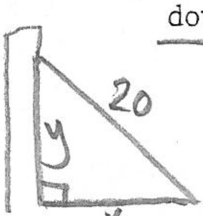
Related Rates In-Class Examples

1. A ladder 20 feet long leans against a vertical building. If the bottom of the ladder slides away from the building horizontally at a rate of 2 ft/sec, how fast is the ladder sliding down the building when the top of the ladder is 12 feet above the ground?



$\frac{dx}{dt} = 2$ ft/sec

$\frac{dy}{dt} = ?$ when $y = 12$ ft

$x^2 + 12^2 = 20^2$

when $x = 16$

$x^2 + y^2 = 20^2$

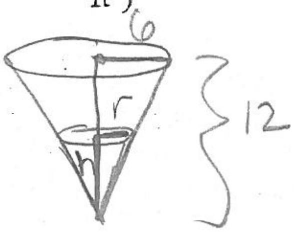
$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(16)(2) + 2(12)\frac{dy}{dt} = 0$

$\frac{8}{3}$ ft/sec down

$\boxed{\frac{dy}{dt} = -\frac{8}{3} \text{ ft/sec}}$

2. A water tank has the shape of an inverted right circular cone of altitude 12 feet and base radius 6 feet. If water is being pumped into the tank at a rate of 10 gal/min, approximate the rate at which the water level is rising when the water is 3 feet deep. (1gal = 0.1337 ft³)



$\frac{6}{12} = \frac{r}{h}$

$r = \frac{1}{2}h$

$\frac{dV}{dt} = 10$ gal/min $= 1.337$ ft³/min

$\frac{dh}{dt} = ?$ when $h = 3$ ft

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h = \frac{1}{3}\pi \frac{1}{4}h^2 h = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{12}\pi \cdot 3h^2 \frac{dh}{dt}$

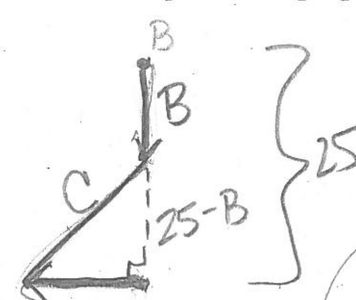
$1.337 = \frac{1}{4}\pi(3)^2\frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = 0.189 \text{ ft/min}}$

3. At 1:00 pm., ship A is 25 miles due south of ship B. If ship A is sailing west at a rate of 16 mph and ship B is sailing south at a rate of 20 mph, find the rate at which the distance between the ships is changing at 1:30 pm.

$\frac{dA}{dt} = 16$ mph $\quad \frac{dB}{dt} = 20$ mph $\quad \frac{dC}{dt} = ?$ @ 1:30

$\frac{1}{2}$ hr



$A^2 + (25-B)^2 = C^2$

$2A\frac{dA}{dt} + 2(25-B)\left(-\frac{dB}{dt}\right) = 2C\frac{dC}{dt}$

$\frac{1}{2}$ hr

$A = 16\left(\frac{1}{2}\right) = 8$ mi

$B = 20\left(\frac{1}{2}\right) = 10$ mi

$2(8)(16) + 2(15)(-20) = 2(17)\frac{dC}{dt}$

$\boxed{dC/dt = -10.1176 \text{ mph}}$

FORMULAS FROM GEOMETRY

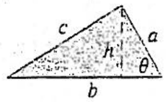
Triangle

$h = a \sin \theta$

$\text{Area} = \frac{1}{2}bh$

(Law of Cosines)

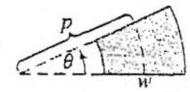
$c^2 = a^2 + b^2 - 2ab\cos\theta$



Sector of Circular Ring

(p = average radius,
w = width of ring,
θ in radians)

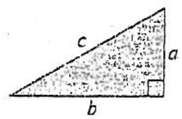
$\text{Area} = \theta p w$



Right Triangle

(Pythagorean Theorem)

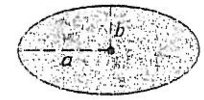
$c^2 = a^2 + b^2$



Ellipse

$\text{Area} = \pi ab$

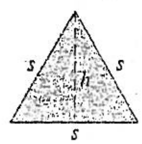
$\text{Circumference} \approx 2\pi\sqrt{\frac{a^2 + b^2}{2}}$



Equilateral Triangle

$h = \frac{\sqrt{3}s}{2}$

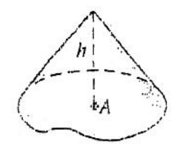
$\text{Area} = \frac{\sqrt{3}s^2}{4}$



Cone

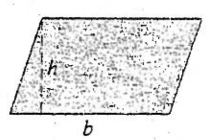
(A = area of base)

$\text{Volume} = \frac{Ah}{3}$



Parallelogram

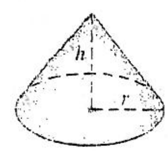
$\text{Area} = bh$



Right Circular Cone

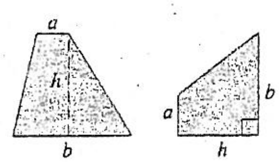
$\text{Volume} = \frac{\pi r^2 h}{3}$

$\text{Lateral Surface Area} = \pi r\sqrt{r^2 + h^2}$



Trapezoid

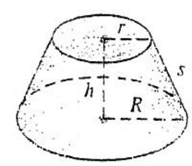
$\text{Area} = \frac{h}{2}(a + b)$



Frustum of Right Circular Cone

$\text{Volume} = \frac{\pi(r^2 + rR + R^2)h}{3}$

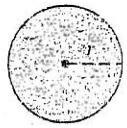
$\text{Lateral Surface Area} = \pi s(R + r)$



Circle

$\text{Area} = \pi r^2$

$\text{Circumference} = 2\pi r$

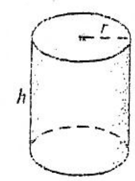


Right Circular Cylinder

$\text{Volume} = \pi r^2 h$

$\text{Lateral Surface Area} = 2\pi rh$

Total Surface Area = $2\pi r^2 + 2\pi rh$

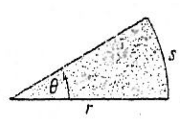


Sector of Circle

(θ in radians)

$\text{Area} = \frac{\theta r^2}{2}$

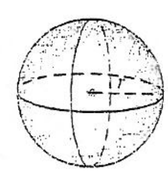
$s = r\theta$



Sphere

$\text{Volume} = \frac{4}{3}\pi r^3$

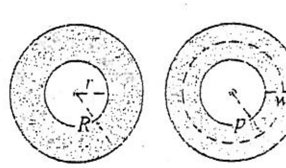
$\text{Surface Area} = 4\pi r^2$



Circular Ring

(p = average radius,
w = width of ring)

$\text{Area} = \pi(R^2 - r^2)$
$= 2\pi pw$



Wedge

(A = area of upper face,
B = area of base)

$A = B\sec\theta$