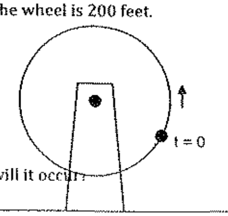
Precalculus

Sinusoidal Applications

Name: Key

1. As you ride the Texas Star Ferris wheel at the State Fair of Texas, your distance from the ground varies sinusoidally with time. When the Ferris wheel starts, your seat is at the position shown in the picture. Let t be the number of seconds that have elapsed since the Ferris wheel started. You find that it takes you 30 seconds to reach the top, 212 feet above the ground, and that the wheel makes a revolution once every 80 seconds. The diameter of the wheel is 200 feet.

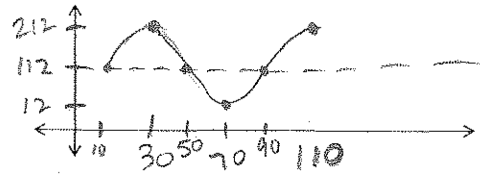


a) How long does it take to complete one cycle? 80 sec

b) When will the graph reach its first maximum? 30 sec

c) What is the lowest you will go as the Ferris wheel turns and when will it occur? 12 ft @ 70 sec

d) Choose appropriate scales and plot these maximum and minimum points on the graph. Then, use those points to finish sketching in the complete graph.



$A = 100$
per $= 80$ sec $\Rightarrow B = \frac{\pi}{40}$
V.S $= 112$

e) Using these maximums and minimums, find the horizontal axis for the graph. $y = 112$

f) Write an equation for this sinusoid. $y = 100 \sin \frac{\pi}{40}(x-10) + 112$ OR $y = 100 \cos \frac{\pi}{40}(x-30) + 112$

g) Predict your height above the ground when (i) t = 0, (ii) t = 50, (iii) t = 70, (iv) t = 110.

41.29 ft   112 ft   12 ft   212 ft

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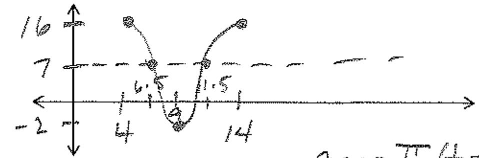
2. Betty Lou was sitting on the front porch of her plantation when the riverboat went by. As the paddlewheel turned, a point on the paddle blade moved in such a way that its distance, d from the water's surface was a sinusoidal function of time. Four seconds later, the point was at its highest, 16 feet above the water's surface. The diameter of the wheel was 18 feet, and it completed a revolution every 10 seconds.

period = 10 sec
$B = \frac{\pi}{5}$
Amp = 9
V.S = 7
P.S = 4

a) Sketch a graph and find the period, phase shift, amplitude and ~~horizontal~~ Vert. shift.



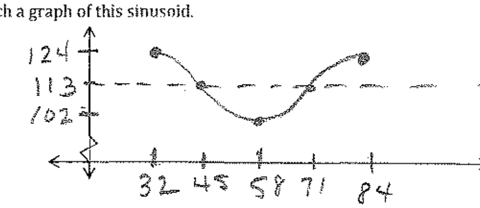
b) Write the equation of the sinusoid. $y = 9\cos\frac{\pi}{5}(t-4) + 7$
$y = -9\sin\frac{\pi}{5}(t-6.5) + 7$

c) How far above the surface was the point after (i) 5 seconds, (ii) 17 seconds?

14.28 ft   4.22 ft

3. Researchers find a creature from an alien planet. Its body temperature is varying sinusoidally with time. 32 minutes after they start timing, it reaches a high of 124° F. 26 minutes after that it reaches its next low, 102° F.

a) Sketch a graph of this sinusoid.



b) Write an equation expressing the temperature in terms of the number of minutes since they started timing. $y = 11\cos\frac{\pi}{26}(t-32) + 113$

c) What was its temperature when they first started timing? 104.77°F

A = 11
period = 52
$B = \frac{2\pi}{52} = \frac{\pi}{26}$

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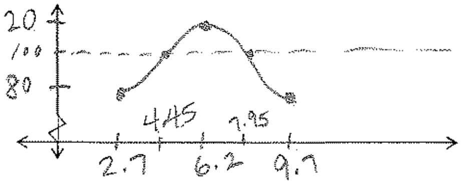
4. Naturalists find that the populations of some kinds of predatory animals vary periodically. Assume that the population of bobcats in a certain forest varies sinusoidally with time. Records started being kept when time t = 0 years. A minimum number, 80 bobcats, occurred when t = 2.7 years. The next maximum, 120 bobcats, occurred at t = 6.2 years.

a) Sketch a graph of this sinusoid.



