14. Which of the following series diverges?

(A) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ (B) $\sum_{n=1}^{\infty} \frac{n+1}{n!}$ (C) $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ (D) $\sum_{n=1}^{\infty} \frac{\ln n}{2^n}$

(E) $\sum_{n=1}^{\infty} \frac{n}{2^n}$

15. Which of the following series diverges?

(A) $\sum \frac{1}{n^2}$ (B) $\sum \frac{1}{n^2+n}$ (C) $\sum \frac{n}{n^3+1}$ (D) $\sum \frac{n}{\sqrt{4n^2-1}}$

(E) none of the preceding

16. For which of the following series does the Ratio Test fail?

(A) $\sum \frac{1}{n!}$ (B) $\sum \frac{n}{2^n}$ (C) $1 + \frac{1}{2^{3/2}} + \frac{1}{3^{3/2}} + \frac{1}{4^{3/2}} + \cdots$

(D) $\frac{\ln 2}{2^2} + \frac{\ln 3}{2^3} + \frac{\ln 4}{2^4} + \cdots$ (E) $\sum \frac{n^n}{n!}$

17. Which of the following alternating series diverges?

(A) $\sum \frac{(-1)^{n-1}}{n}$ (B) $\sum \frac{(-1)^{n+1}(n-1)}{n+1}$ (C) $\sum \frac{(-1)^{n+1}}{\ln(n+1)}$

(D) $\sum \frac{(-1)^{n-1}}{\sqrt{n}}$ (E) $\sum \frac{(-1)^{n-1}(n)}{n^2+1}$

18. Which of the following series converges conditionally?

(A) $3 - 1 + \frac{1}{9} - \frac{1}{27} + \cdots$ (B) $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} - \cdots$

(C) $\frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{4^2} - \cdots$ (D) $1 - 1.1 + 1.21 - 1.331 + \cdots$

(E) $\frac{1}{1 \cdot 2} - \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} - \frac{1}{4 \cdot 5} + \cdots$

19. Let $S = \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$; then S equals

(A) 1 (B) $\frac{3}{2}$ (C) $\frac{4}{3}$ (D) 2 (E) 3

20. Which of the following statements is true?

(A) If a series converges, then it converges absolutely.
(B) If a series is truncated after the nth term, then the error is less than the first term omitted.
(C) If the terms of an alternating series decrease, then the series converges.
(D) If $r < 1$, then the series Σr^n converges.
(E) None of the preceding.

21. Which of the following expansions is impossible?

(A) $\sqrt{x-1}$ in powers of x **(B)** $\sqrt{x+1}$ in powers of x

(C) $\ln x$ in powers of $(x-1)$ **(D)** $\tan x$ in powers of $\left(x - \frac{\pi}{4}\right)$

(E) $\ln(1-x)$ in powers of x

22. The power series $x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots + \frac{x^n}{n} + \cdots$ converges if and only if

(A) $-1 < x < 1$ **(B)** $-1 \leqq x \leqq 1$ **(C)** $-1 \leqq x < 1$
(D) $-1 < x \leqq 1$ **(E)** $x = 0$

23. The power series

$$(x+1) - \frac{(x+1)^2}{2!} + \frac{(x+1)^3}{3!} - \frac{(x+1)^4}{4!} + \cdots$$

diverges

(A) for no real x **(B)** if $-2 < x \leqq 0$ **(C)** if $x < -2$ or $x > 0$
(D) if $-2 \leqq x < 0$ **(E)** if $x \neq -1$

24. The series $\sum_{n=0}^{\infty} n!(x-3)^n$ converges if and only if

(A) $x = 0$ **(B)** $2 < x < 4$ **(C)** $x = 3$ **(D)** $2 \leqq x \leqq 4$
(E) $x < 2$ or $x > 4$

25. The interval of convergence of the series obtained by differentiating term by term the series

$$(x-2) + \frac{(x-2)^2}{4} + \frac{(x-2)^3}{9} + \frac{(x-2)^4}{16} + \cdots$$

is

(A) $1 \leqq x \leqq 3$ **(B)** $1 \leqq x < 3$ **(C)** $1 < x \leqq 3$ **(D)** $0 \leqq x \leqq 4$
(E) none of the preceding

26. Let $f(x) = \sum_{n=0}^{\infty} x^n$. The interval of convergence of $\int_0^x f(t)\,dt$ is

(A) $x = 0$ only **(B)** $|x| \leqq 1$ **(C)** $-\infty < x < \infty$
(D) $-1 \leqq x < 1$ **(E)** $-1 < x < 1$

27. The coefficient of x^4 in the Maclaurin series for $f(x) = e^{-x/2}$ is

(A) $-\frac{1}{24}$ **(B)** $\frac{1}{24}$ **(C)** $\frac{1}{96}$ **(D)** $-\frac{1}{384}$ **(E)** $\frac{1}{384}$

28. The Maclaurin polynomial of order 3 for $f(x) = \sqrt{1+x}$ is

(A) $1 + \frac{x}{2} - \frac{x^2}{4} + \frac{3x^3}{8}$ **(B)** $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}$

(C) $1 - \frac{x}{2} + \frac{x^2}{8} - \frac{x^3}{16}$ **(D)** $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{8}$

(E) $1 - \frac{x}{2} + \frac{x^2}{4} - \frac{3x^3}{8}$

29. The Taylor polynomial of order 3 at $x = 1$ for e^x is

(A) $1 + (x-1) + \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}$

(B) $e\left[1 + (x-1) + \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3!}\right]$

(C) $e\left[1 + (x+1) + \frac{(x+1)^2}{2!} + \frac{(x+1)^3}{3!}\right]$

(D) $e\left[1 + (x-1) + \frac{(x-1)^2}{2!} + \frac{(x-1)^3}{3!}\right]$

(E) $e\left[1 - (x-1) + \frac{(x-1)^2}{2!} - \frac{(x-1)^3}{3!}\right]$

30. The coefficient of $\left(x - \frac{\pi}{4}\right)^3$ in the Taylor series about $\frac{\pi}{4}$ of $f(x) = \cos x$ is

(A) $\frac{\sqrt{3}}{12}$ **(B)** $-\frac{1}{12}$ **(C)** $\frac{1}{12}$ **(D)** $\frac{1}{6\sqrt{2}}$ **(E)** $-\frac{1}{3\sqrt{2}}$

31. Which of the following series can be used to compute ln 0.8?

(A) $\ln(x-1)$ expanded about $x = 0$
(B) $\ln x$ about $x = 0$
(C) $\ln x$ in powers of $(x-1)$
(D) $\ln(x-1)$ in powers of $(x-1)$
(E) none of the preceding

32. If $e^{-0.1}$ is computed using series, then, correct to three decimal places, it equals

(A) 0.905 **(B)** 0.950 **(C)** 0.904 **(D)** 0.900 **(E)** 0.949

33. The coefficient of x^2 in the Maclaurin series for $e^{\sin x}$ is

(A) 0 **(B)** 1 **(C)** $\frac{1}{2!}$ **(D)** -1 **(E)** $\frac{1}{4}$