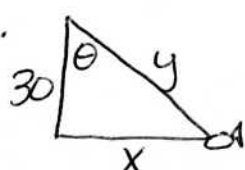


Related Rates Practice

1.  $\frac{dy}{dt} = -2 \text{ ft/sec}$

a) $\frac{dx}{dt} = ?$ when $y = 50 \text{ ft}$
 $x^2 + 30^2 = 50^2$
 $x = 40$

$$x^2 + 30^2 = y^2$$

$$2x \frac{dx}{dt} + 0 = 2y \frac{dy}{dt}$$

$$2(40) \frac{dx}{dt} = 2(50)(-2)$$

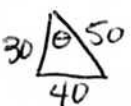
$$\boxed{\frac{dx}{dt} = -2.5 \text{ ft/s}}$$

b) $\frac{d\theta}{dt} = ?$ when $y = 50 \text{ ft}$
 $x = 40$

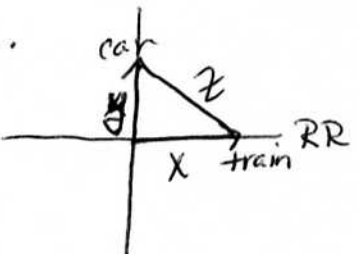
$$\cos \theta = \frac{30}{y}$$

$$\frac{d\theta}{dt} (-\sin \theta) = -\frac{30}{y^2} \frac{dy}{dt}$$

$$\frac{d\theta}{dt} \left(-\frac{40}{50}\right) = -\frac{30}{50^2} (-2)$$


 $\sin \theta = \frac{40}{50}$

$$\boxed{\frac{d\theta}{dt} = -0.03 \text{ rad/sec}}$$

2.  $\frac{dy}{dt} = 60 \text{ mph}$
 $\frac{dx}{dt} = 80 \text{ mph}$

$$x^2 + y^2 = z^2$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

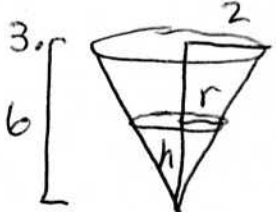
$$2(40)(80) + 2(45)(60) = 2\sqrt{3625} \frac{dz}{dt}$$

$$\boxed{\frac{dz}{dt} = 97.994 \text{ mph}}$$

$\frac{dz}{dt} = ?$ when $\frac{1}{2} \text{ hr} \Rightarrow \text{train } x = 80(\frac{1}{2}) = 40$
 $\frac{3}{4} \text{ hr} \Rightarrow \text{car } y = 60(\frac{3}{4}) = 45$

$$40^2 + 45^2 = z^2$$

$$z = \sqrt{3625}$$

3.  $\frac{dV}{dt} = 2 \text{ ft}^3/\text{min}$
 $\frac{dh}{dt} = ?$ when $\frac{1}{2} V \Rightarrow V = \frac{1}{3} \pi (r)^2 (6) = 8\pi$

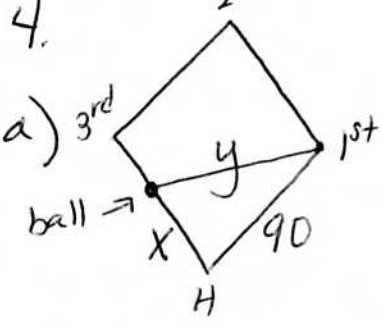
$\frac{1}{2} V = 4\pi = \frac{1}{27} \pi h^3 \Rightarrow$ when $h = \sqrt[3]{108}$

$$V = \frac{1}{3} \pi r^2 h$$

$\frac{z}{6} = \frac{r}{h} \Rightarrow r = \frac{1}{3} h$
 $V = \frac{1}{3} \pi \left(\frac{1}{3} h\right)^2 h = \frac{1}{27} \pi h^3$

$$\frac{dV}{dt} = \frac{1}{9} \pi h^2 \frac{dh}{dt} \quad 2 = \frac{1}{9} \pi (\sqrt[3]{108})^2 \frac{dh}{dt}$$