$A = 20$
period = 7 years
$V.S = 100$

b) Write an equation expressing the number of bobcats as a function of time.

$y = -20\cos\frac{2\pi}{7}(t-2.7)+100$

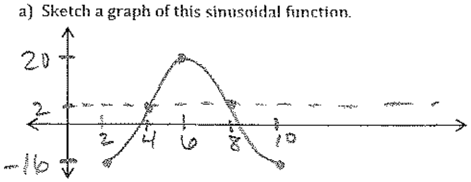
$y = 20\sin\frac{2\pi}{7}(t-4.45)+100$

c) Predict the population when t = 7.

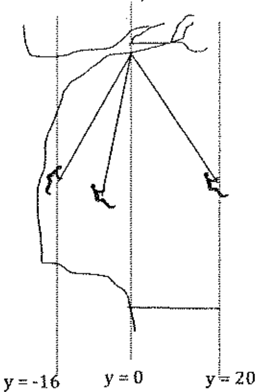
115 bobcats

5. Tarzan is swinging back and forth on his grapevine. As he swings, he goes back and forth across the riverbank, going alternately over land and water. Jane decided to model mathematically his motion and starts her stopwatch. Let t be the number of seconds the stopwatch reads and let y be the number of meters Tarzan is from the riverbank. Assume that y varies sinusoidally with t, and that y is positive when Tarzan is over water and negative when he is over land. Jane finds that when t = 2, Tarzan is at one end of his swing, where y = -16. She finds that when t = 6, he reaches the other end of his swing and y = 20.

a) Sketch a graph of this sinusoidal function.



$r = 18$
$2r = 8$ sec
$B = \pi/4$
$V.S = 2$



b) Write an equation expressing Tarzan's distance from the riverbank in terms of t.

$y = -18\cos\frac{\pi}{4}(t-2)+2$

$y = 18\sin\frac{\pi}{4}(t-4)+2$

$C = \pi(24) = 24\pi$
period $= 24\pi$

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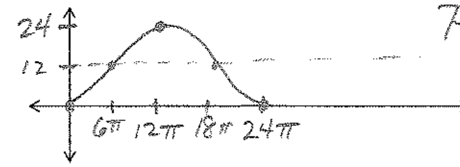
6. As Bobby Joe is driving his Mustang down Main St., he runs over a splotch of red paint. When he starts off, the distance of the splotch from the pavement varies sinusoidally with the distance he has traveled. The period is, of course, the circumference of the tire. Assume that the diameter of the tire is 24 inches.

a) Sketch a graph of this function.



$A = 12$
per $= 24\pi$
$B = \frac{2\pi}{24\pi} = \frac{1}{12}$
$V.S = 12$

b) Write an equation of this function.

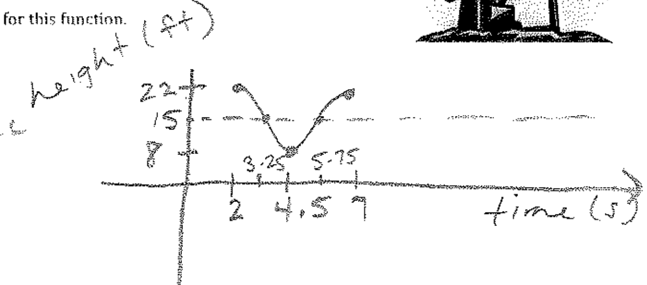
$y = -12\cos\frac{1}{12}(t)+12$

$y = 12\sin\frac{1}{12}(t-6\pi)+12$

7. As an oil well like the one shown pumps, the height of its cathead varies sinusoidally with time. Suppose that the pump is started at time t = 0 sec. Two seconds later, it is at its highest point above the ground, 22 ft. It is at its next low point (8 ft) 2.5 seconds after that.

a) Sketch a graph of this function.

b) Write an equation for this function.



$A = 7$
period = 5 sec
$B = \frac{2\pi}{5}$
$V.S = 15$

$y = 7\cos\frac{2\pi}{5}(t-2)+15$

$y = -7\sin\frac{2\pi}{5}(t-3.25)+15$