AP Calculus BC - Practice 8.1 & 8.2

Determine whether the sequence converges or diverges & if it converges, find the limit.

1. $a_n = (-1)^{n+1}\frac{3n}{n^2+4n+5}$ 2. $a_n = 1-\frac{1}{2^n}$ 3. $a_n = 1+(-1)^{n-1}$

4. $a_n = \frac{n^2}{\ln(n+1)}$ 5. $a_n = e^{-n}\ln n$ 6. $a_n = (-1)^n n^3 3^{-n}$ 7. $a_n = n\sin n$

8. $a_n = \frac{n^2}{2n-1} - \frac{n^2}{2n+1}$ 9. $a_n = 2^{-n}\sin n$ 10. $a_n = \frac{4n^3+5n+1}{2n^3-n^2+5}$

11. $a_n = \frac{3^n}{n!}$ 12. $a_n = \frac{(2n+2)!}{(2n+4)!}$ 13. $a_n = \frac{(n+2)!}{n!}$

14. Find S_1, S_2, S_3, S_n, & the sum of the series if it converges:
$\sum_{n=1}^{\infty}\frac{1}{4n^2-1}$

Determine whether the following series converge or diverge. If converges, find the sum.

15. $\frac{1}{4\cdot5}+\frac{1}{5\cdot6}+\cdots+\frac{1}{(n+3)(n+4)}+\cdots$ 16. $\sum_{n=1}^{\infty}\frac{3}{4^{n-1}}$ 17. $\sum_{n=1}^{\infty}\frac{3n}{5n-1}$

18. $\sum_{n=1}^{\infty}2^{-n}3^{n-1}$ 19. $\sum_{n=1}^{\infty}\left(\frac{e}{3}\right)^{n-1}$ 20. $\sum_{n=1}^{\infty}\left[\frac{1}{8^n}+\frac{1}{n(n+1)}\right]$

21. $\sum_{n=1}^{\infty}\left(\frac{1}{3^n}-\frac{1}{4^n}\right)$ 22. $\sum_{n=1}^{\infty}\frac{n}{\ln(n+1)}$ 23. $\sum_{n=1}^{\infty}\left(\frac{5}{n+2}-\frac{5}{n+3}\right)$

24. $\sum_{n=3}^{\infty}\frac{3}{(-4)^{n-1}}$ 25. $\sum_{n=1}^{\infty}\left(2^{-n}-2^{-3n}\right)$ 26. $\sum_{n=1}^{\infty}\left[\left(\frac{3}{2}\right)^n+\left(\frac{2}{3}\right)^n\right]$

Determine all values of x for which the series converges & find the sum

27. $1+x^2+x^4+\cdots+x^{2n}+\cdots$ 28. $\frac{1}{2}+\frac{(x-3)}{4}+\frac{(x-3)^2}{8}+\cdots+\frac{(x-3)^n}{2^{n+1}}+\cdots$

Ans.

1. converges to 0 2. conv. to 1 3. diverges 4. diverges 5. conv. to 0

6. conv. to 0 7. conv. to 1 8. conv. to $\frac{1}{2}$ 9. conv. to 0 10. conv. to 2

11. conv. to 0 12. conv. to 0 13. diverges

14. $S_1 = \frac{1}{3}$ $S_2 = \frac{2}{5}$ $S_3 = \frac{3}{7}$ .. $S_n = \frac{n}{2n+1}$ Sum $= \lim_{n\to\infty} \frac{n}{2n+1} = \boxed{\frac{1}{2}}$

15. converge sum $= \frac{1}{4}$ 16. $\sum_{n=1}^{\infty} 3\left(\frac{1}{4}\right)^{n-1} = \frac{3}{1-\frac{1}{4}} = \boxed{4}$ 17. diverges by nth term test

18. $\sum_{n=1}^{\infty} \frac{1}{3}\left(\frac{3}{2}\right)^n$ diverges b/c geom. & $r = \frac{3}{2} > 1$ 19. Converges sum $= \frac{1}{1-\frac{e}{3}} = \boxed{\frac{3}{3-e}}$

20. $\sum_{n=1}^{\infty} \left(\frac{1}{8}\right)^n + \sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \frac{\frac{1}{8}}{1-\frac{1}{8}} + 1 = 1\frac{1}{7} = \boxed{\frac{8}{7}}$ 21. conv. $\frac{\frac{1}{3}}{1-\frac{1}{3}} - \frac{\frac{1}{4}}{1-\frac{1}{4}}$
$= \frac{1}{2} - \frac{1}{3} = \boxed{\frac{1}{6}}$

22. diverges by nth term test
$\lim_{n\to\infty} \frac{n}{\ln(n+1)} = \infty \neq 0$

23. converges
Sum $= \frac{5}{3}$

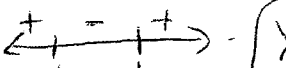
24. $\sum_{n=3}^{\infty} 3\left(-\frac{1}{4}\right)^{n-1}$ converges $\to$ sum $= \frac{\frac{3}{16}}{1--\frac{1}{4}} = \frac{\frac{3}{16}}{\frac{5}{4}} = \boxed{\frac{3}{20}}$

25. $\sum_{n=1}^{\infty}\left[\left(\frac{1}{2}\right)^n - \left(\frac{1}{8}\right)^n\right]$ converges Sum $= \frac{\frac{1}{2}}{1-\frac{1}{2}} - \frac{\frac{1}{8}}{1-\frac{1}{8}} = 1 - \frac{1}{7} = \boxed{\frac{6}{7}}$

26. diverges b/c $\left(\frac{3}{2}\right)^n$ is geom. w/ $r = \frac{3}{2} > 1$.

27. geometric $\sum_{n=0}^{\infty} x^{2n}$ $r = x^2$ converges if $|x^2| < 1$

$x^2 - 1 < 0$
$(x-1)(x+1) < 0$



$\boxed{x \in (-1,1) \text{ to converge}}$

$\boxed{\text{Sum} = \frac{1}{1-x^2}}$

28. $\sum_{n=0}^{\infty} \frac{(x-3)^n}{2^{n+1}} = \sum_{n=0}^{\infty} \frac{1}{2}\left(\frac{x-3}{2}\right)^n$ converges if $\left|\frac{x-3}{2}\right| < 1 \Rightarrow \frac{x-3}{2} < 1$ & $\frac{x-3}{2} > -1$

$x < 5$ & $x > 1$ $\boxed{x \in (1,5}$

Sum $= \frac{\frac{1}{2}}{1-\frac{x-3}{2}} = \boxed{\frac{1}{5-x}}$