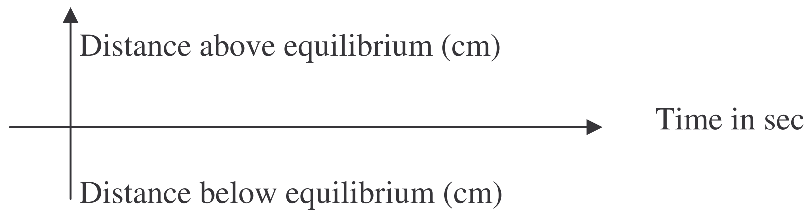
Sinusoidal Applications

1. A weight is suspended from a spring. Assuming no friction or air resistance, when the weight is pulled down a small distance, it will oscillate indefinitely about the equilibrium position. If the weight is pulled down 3 cm, then after 1 second it will be back at the equilibrium position, at 2 seconds it will be 3 cm above the equilibrium position, and 3 seconds it will be back at equilibrium and at 4 seconds it will be 3 cm below.



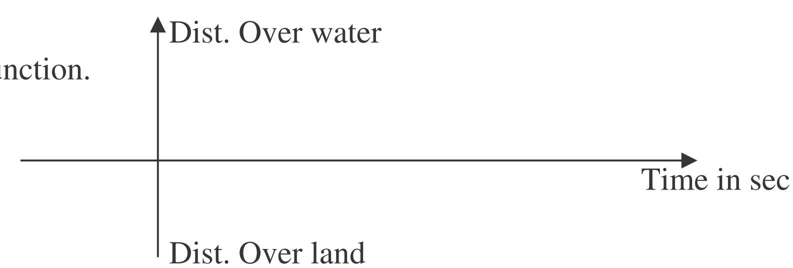
a) Find the equation of a sinusoidal function that will model this movement.

b) Find the distance of the weight from its equilibrium position, 1.5 seconds after release and 15 seconds after release.

2. Tarzan is swinging back and forth on his grapevine. As he swings, he goes back and forth across the riverbank, going alternately over land and water. Jane decides to model his movement mathematically and starts her stopwatch. Let t be the number of seconds the stopwatch reads and let y be the number of meters Tarzan is from the riverbank. Assume that y varies sinusoidally with t and that y is positive when Tarzan is over water and negative when he is over land.

Jane finds that when $t = 2$, Tarzan is at the end of his swing, where $y = -23$. She finds that when $t = 5$, he reaches the other end of his swing and $y = 17$.

a. Sketch a graph of this function.



b. Write an equation expressing Tarzan's distance from the river bank in terms of t.

c. Find y when $t = 2.8$, $t = 6.3$ and $t = 1.5$

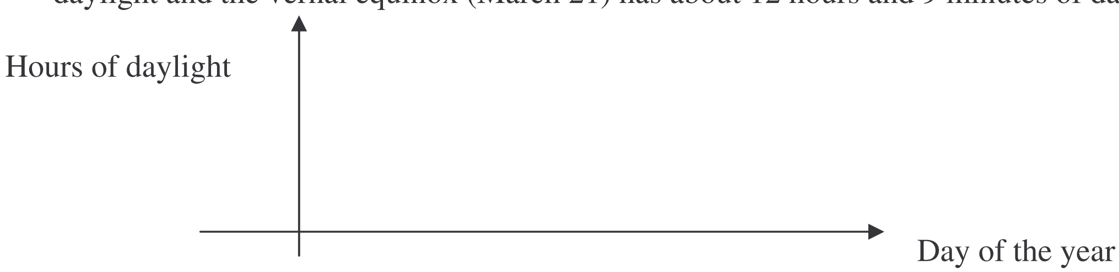
d. Where was Tarzan when Jane started her stopwatch?

3. A satellite is deployed from a space shuttle into an orbit which goes alternately north and south of the equator. Its distance from the equator over time can be approximated by a sine wave. It reaches 4500 km, its farthest point north of the equator, 15 minutes after the launch. Half an orbit later it is 4500 km south of the equator, its farthest point south. One complete orbit takes 2 hours.

a. Find an equation of a sinusoidal function that models the distance of the satellite from the equator.

b. How far away from the equator is the satellite 1 hour after launch?

4. In New Orleans, the longest day of the year (June 21) has approximately 14 hours and 5 minutes of daylight. The shortest day (December 21) has about 10 hours and 13 minutes of daylight and the vernal equinox (March 21) has about 12 hours and 9 minutes of daylight.



a. Assuming a sinusoidal model for the number of hours of daylight, find an equation that gives the number of minutes of daylight in New Orleans as a function of the day of the year.

b. Find the number of daylight hours on November 1.