$$\boxed{\frac{dh}{dt} = 0.2526 \text{ ft/min}}$$



$$\frac{dx}{dt} = 100 \text{ ft/s}$$

$$\frac{dy}{dt} = ? \text{ when } x = 45 \text{ ft}$$

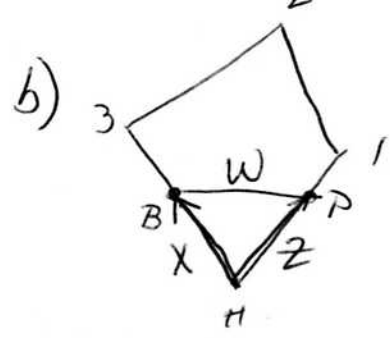
$$y = \sqrt{10125}$$

$$x^2 + 90^2 = y^2$$

$$2x \frac{dx}{dt} = 2y \frac{dy}{dt}$$

$$2(45)(100) = 2(\sqrt{10125}) \frac{dy}{dt}$$

$$\frac{dy}{dt} = 44.721 \text{ ft/s}$$



$$x^2 + z^2 = w^2$$

$$2x \frac{dx}{dt} + 2z \frac{dz}{dt} = 2w \frac{dw}{dt}$$

$$2(45)(100) + 2(11.25)(25) = 2w \frac{dw}{dt}$$

$$= 2(46.387) \frac{dw}{dt}$$

$$\frac{dw}{dt} = 103.0776 \text{ ft/s}$$

$$\frac{dx}{dt} = 100 \text{ ft/s}$$

$$\frac{dz}{dt} = 25 \text{ ft/s}$$

$$\frac{dw}{dt} = ? \text{ when } x = 45$$

$$* \text{ When } x = 45$$

$$45 = 100t$$

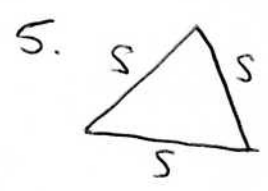
$$t = .45 \text{ sec}$$

$$z = (25)(.45)$$

$$= 11.25 \text{ ft}$$

$$w \Rightarrow 46.387$$

$$45^2 + (11.25)^2 =$$



$$\frac{ds}{dt} = 1 \text{ mm/sec}$$

$$\frac{dA}{dt} = ? \text{ when } A = 14$$

$$14 = \frac{\sqrt{3}}{4} s^2$$

$$s = \sqrt{\frac{56}{\sqrt{3}}}$$

$$\approx 5.6861$$

$$A = \frac{\sqrt{3}}{4} s^2$$

$$\frac{dA}{dt} = \frac{\sqrt{3}}{2} s \frac{ds}{dt}$$

$$= \frac{\sqrt{3}}{2} (5.6861)(1) = 4.924 \text{ mm}^2/\text{sec}$$

6. $\frac{dV}{dt} = 300 \text{ cm}^3/\text{min}$

a) $\frac{dr}{dt} = ?$ when $r = 30 \text{ cm}$

b) $\frac{dA}{dt} = ?$ when $r = 30$

a) $V = \frac{4}{3} \pi r^3$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$300 = 4\pi (30)^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = 0.0265 \text{ cm/min}$$

b) $A = 4\pi r^2$

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

$$= 8\pi (30)(0.0265) = 20 \text{ cm}^2/\text{min}$$

7.



$$\frac{dV}{dt} = 10 \text{ m}^3/\text{sec}$$

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi r^2 \left(\frac{3}{4}r\right)$$

$$V = \frac{1}{4} \pi r^3$$

$$\frac{dV}{dt} = \frac{3}{4} \pi r^2 \frac{dr}{dt}$$

$$10 = \frac{3}{4} \pi \left(\frac{16}{3}\right)^2 \frac{dr}{dt}$$

$$b) \frac{dr}{dt} = 0.1492 \text{ m/sec}$$

$$a) \frac{dh}{dt} = ?$$

$$b) \frac{dr}{dt} = ?$$

when $h = 4 \text{ m}$

$$4 = \frac{3}{4}r$$

$$r = \frac{16}{3} \text{ m}$$

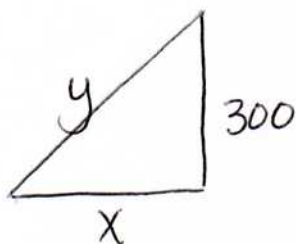
$$a) h = \frac{3}{4}r$$

$$\frac{dh}{dt} = \frac{3}{4} \frac{dr}{dt}$$

$$= \frac{3}{4} (0.1492) = 0.1119 \text{ m/sec}$$

* * *
 * could have *
 done (a) first
 by using $r = \frac{4}{3}h$
 & writing volume eq.
 with only h 's.

8.



$$\frac{dx}{dt} = 25 \text{ ft/s}$$

$$\frac{dy}{dt} = ? \text{ when } y = 500 \text{ ft}$$

$$x = 400 \text{ ft}$$

$$x^2 + 300^2 = y^2$$

$$2x \frac{dx}{dt} = 2y \frac{dy}{dt}$$

$$2(400)(25) = 2(500) \frac{dy}{dt}$$

$$\frac{dy}{dt} = 20 \text{ ft/s}$$