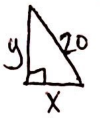
Related Rates In class Examples:

1. A ladder 20 feet long leans against a vertical building. If the bottom of the ladder slides away from the building horizontally at a rate of 2 ft/sec, how fast is the ladder sliding down the building when the top of the ladder is 12 feet above the ground?



$\frac{dx}{dt} = 2\text{ ft/sec}$

$\frac{dy}{dt} = ?$ when $y = 12$

$x^2 + 12^2 = 20^2$

when $x = 16$

$x^2 + y^2 = 20^2$

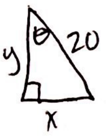
$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(16)(2) + 2(12)\frac{dy}{dt} = 0$

$\frac{dy}{dt} = -\frac{8}{3}$ ft/sec.

When the ladder is 12 ft above ground, the ladder is sliding down building @ rate of 8/3 ft/sec.

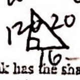
b) At what rate is the angle between the ladder and the wall changing when the ladder is 12 feet above the ground?



$\frac{dx}{dt} = 2\text{ ft/sec.}$

$\frac{d\theta}{dt} = ?$ when $y = 12$ ft, when $x = 16$ ft

when $\frac{dy}{dt} = -\frac{8}{3}$ ft/sec.



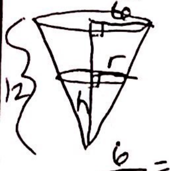
$\sin\theta = \frac{x}{20} = \frac{1}{20}x$

$\cos\theta \cdot \frac{d\theta}{dt} = \frac{1}{20}\frac{dx}{dt}$

$\left(\frac{12}{20}\right)\frac{d\theta}{dt} = \frac{1}{20}(2)$

$\frac{d\theta}{dt} = \frac{1}{6}$ rad/sec.

2. A water tank has the shape of an inverted right circular cone of altitude 12 feet and base radius 6 feet. If water is being pumped into the tank at a rate of 10 gal/min, approximate the rate at which the water level is rising when the water is 3 feet deep. (1gal = 0.1337 ft³)



$\frac{dV}{dt} = 10\text{ gal/min} = 1.337\text{ ft}^3/\text{min}$

$\frac{dh}{dt} = ?$ when $h = 3$ ft

$\frac{6}{12} = \frac{r}{h}$ $\quad r = \frac{1}{2}h$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h$

$V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{12}\pi\, 3h^2\frac{dh}{dt}$

$1.337 = \frac{1}{12}\pi(3)(3)^2\frac{dh}{dt}$

$\frac{dh}{dt} = .189$ ft/min

When water is 3 ft deep, the water level is rising @ rate of .189 ft/min

b) At what rate is the radius of the water in the tank changing at this instant?

$\frac{dr}{dt} = ?$

$r = \frac{1}{2}h$

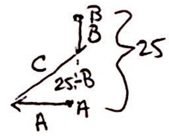
$\frac{dr}{dt} = \frac{1}{2}\frac{dh}{dt}$

$= \frac{1}{2}(.189)$

$\frac{dr}{dt} = .0946$ ft/min

The radius is increasing @ rate of .0946 ft/min when 3 ft deep water.

3. At 1:00 pm., ship A is 25 miles due south of ship B. If ship A is sailing west at a rate of 16 mph and ship B is sailing south at a rate of 20 mph, find the rate at which the distance between the ships is changing at 1:30 pm.



$\frac{dA}{dt} = 16$ mph

$\frac{dB}{dt} = 20$ mph

$\frac{dC}{dt} = ?$ when $t = \frac{1}{2}$ hr

when $A = 8$ mi

$B = 10$ mi

$8^2 + 15^2 = c^2$ $\quad C = 17$ mi

$A^2 + (25 - B)^2 = C^2$

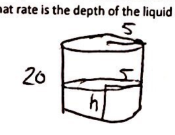
$2A\frac{dA}{dt} + 2(25 - B)\left(-\frac{dB}{dt}\right) = 2C\frac{dC}{dt}$

$2(8)(16) + 2(15)(-20) = 2(17)\frac{dC}{dt}$

$\frac{dC}{dt} = -10.1176$ mph

The distance between the ships is decreasing at a rate of 10.1176 mph at 1:30 pm.

4. A right circular cylindrical container with a height of 20 ft and radius of 5 ft is being drained at the rate of 30 cubic ft per hour. At what rate is the depth of the liquid changing when it is half full?



* Note with a cylinder the radius is constant

$\frac{dV}{dt} = -30\text{ ft}^3/\text{hr}$

$\frac{dh}{dt} = ?$ when $\frac{1}{2}V \Rightarrow V = 250\pi$

$V = \pi r^2 h$ $\quad V = \pi(5)^2(20) = 500\pi$

Notice don't even use this @ end

$V = \pi(5)^2 h$

$V = 25\pi h$

$\frac{dV}{dt} = 25\pi\frac{dh}{dt}$

$-30 = 25\pi\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{-30}{25\pi} = -\frac{6}{5\pi}$ ft/hr

The depth of water is decreasing at rate of $\frac{6}{5\pi}$ ft/hr when 1/2 full.