5. You are riding a Ferris wheel with a diameter of 40 feet. When the last seat is filled and the ride begins, your seat is at *approximately* 4 o'clock. It takes 3 seconds for you to reach the top, which is 43 feet above the ground. The wheel makes one revolution every 8 seconds.

   a. Write the equation of your height above ground as a function of time.

   b. What is the lowest you go as the wheel turns?

   c. Find your height above ground at 6 seconds, 4 1/3 seconds, 9 seconds and 0 seconds.

6. Suppose that one day all 300 million people in the US climb up on tables. At time $t = 0$, we all jump off. The resulting shock, as we hit the Earth's surface, will start the entire Earth vibrating in such a way that its surface first moves *down* from its normal position and then moves up an equal distance *above* its normal position. The displacement y of the surface is a sinusoidal function of time with a period of about 54 minutes. Assume that the amplitude is 50 meters.

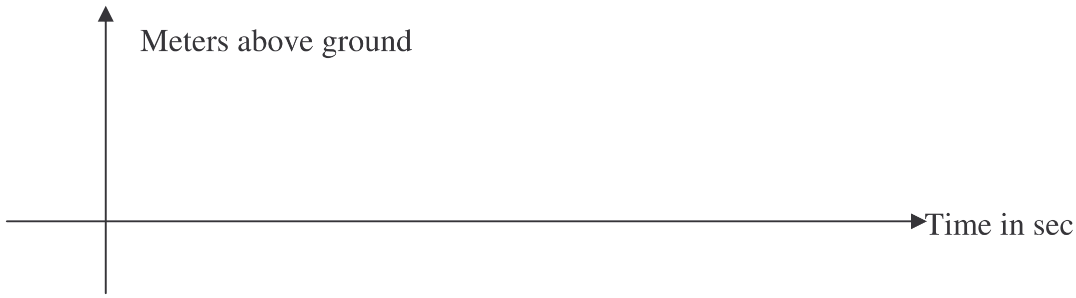
   a. At what time will the first maximum (i.e. the greatest distance above the normal position) occur?

   b. Write an equation expressing displacement in terms of time lapsed since the people jumped.

   c. Predict the displacement when $t = 21$.

   d. What are the first three times at which the displacement is –37 meters?

7. A Ferris wheel rotates counter-clockwise at a uniform speed of 2.5 revolutions per minute. It has a radius of 20 meters and its center is 22 meters above the ground. After the ride starts, your seat takes 4 seconds to reach the lowest point on the wheel.



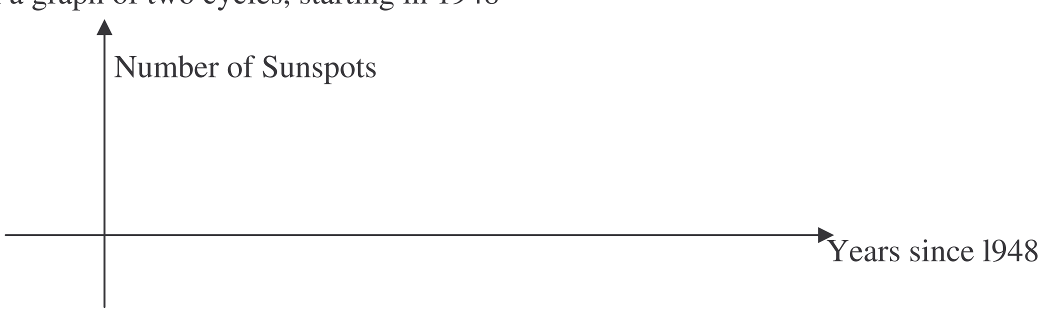
a) Find an equation for *h* the height (in meters) of your seat above the ground *t* seconds after the ride starts. Begin by drawing a sketch of the function and then determining the amplitude, period, vertical shift and phase shift.

b) What will the height of your seat be 30 seconds after the ride starts? Round to the nearest hundredth of a meter. What is the height of your seat before the ride starts? Round your answers to the nearest hundredth of a meter.

8. For several hundred years, astronomers have kept track of the number of solar flares, or "sunspots," which occur on the surface of the sun. The number of sunspots counted in a given year varies periodically from a minimum of about 10 per years to a maximum of about 110 per year. Between the maximums that occur in the years 1750 and 1948, there were 18 complete cycles.

a) What is the period of the sunspot cycle?

b) Assume that the number of sunspots counted in a year varies sinusoidally with the year. Sketch a graph of two cycles, starting in 1948



c) Write an equation expressing the number of sunspots per year in terms of the year

d) How many sunspots would you expect in the year 2000?

e) What is the first year after 2000 in which the number of sunspots will be about 35?

f) What is the first year after 2000 in which the number of sunspots will be a maximum?