sqrt{x-6}=8$

Graph, state the domain and range.

22. $y=\sqrt{x+2}-1$
23. $y=2\sqrt{x}+3$
24. $y=-\sqrt{x-2}-2$

Simplify:

25. $\frac{2+i}{3-i}$
26. $\frac{3-i\sqrt{2}}{3+i\sqrt{2}}$

REVIEW 2 **HONORS**

Simplest radical form unless specified:

1. $\frac{4}{\sqrt{5}}$

2. $\sqrt{9x^5 - 24x^4 + 16x^3}$

3. $\sqrt[3]{-27}$

4. $-3\sqrt{18} - 2\sqrt{27} + \sqrt{32}$

5. $\left(3^{\frac{1}{3}}\right)^{\frac{2}{5}}$

6. $\sqrt{25} \bullet \sqrt[3]{25}$

7. $27^{-\frac{2}{3}}$

8. $\sqrt[7]{x^{-8}}$

9. $\left(\sqrt[3]{7} \cdot \sqrt[4]{7}\right)^2$

10. $m^{\frac{3}{4}} n^{\frac{1}{3}}$

11. $\left(\sqrt{2}\right)\left(\sqrt[3]{2^4}\right)$

12. $\left(8x^6 y^{-9}\right)^{\frac{-2}{3}}$

13. $\frac{12}{4^{\frac{2}{3}}}$

14. $\frac{3 - \sqrt{5}}{6 + \sqrt{5}}$

15. $\sqrt[6]{125}$

16. $\left(\sqrt[3]{15x^2y^4z^5}\right)\left(\sqrt[3]{-9x^6yz^2}\right)$

17. $\left(\sqrt{3} - \sqrt{5}\right)\left(3\sqrt{3} + 2\sqrt{5}\right)$

18. $\frac{\sqrt[5]{x^4}}{\sqrt[8]{x^3}}$

19. $\sqrt{a}\left(\sqrt{a} + a\sqrt{b}\right)$

20. (exp. form) $\frac{y}{y^{\frac{1}{2}} - n^{\frac{1}{2}}}$

21. $\sqrt{\frac{2}{5}} + \sqrt{90}$

22. $\dfrac{6}{3-\sqrt{2}}$

23. $\dfrac{\sqrt[3]{81}}{\sqrt[3]{9}}$

24. $64^{\frac{1}{6}} - 64^{-\frac{1}{2}}$

25. $3\sqrt[3]{40x^3} - 7\sqrt[3]{5x^3}$

26. $\sqrt[4]{32x^{11}y^{14}z^{12}}$

27. (exp. form) $\dfrac{u^{-\frac{2}{3}}w^{\frac{1}{2}}}{\sqrt[3]{u}\cdot w^{-\frac{1}{2}}}$

28. $\left(\sqrt{2}\right)\left(\sqrt[4]{5}\right)$

29. $\left(2\sqrt{3x}\right)^2$

30. $-\sqrt{36x^3y^4}$

31. $\dfrac{15\sqrt{12}}{5\sqrt{3}}$

32. $3\sqrt{3} - 2\sqrt{12} + 4\sqrt{\dfrac{1}{3}}$

33. $5\sqrt{3}\left(\sqrt{6} + 2\sqrt{8}\right)$

34. $\left(\sqrt{5} - \sqrt{7}\right)\left(\sqrt{5} + \sqrt{7}\right)$

35. $\left(3\sqrt{2} + \sqrt{5}\right)\left(\sqrt{2} + 2\sqrt{5}\right)$

36. $\sqrt[3]{-54x^4y^5}$

Graph, state the domain & range.

37. $y = \sqrt{x-4}$

38. $y = \sqrt[3]{x} + 3$

39. $y = -\sqrt{x+3} - 2$

Solve for x.

40. $x^2 - 64 = 0$

41. $3x^2 + 16 = 0$

42. $x^{-\frac{4}{3}} = 16$

43. $3(x-3)^{\frac{2}{3}} + 4 = 52$

44. $c^4 = \sqrt{\dfrac{a^2b}{7x}}$

45. $\sqrt{5-x} = 5 - x$

46. $\sqrt{5+x} - \sqrt{x} = 1$

47. $10 - 3x = x\sqrt{2}$